This homework is designed as practice for the final and is optional. You will not have to submit anything to Gradescope.

1 Midterm Redos

a) Redo MT1: `http://www.eecs16b.org/exam/mt1.pdf`

b) Redo MT2: `http://www.eecs16b.org/exam/mt2.pdf`

2 Stability of State Space Systems (X points)

Consider a discrete time state space system

$$\vec{x}[n+1] = \mathbf{A}\vec{x}[n].$$

For which of the following possible matrices $\mathbf{A}$ is the system stable? Explain your answers.

a)
$$\mathbf{A} = \frac{1}{4}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$
Stable? Yes / No
Explanation:

b)
$$\mathbf{A} = \frac{1}{2}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$
Stable? Yes / No
Explanation:

For parts (c) and (d), consider a continuous time system

$$\frac{\mathrm{d}\vec{x}(t)}{\mathrm{d}t} = \mathbf{A}\vec{x}(t).$$

c)
$$\mathbf{A} = \begin{bmatrix} -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$$
Stable? Yes / No
Explanation:

d) Recall that we are still considering the continuous time system.

$$\mathbf{A} = \begin{bmatrix} -2 & 1 & 0 & -1 \\ -1 & -2 & 1 & 0 \\ 0 & -1 & -2 & 1 \\ 1 & 0 & -1 & -2 \end{bmatrix}$$
Stable? Yes / No
Explanation:

3 Sampling Theorem

Consider the following signal, $x(t)$ defined as,

$$x(t) = \cos(2\pi t) + \sin(4\pi t)$$

a) Find the maximum frequency, ω_{max}, in radians per second? In Hertz? (From now on, frequencies will refer to radians per second.)

b) If I sample every T seconds, what is the sampling frequency?

c) What is the smallest sampling period T that would result in an imperfect reconstruction?

4 Aliasing

Watch the following video: `https://www.youtube.com/watch?v=jQDjJRYmeWg`.

Assume the video camera running at 30 frames per second. That is to say, the camera takes 30 photos within a second, with the time between photos being constant.

a) Given that the main rotor has 5 blades, list *all* the possible rates at which the main rotor is spinning in revolutions per second assuming no physical limitations.
Hint: Your answer should depend on k where k can be any integer.

b) Given that the back rotor has 3 blades and completes 2 revolutions in 1 second **in the video**, list *all* the possible rates at which the back rotor is spinning in revolutions per second assuming no physical limitations.
Hint: Your answer should depend on k where k can be any integer.