6-24, Lecture 3: Differential Eqns and RC Transients

Wednesday, June 24, 2020 10:45 AM

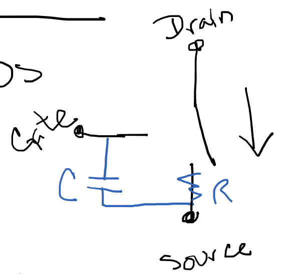
(post-lecture edits in purple)

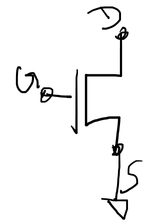
(useful eqns in green boxes)

Last time on EE16B...

Transistors

NMOS



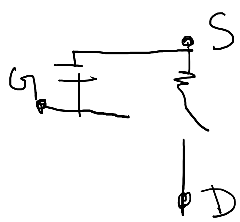


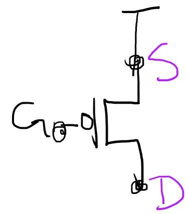
Swt closed when

$V_G - \cancel{V_S} \geq V_{TH}$, $V_S \to 0$

$\boxed{V_G \geq V_{TH}}$

PMOS



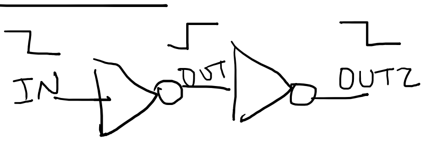


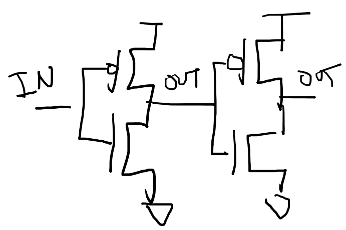
$\cancel{V_S} - V_G \geq V_{TH} = \frac{VDD}{2}$, $V_S \to VDD$

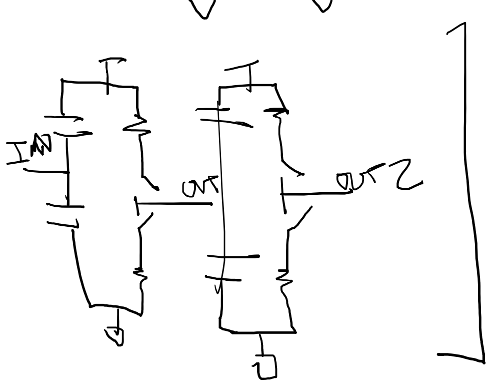
$\frac{VDD}{2} \geq V_G$

$\boxed{V_G \leq V_{TH}}$

Inverter







Energy / Transition

$\boxed{E = \frac{1}{2} C V_{DD}^2}$

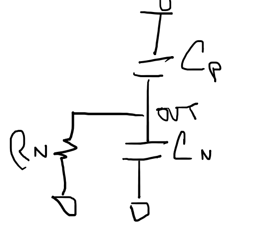
If IN: VDD → 0 @ t_0

Then PMOS open → closed

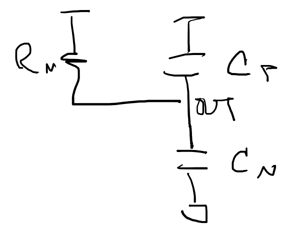
NMOS closed → open

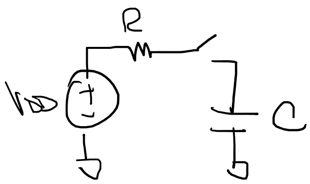
OUT $0 \rightarrow VDD$ @ $t_0 + \Delta$

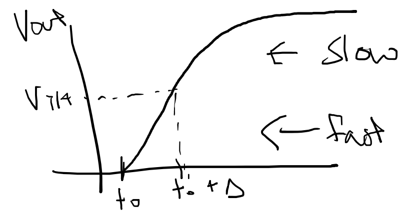
Discharge C_N



Charging C_N







Delay

Motivation: Why care about delay?

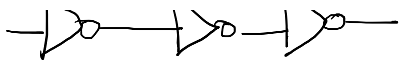
Today

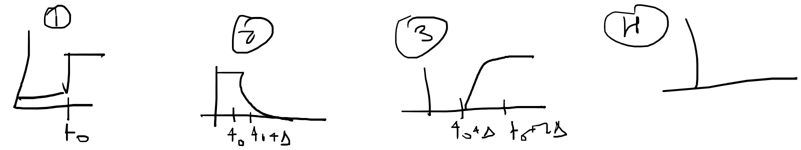
Differential Eqns

- Homogeneous Eqns
- Uniqueness & Existence Thm
- Soln to $V_c(t)$

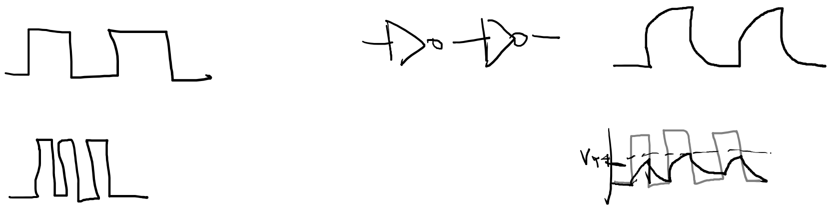
Transients

- Time Constants



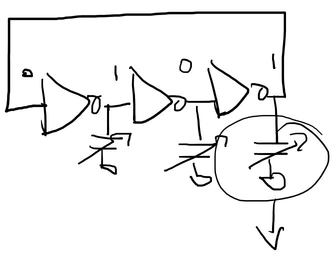


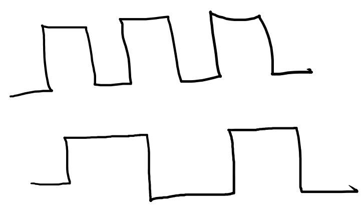
Want a fast CPU



Application Tangent

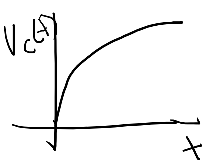
Good engineers make lemonade





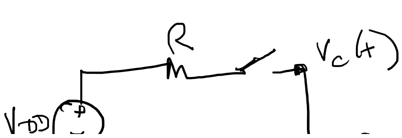
sensor – touch screen

– irrigation systems



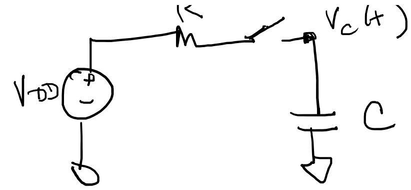
← want to describe accurately

Goal I. Math model of RC circuit



$$I_R = I_C$$

$$C\frac{d}{dt}V(t) = V_R(t)/R$$



$$I_R = I_C$$

$$C\frac{d}{dt}V_c(t) = V_R(t)/R$$

$$\frac{d}{dt}V_c(t) = (V_{DD} - V_c(t))/RC$$

Diff Eqn $\longrightarrow$

$$\frac{d}{dt}V_c(t) + \frac{1}{RC}V_c(t) = \frac{V_{DD}}{RC}$$

* Math Break *

Subgoal A: Solve for $V_c(t)$

Need: Diff Eqn

Diff Eq Warm Up

$$\frac{d}{dt}x(t) = b$$

$$\int \frac{d}{dt}x(t)\,dt = \int b\,dt$$

$$x(t) = bt + k_c$$

k_c = some const set by $x(0) = x_0$

General version

$$\frac{d}{dt}x(t) + ax(t) = b \qquad \left(\frac{d}{dt}V_c(t) + \frac{1}{RC}V_c(t) = \frac{V_{DD}}{RC}\right)$$

Homogenous part

$$\frac{d}{dt}x(t) + ax(t) = 0$$

Homogenous Eqn = diff eqn where $x(t) = 0$ is valid

Thm :

$$x(t) = x_G(t) + x_p(t)$$

general sol'n homogenous

particular sol'n to partic

subsubgoal i: Get an idea of $x_G(t)$ to check $x(t)$

$$\frac{d}{dt} x(t) + a\,x(t) = 0$$

$$\frac{d}{dt} x(t) = -a\,x(t)$$

[side note:

$\frac{d}{dt} f(x) = \lambda f(x)$ eigenfunction!]

Table of Basic Integrals[1]

(1) $\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$

(2) $\int \frac{1}{x}\,dx = \ln|x|$

(3) $\int u\,dv = uv - \int v\,du$

(4) $\int e^x\,dx = e^x$

(5) $\int a^x\,dx = \frac{1}{\ln a}a^x$

(6) $\int \ln x\,dx = x\ln x - x$

(7) $\int \sin x\,dx = -\cos x$

(8) $\int \cos x\,dx = \sin x$

(9) $\int \tan x\,dx = \ln|\sec x|$

(10) $\int \sec x\,dx = \ln|\sec x + \tan x|$

(11) $\int \sec^2 x\,dx = \tan x$

(12) $\int \sec x \tan x\,dx = \sec x$

(13) $\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\frac{x}{a}$

(14) $\int \frac{a}{a^2-x^2}\,dx = \frac{1}{2}\ln\left|\frac{x+a}{x-a}\right|$

(15) $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}$

(16) $\int \frac{a}{x\sqrt{x^2-a^2}}\,dx = \sec^{-1}\frac{x}{a}$

(17) $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \cosh^{-1}\frac{x}{a} = \ln(x+\sqrt{x^2-a^2})$

(18) $\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\frac{x}{a} = \ln(x+\sqrt{x^2+a^2})$

Sol'n homogenous part

$$x(t) = C_0 e^{-at} \quad , \; x(0) = x_0$$

$$x(t) = x_0 e^{-at}$$

$$\frac{d}{dt} x(t) = a x_0 e^{-at}$$

Uniqueness & Existence Thm

if a function solves a diff eqn then it is <u>correct</u> & <u>unique</u>, as long as

(1) initial condition $x(t_0) = x_0$

(2) the sol'n of $x(t)$ is cont over the values we care about, & contains that t_0

subsubsubgoal a.

Prove uniqueness thm → HW, Note 1 Fa19, pg 11

Have an idea of $x(t)$ what it should look like

$$x(t) = \underbrace{x_g(t)}_{x_0 e^{-at}} + x_p(t)$$

Solve $\frac{d}{dt}x(t) + ax(t) = b \qquad x(0) = x_0$

$$\frac{d}{dt}x(t) = b - ax(t)$$

$$\int \frac{\frac{d}{dt}x(t)}{x(t) - b/a} = \int -a$$

$$\ln\left|x(t) - b/a\right| = -at + C \quad , \; C = \text{some const}$$

$$x(t) = b/a + C' e^{-at} \quad , \; C' = \text{some const}$$

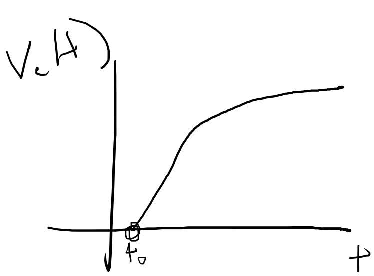
Plug in $x(0) = x_0$ to check

$$\boxed{x(t) = b/a + (x_0 - b/a)e^{-at}}$$

Notes Fa, pg 13

* Math Break Over! *

Solve for $V_c(t)$



Math Form

$$\frac{d}{dt}x(t) + a x(t) = b$$

$$x(t) = b/a + (x_0 - b/a)e^{-at}$$

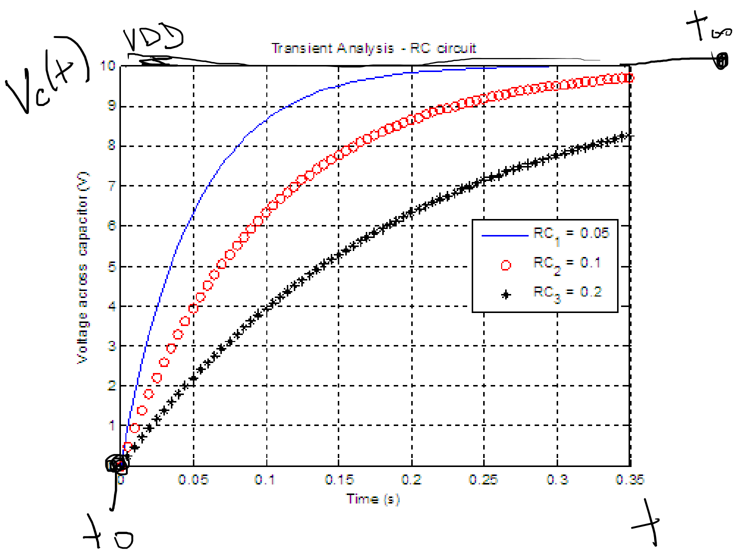
RC Form

$$\frac{d}{dt}V_c(t) + \frac{1}{RC}V_c(t) = \frac{VDD}{RC}, \quad x(0) = x_0$$

$$V_c(t) = \frac{V_{DD}}{RC}\Big/\frac{1}{RC} + \left[0 - \frac{VDD}{RC}\Big/\frac{1}{RC}\right]e^{-t/RC}$$

$$\boxed{V_c(t) = V_{DD} - V_{DD}e^{-t/RC}}$$

charging RC mathematical model



1. charges fast then slow

2. $x(t_0) = 0$

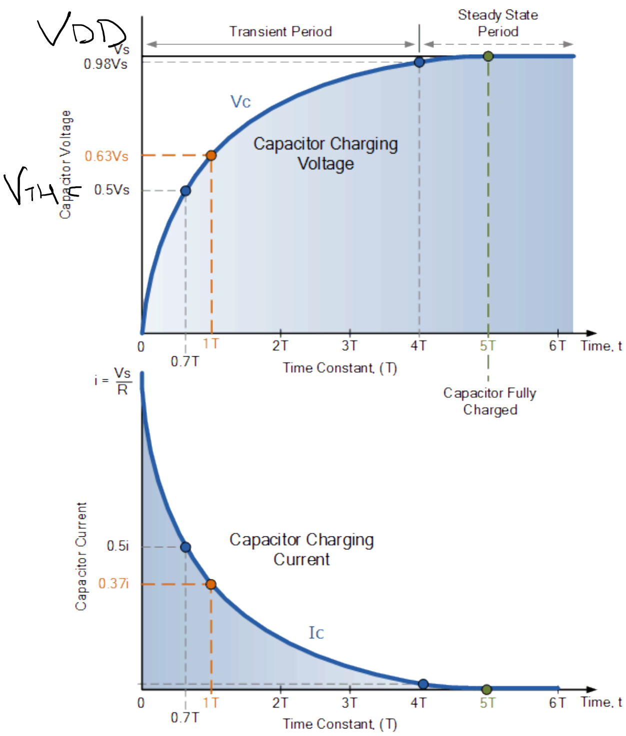
$$V_c(0) = V_{DD} - V_{DD}e^{-0/RC} = VDD - VDD$$

3. $x(t_\infty) = VDD$

$$V_c(\infty) = V_{DD} - V_{DD}e^{-\infty/RC} = VDD - 0$$

Goal II. Explore Math Model

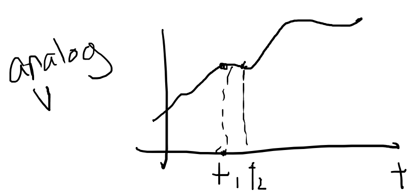
$$x(t) = A - Be^{-t/\tau} \qquad \boxed{\tau = RC}$$

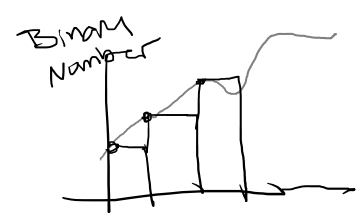


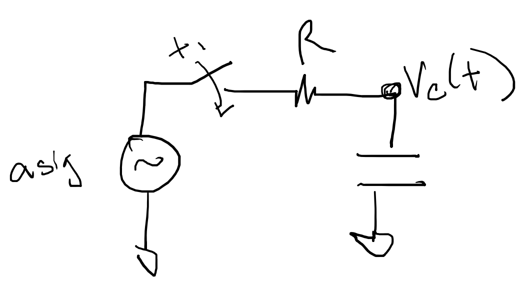
Time Constant	RC Value	Percentage of Maximum
		Voltage
0.5 time constant	0.5T = 0.5RC	39.3%
0.7 time constant	0.7T = 0.7RC	50.3%
1.0 time constant	1T = 1RC	63.2%
2.0 time constants	2T = 2RC	86.5%
3.0 time constants	3T = 3RC	95.0%
4.0 time constants	4T = 4RC	98.2%
5.0 time constants	5T = 5RC	99.3%

Meta Eng Concept: When is good enough?

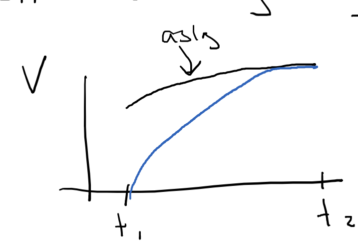
Application Tangent







Zoom in on asig

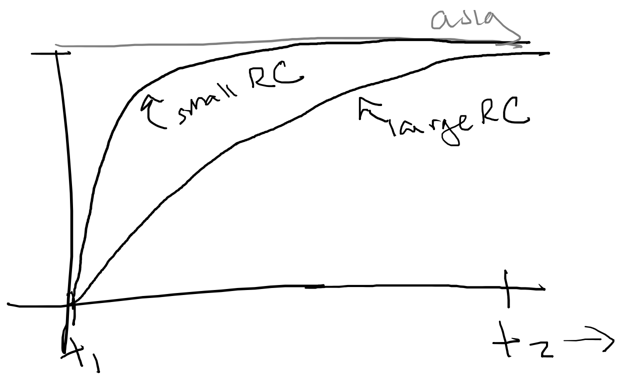


t_1 - swt closes
ADC waits

t_2 - swt opens
ADC guesses

If need a signal...

$\tau \approx 2$ or 3

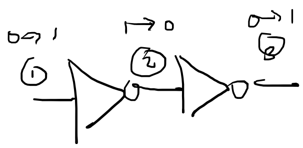


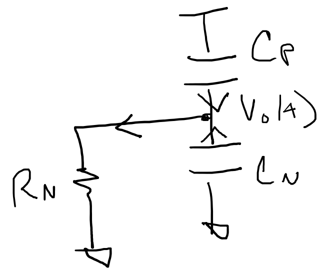
fast, not accurate : $\tau \simeq 2$ or 3

accurate, not fast : $\tau \simeq 5$ or larger

fast & accurate: RC smaller

Discharge an Inverter Output

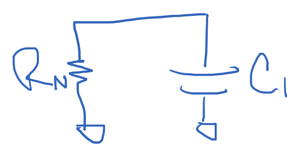




Impt Eqns

$$i_R(t) = \frac{V_R(t)}{R}$$

$$C\frac{d}{dt}V_C(t) = i_C(t)$$



(KCL)

$$i_{C_P} + i_{C_N} - i_{R_N} = 0$$

$$C_P\frac{d}{dt}\left(\cancel{V_{DD}}^{\,0} - V_o(t)\right) + C_N\frac{d}{dt}\left(0 - V_o(t)\right) - \frac{(V_o(t) - 0)}{R} = 0$$

$$-C_P\frac{d}{dt}V_o(t) + -C_N\frac{d}{dt}V_o(t) - \frac{V_o(t)}{R} = 0$$

$$\frac{d}{dt}V_o(t)\underbrace{(C_N + C_P)}_{C_{11}} = \frac{-V_o(t)}{R}$$ ← homogenous!

$$V_o(t) = K e^{-t/R(C_N + C_P)}, \quad V_o(0) = V_{DD}$$

$$\boxed{V_o(t) = V_{DD} e^{-t/R(C_N + C_P)}}$$

Tomorrow: Time variant inputs through an RC

Scroll Move