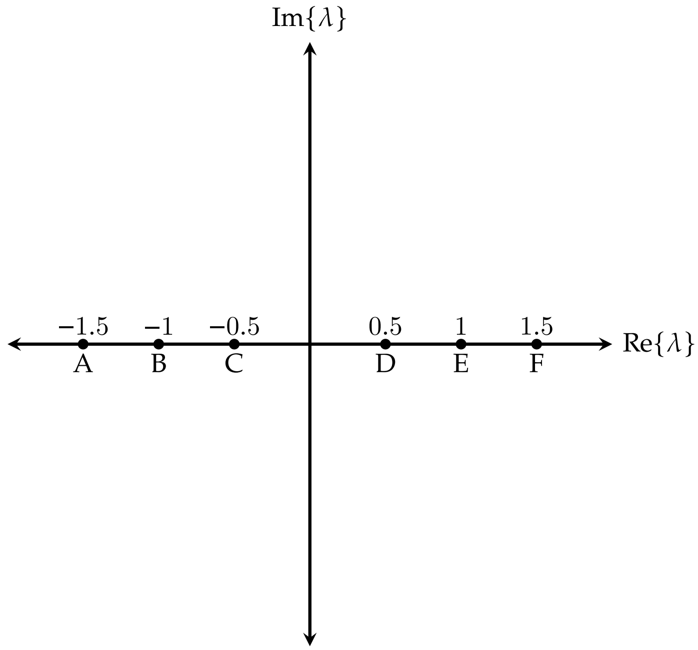
1 Discrete time system responses

We have a system $x[k+1] = \lambda x[k]$. For each λ value plotted on the real-imaginary axis, sketch $x[k]$ with an initial condition of $x[0] = 1$. Determine if each system is stable.



2 Eigenvalues Placement in Discrete Time

Consider the following linear discrete time system

$$\vec{x}[t+1] = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \vec{x}[t] + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u[t] + \vec{w}[t] \tag{1}$$

a) Is this system controllable?

b) Is the linear discrete time system stable?

c) Derive a state space representation of the resulting closed loop system using state feedback of the form $u[t] = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \vec{x}[t]$

d) Find the appropriate state feedback constants, k_1, k_2 in order the state space representation of the resulting closed loop system to place the eigenvalues at $\lambda_1 = -\frac{1}{2}, \lambda_2 = \frac{1}{2}$

e) Suppose that instead of $\begin{bmatrix} 1 \\ 0 \end{bmatrix} u[t]$ in (1), we had $\begin{bmatrix} 1 \\ 1 \end{bmatrix} u[t]$ as the way that the discrete-time control acted on the system. Is this system controllable from $u[t]$?

f) For the part above, suppose we used $[k_1, k_2]$ to try and control the system. What would the eigenvalues be? Can you move all the eigenvalues to where you want? Give an intuitive explanation of what is going on.

3 Eigenvalue Placement

Consider the following linear discrete time system

$$\vec{x}(t+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -9 & -6 \end{bmatrix} \vec{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

a) **What are the eigenvalues of the A matrix? Is this system stable?**

b) **Using state feedback $u(t) = K\vec{x}(t) = \begin{bmatrix} k_0 & k_1 & k_2 \end{bmatrix} \vec{x}(t)$ place the eigenvalues at $0, 1/2, -1/2$.**

c) **Suppose we now have a limitation on how much our controller can amplify $\vec{x}$ and all of our k values must be in between -5 and 5. Is it possible to pick a set of eigenvalues that will make the system stable?**