8-11, Lecture 30: Interpolation

Tuesday, August 11, 2020 10:15 AM

I. Imperfect Interpolation
   a. ~~Polynomial~~
   b. Zero Order Hold
   c. Linear

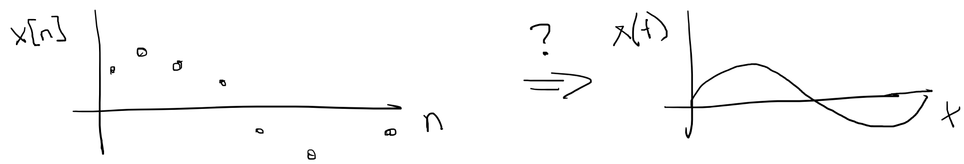
II. Frequency Analysis of Interpolation
   a. Useful Fourier Properties
   b. Zero Order Hold in Frequency Domain
   c. Perfect Reconstruction

III. Anti-Aliasing Filter

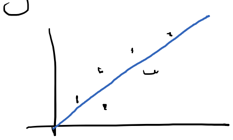
(Post lecture notes in purple)
(Impt equations boxed in green)

I. Imperfect Interpolation

Sampled Signal

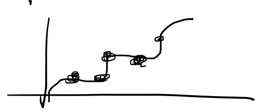


Regression



- Least Squares
- Good for noisy data
  Ignore exact points

Interpolation

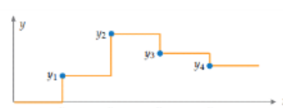


- Seen today!
- Trust our data is accurate

a. Polynomial Interpolation

$$y = mx + b$$

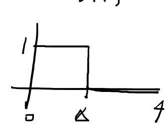
b. Zero-Order Hold Interpolation

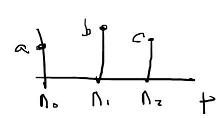


- Hold previous value until the next sample arrives

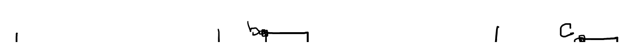
Represent with :

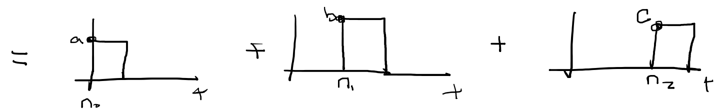
$u(t)$ $\quad * \quad$ $f(n)$



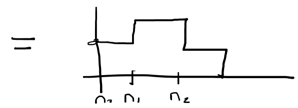


$\Delta = \frac{1}{f_s}$





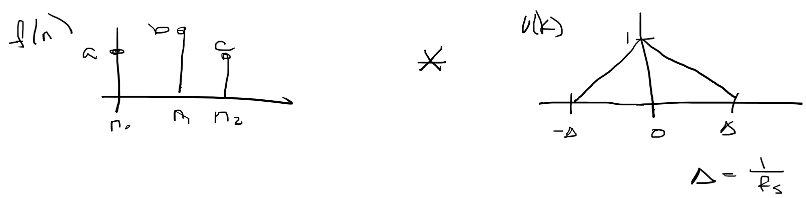
- Apply the scale and delay of each sample to the function u(t) and sum

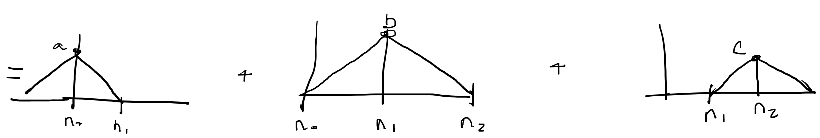


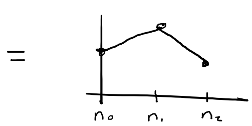
c. Linear Interpolation

- Draw a line b/w each point

Convolve our signal







II. Frequency Anaylsis of Interpolation

a. Useful Properties

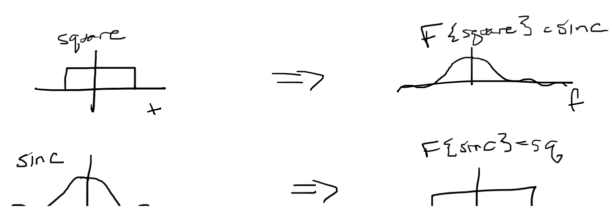
- Convolution in time domain is multiplication in freq domain (element wise multiplication in discrete)

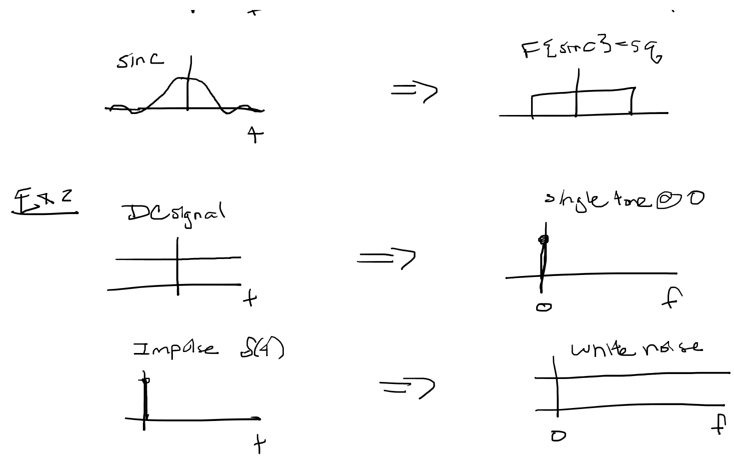
$$f * h \overset{F}{\Longrightarrow} F\{f\} \cdot F\{h\}$$

- Sampled signals have aliasing, but continuous time signals do not

- Duality of a function's transform in time of freq

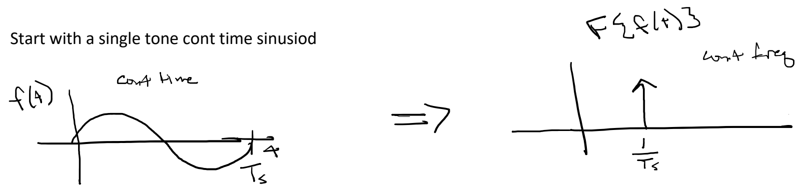
Ex)



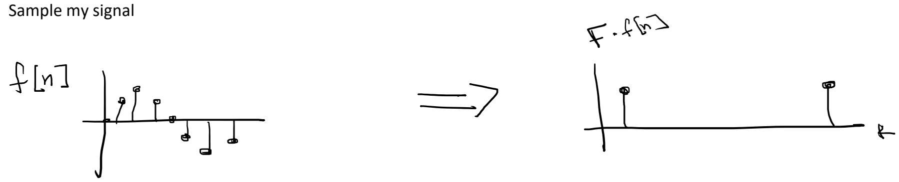


b. ZOH in Freq Domain

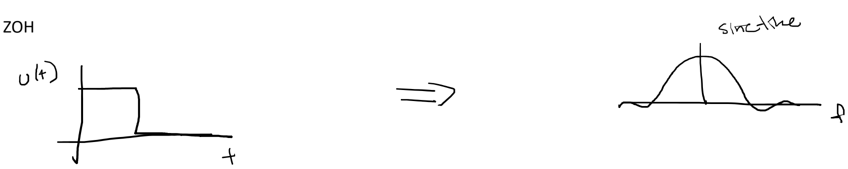
Start with a single tone cont time sinusiod



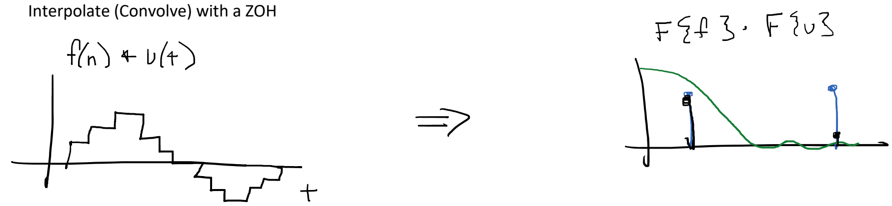
Sample my signal



ZOH



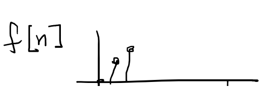
Interpolate (Convolve) with a ZOH

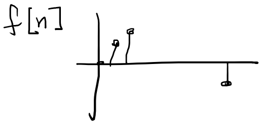


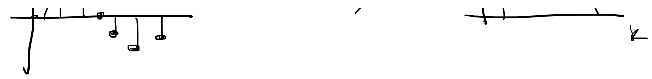
Gotten pretty close to our original
Signal in frequency, just with that extra
From aliasing when we sampled

c. Perfect Reconstruction

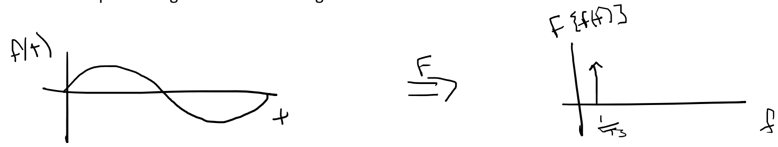
Have same sampled function with aliasing in the freq domain



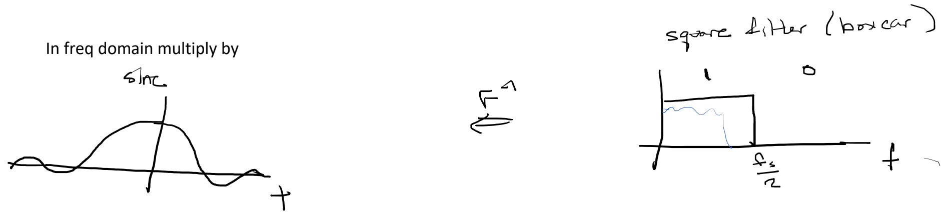




Want an interpolated signal with the aliasing removed

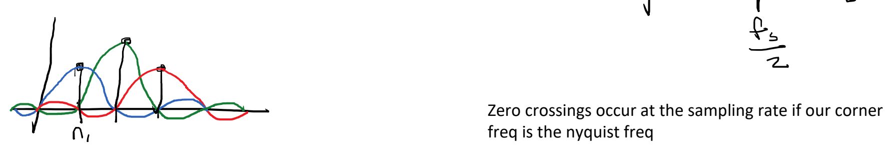


In freq domain multiply by



Interpolate

$f(n) * \mathrm{sinc}(t)$

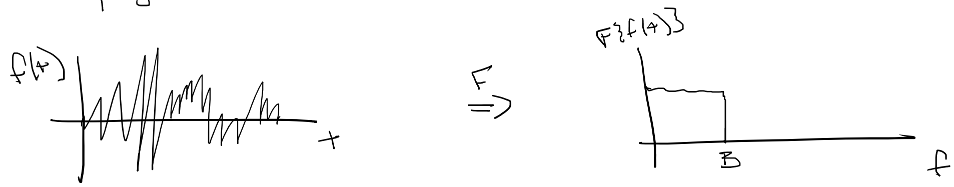


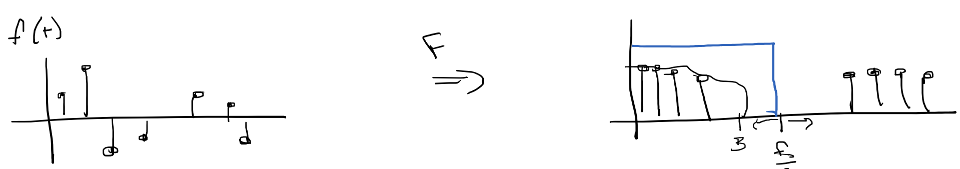
Zero crossings occur at the sampling rate if our corner freq is the nyquist freq

Nyquist Sampling Theorem:
A bandlimited, cont time signal can be sampled and perfectly represented/ **perfectly reconstructed** from its samples, IF the sampling freq (f_s) is over twice as fast as the signals highest freq component (B)

$$\boxed{f_s > 2B}$$

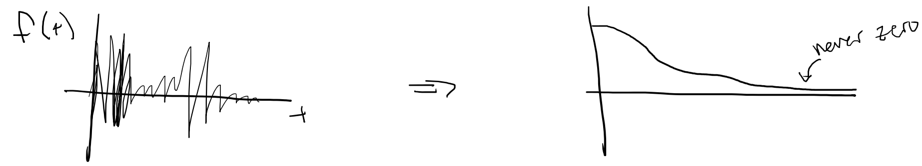
Any signal f(t)



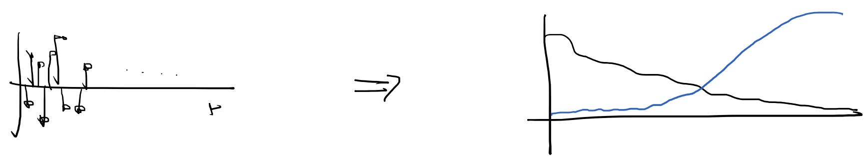


III. Anti-Aliasing Filter

What if our signal isn't bandlimited?

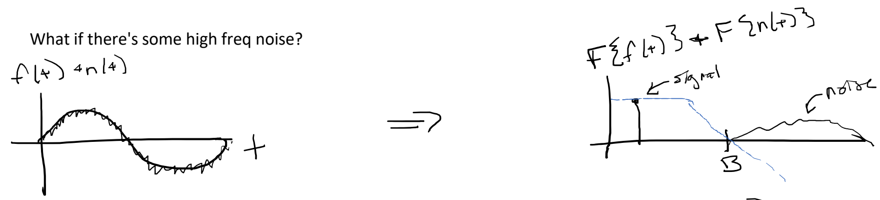


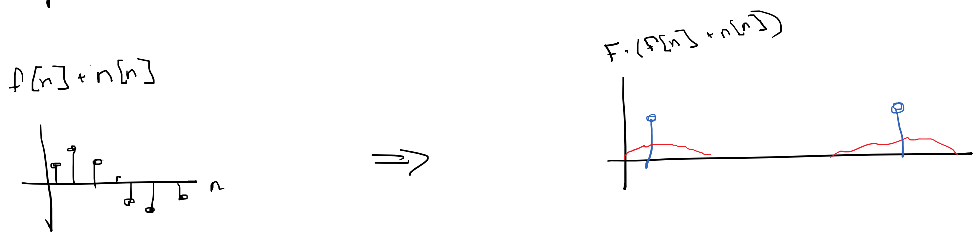
When we sample, we can't possibly choose a sampling freq that's high enough!



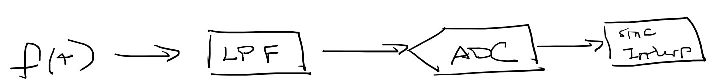
Perfect reconstruction REQUIRES signal be bandlimited

What if there's some high freq noise?





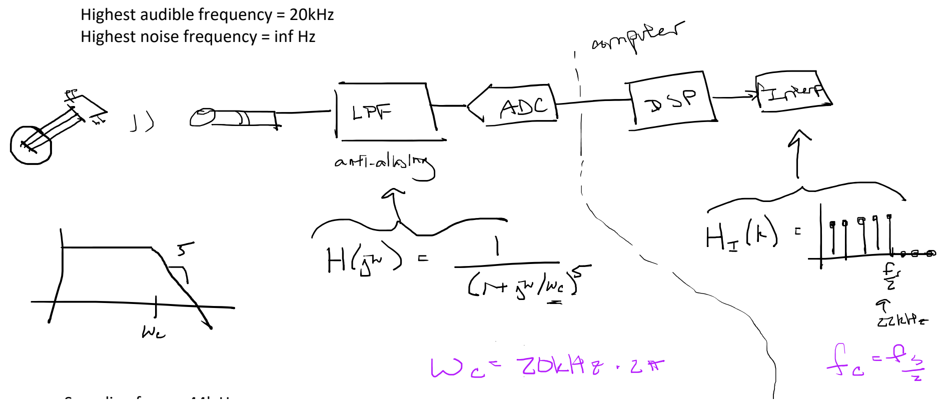
Filter before sampling to prevent noise from aliasing into our signal band

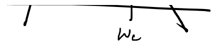


Example

Recording studio, want as good of a recording as they can get

Highest audible frequency = 20kHz
Highest noise frequency = inf Hz





Sampling freq -> 44k Hz
Nyquist Freq -> 22k Hz

$\omega_c = 20kHz \cdot 2\pi$

22kHz

$f_c = \frac{f_s}{2}$

$= 22kHz$