7-1, Lecture 7: Frequency Response and Resonance

Wednesday, July 1, 2020 10:59 AM

Last time on EE16B…..

(Post lecture notes in purple)
(Impt equations boxed in green)

Transfer function will represent the magnitude and phase alteration, which are fully known given all values of passive devices (R, L, C) and the frequency w

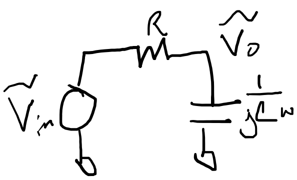
$$H(j\omega) = |H(j\omega)| e^{j \angle H(j\omega)}$$

Frequency Response

examine $H(j\omega)$ over freq $\omega = 0 \to \infty$

Filters

Low Pass Filters

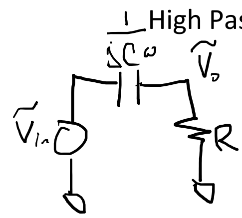


$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$H(j0) \to \frac{1}{1} = 1$$

$$H(j\infty) \to \frac{1}{j\infty} = -j0$$

High Pass Filters



$$H(j\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

$$H(j0) \to \frac{j0}{1} = j0$$

$$H(j\infty) \to \frac{j\infty}{j\infty} = 1$$

Impedance over Frequency

$\underline{\omega = 0}$

$|Z_R| = R$

$|Z_C| = \infty \leftarrow$ open

$\underline{\omega = \infty}$

$|Z_R| = R$

$|Z_C| = 0 \leftarrow$ short

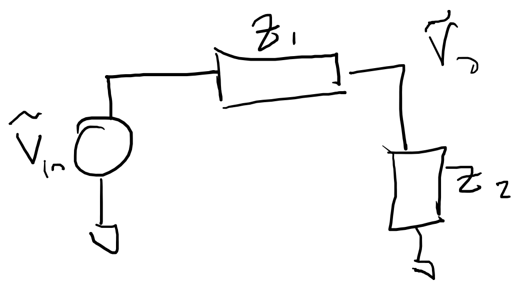
$|Z_C| = \infty \leftarrow$ open

$|Z_L| = 0 \leftarrow$ short

$|Z_C| = 0 \leftarrow$ short

$|Z_L| = \infty \leftarrow$ open

Predict filter type given a voltage divider shape

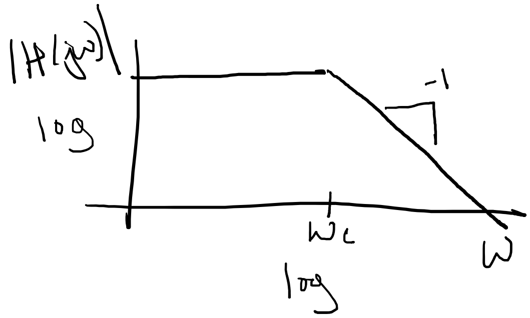


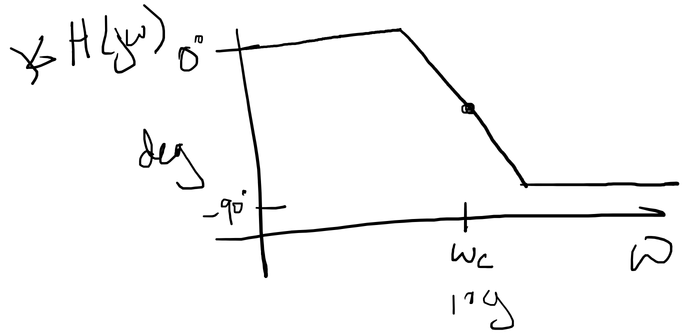
$$\frac{\tilde{V}_{Z1}}{\tilde{V}_{Z2}} = \frac{Z_1}{Z_2}$$

$(\tilde{V}_o = \tilde{V}_{Z2})$

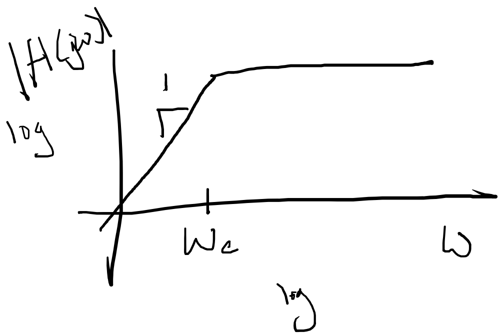
Plot Frequency Response with Bode Plots

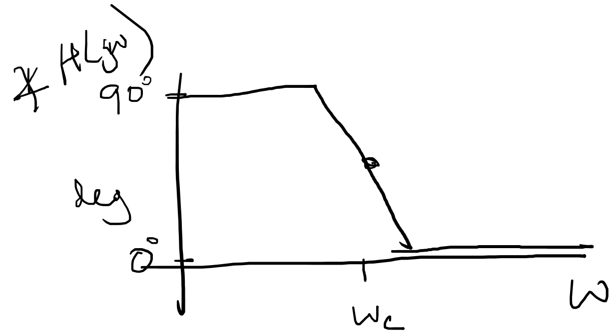
LowPass





HighPass





General Factored H(jw)

$$\frac{(j\omega)^n\,(1+j\omega/\omega_{z1})(1+j\omega/\omega_{z2})\cdots}{(j\omega)^m\,(1+j\omega/\omega_{p1})(1+j\omega/\omega_{p2})\cdots}$$

← zeros

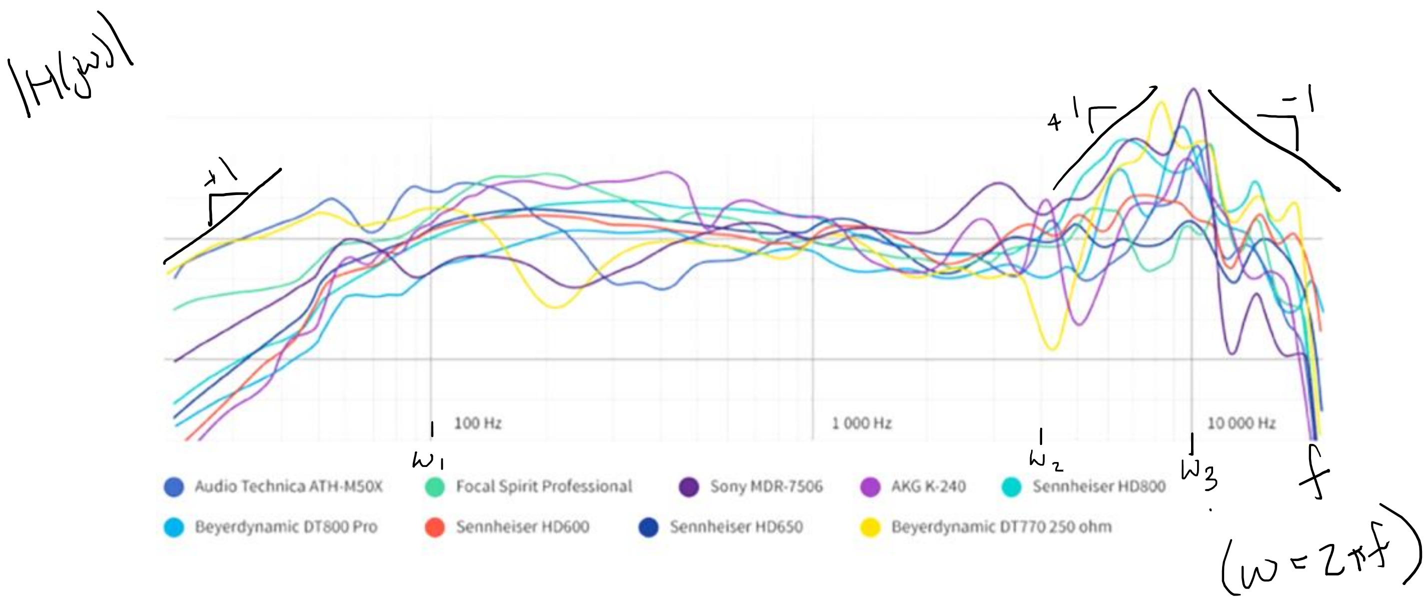
← poles

or use python to plot

Today:

0. Application Tangent: Headphone Frequency Response
I. Second Order Filters:
   a. Filtering Interference with a First Order Filter
   b. Filtering with a Second Order Filter
   c. Second Order Bandpass Filter
II. Resonance
   a. Complex Impedance Practice
   b. Resonant Frequency
   c. Resonant Bandpass Filter
III. Application Tangent: Radio Station Tuning

0. Application Tangent: Headphone Frequency Response



$$H(j\omega) = \frac{(j\omega/\omega_1)(1+j\omega/\omega_2)}{\cdots}$$

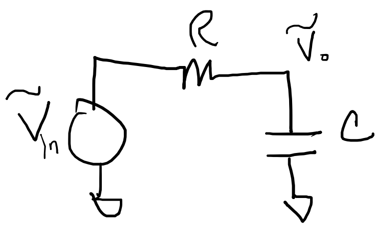
← zeros

$$H(j\omega) = \frac{(j\omega/\omega_1)(1 + j\omega/\omega_2)}{(1 + j\omega/\omega_1)(1 + j\omega/\omega_3)(1 + j\omega/\omega_3)} \begin{array}{l} \leftarrow \text{zeros} \\ \leftarrow \text{poles} \end{array}$$

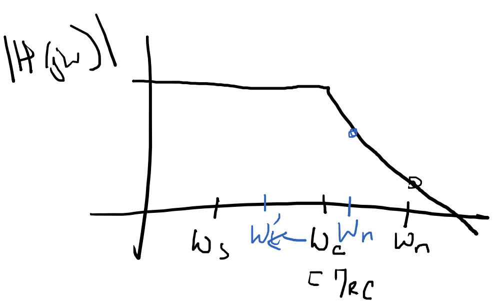
I. Second Order Filters

a. Filtering an Interferer with a First Order Filter

$$V_{in} = \tilde{V}_{in}\Big(\overbrace{e^{j\omega_s t}}^{\text{signal}} + \overbrace{e^{j\omega_n t}}^{\text{noise}}\Big) \qquad \omega_s < \omega_n$$



$$H(j\omega) = \frac{1}{1 + j\omega RC} \leftarrow \begin{array}{l} \text{single pole} \\ W_p = 1/RC \end{array}$$

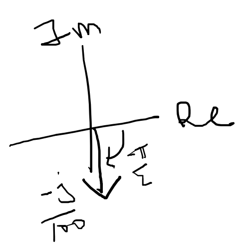


$$\omega_n = 100\,\omega_c$$

$$H(j\omega_n) = \frac{1}{1 + j\omega_n/\omega_c} = \frac{1}{1 + j\,100\omega_c/\omega_c} = \frac{1}{1 + j100}$$

$$\cong \frac{1}{j100} = \frac{-j}{100}$$

$$|H(j\omega_n)| \cong \frac{1}{100} \qquad \measuredangle H(j\omega_n) \cong \frac{-\pi}{2}$$



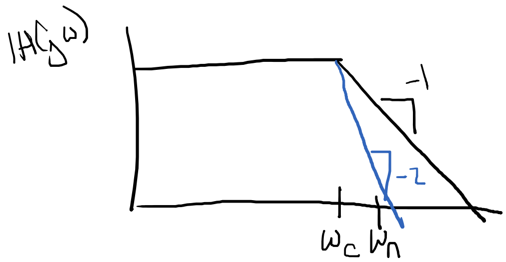
What if noise frequency is lower (closer to our desired signal frequency)

$$\omega_n = 10\,\omega_c$$

$$|H(j\omega)| \cong \frac{1}{10} \qquad \measuredangle H(j\omega) \cong \frac{-\pi}{2}$$

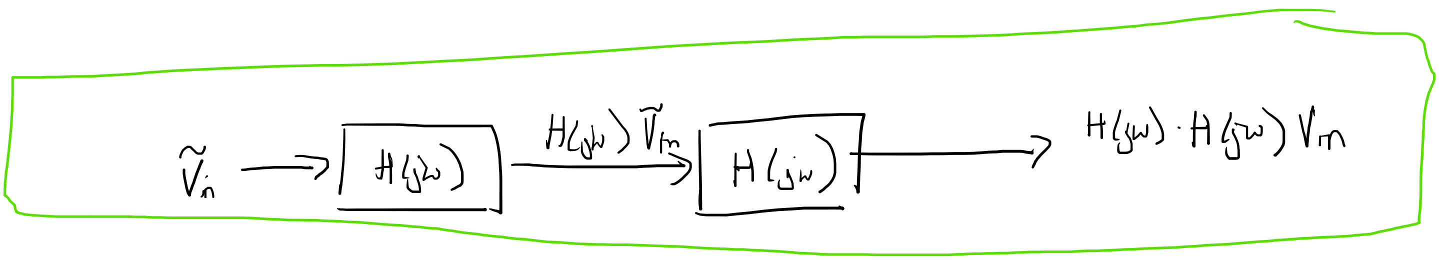
$$|H(j\omega)| \cong \frac{1}{10} \qquad \measuredangle H(j\omega) \cong \frac{-\pi}{2}$$

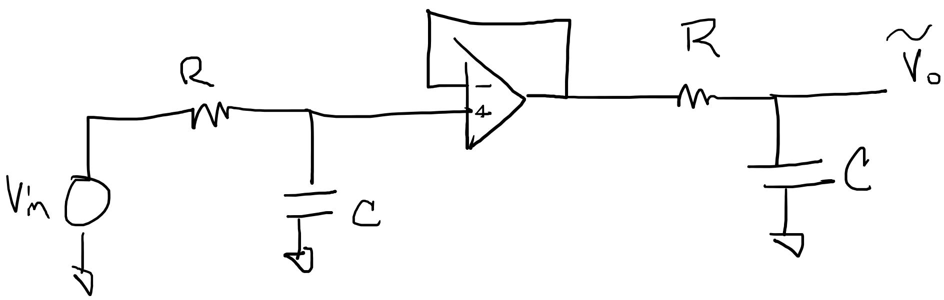
b. Second Order Filter



Design a filter with two poles at w_c

$$H(j\omega) = \frac{1}{(1+j\omega/\omega_c)(1+j\omega/\omega_c)}$$





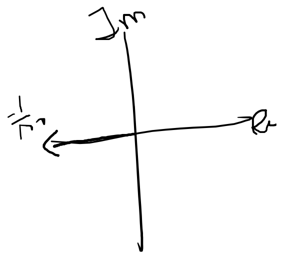
$$H_{so}(j\omega) = \frac{1}{1+j\omega/\omega_c} \cdot \frac{1}{1+j\omega/\omega_c}$$

$$\omega_n = 10\omega_c$$

$$H_{so}(j\omega) = \frac{1}{1+j\,10\omega_c/\omega_c} \cdot \frac{1}{1+j\,10\omega_c/\omega_c} = \frac{1}{1+j10} \cdot \frac{1}{1+j10}$$

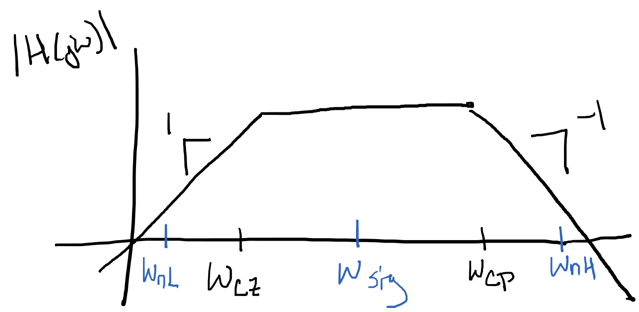
$$\cong \frac{1}{j10} \cdot \frac{1}{j10} = \frac{-j}{10} \cdot \frac{-j}{10} = \frac{-1}{100}$$

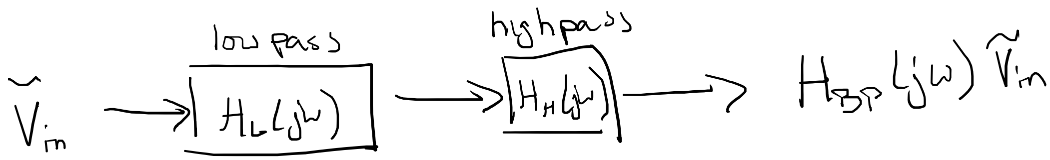
$$\cong \frac{1}{j10}\cdot\frac{1}{j10} = \frac{-j}{10}\cdot\frac{-j}{10} = \frac{-1}{100}$$

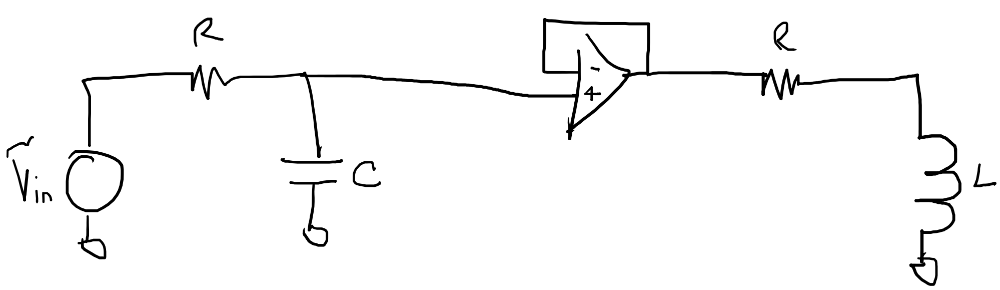


$$|H_{s0}(jw)| \cong \frac{1}{100} \qquad \measuredangle H_{s0}(jw) \cong \pi$$

C. Second Order Bandpass Filter





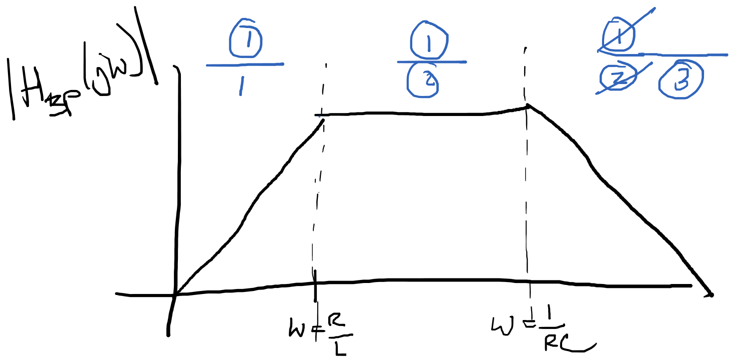


$$H_L(jw) = \frac{\frac{1}{jwC}}{R + \frac{1}{jwC}} \qquad H_H(jw) = \frac{jwL}{R + jwL}\ \frac{\frac{1}{R}}{\frac{1}{R}}$$

$$H_{BP}(jw) = H_L(jw)\cdot H_H(jw) = \frac{1}{(1 + jwRC)}\cdot\frac{jw\frac{L}{R}}{(1 + jw\frac{L}{R})}$$

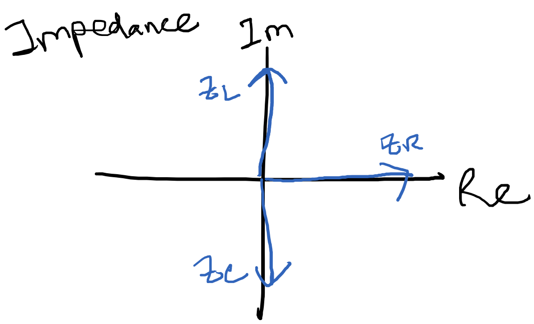
$$H_{BP}(j\omega) = H_L(j\omega)\cdot H_H(j\omega) = \frac{}{(1+j\omega RC)} \cdot \frac{}{(1+j\omega\frac{L}{R})}$$

$$= \frac{j\omega\frac{L}{R} \; (1)}{(3)\,(1+j\omega RC)(1+j\omega\frac{L}{R})\,(2)}$$



II. Resonance

a. Complex Impedance Practice



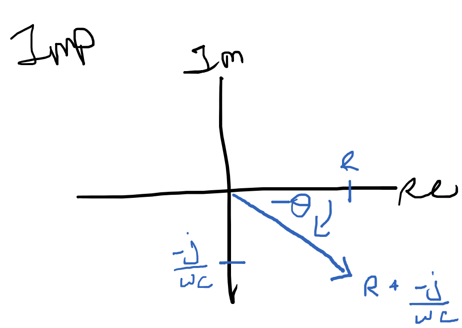
$$Z_R = R$$

$$Z_C = \frac{1}{j\omega C}$$

$$Z_L = j\omega L$$

Combinations

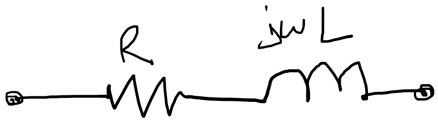
Z_{RC}

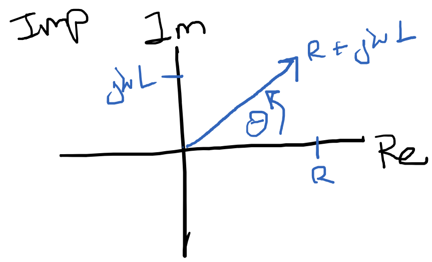


$$\frac{1}{j} = -j$$

$$Z_{RC} = R + \frac{1}{j\omega C}$$

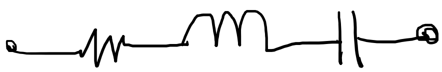
Z_{RL}

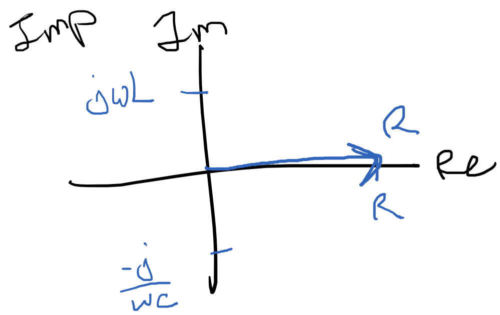




$$Z_{RL} = R + j\omega L$$

Z_{RLC}





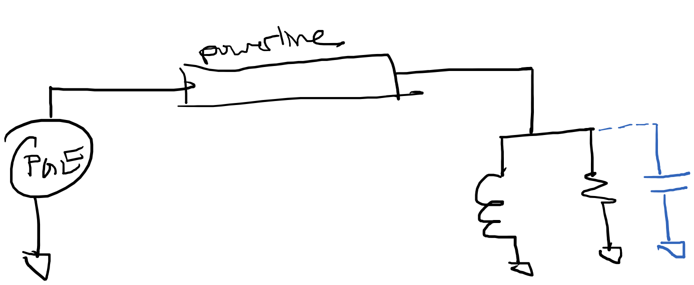
$$Z_{RLC} = R + j\omega L + \frac{-j}{\omega C}$$
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Could choose L and C s.t. for some w_r impedance is purely real

b. Resonant frequency

$$\omega_R \quad \text{where} \quad \omega_r L - \frac{1}{\omega_r C} = 0$$

Return to Power Grid



$$\omega_{\text{wall power}} = 60 \cdot 2\pi \ (\text{rad/sec})$$

$$L = 1H$$

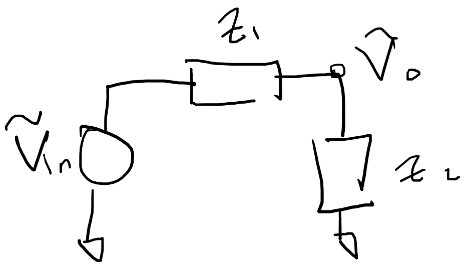
$$\omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r^2 = \frac{1}{\ }$$

$$\boxed{\omega_r^2 = \frac{1}{LC}}$$

$$C = \frac{1}{(60 \cdot 2\pi)^2 \, 1H}$$

c. Resonant bandpass filter

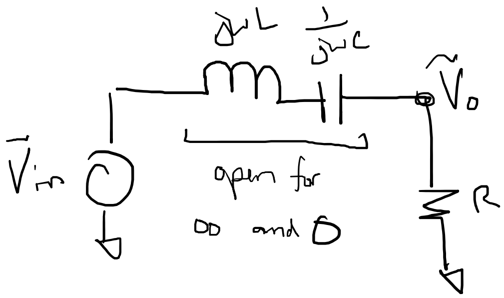


$$H(j\omega) = \frac{\tilde{V}_o}{\tilde{V}_{in}} = \frac{Z_2}{Z_2 + Z_1}$$

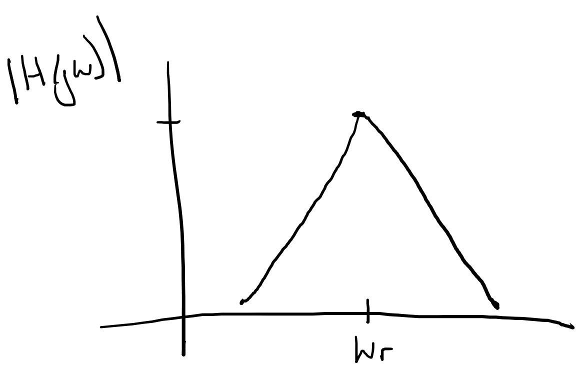
$\omega \neq \omega_r$
$H(j\omega) = 0$

$\omega = \omega_r$
$H(j\omega) = 1$

ω_r

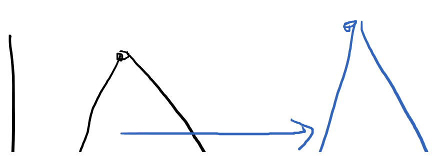


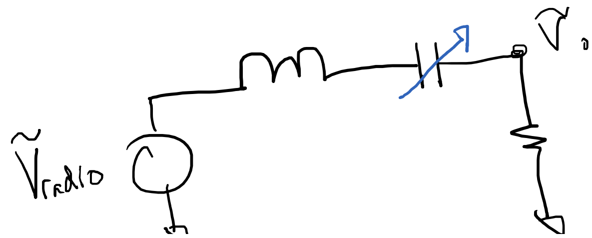
$$H(j\omega) = \frac{R}{R + \underbrace{\left(j\omega L + \frac{1}{j\omega C}\right)}_{\text{dominates @ } \infty \text{ \& } 0}}$$



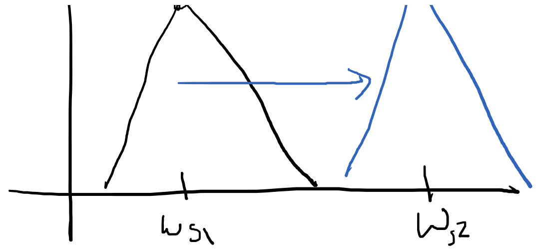
d. Application Tangent: Radio Stations

Radio
Reciever





Radio
Reciever
Filter



$$\omega_r^2 = \frac{1}{LC_{\text{variable}}}$$