1 Single-dimensional linearization

This is an exercise in linearizing a scalar system. The scalar nonlinear differential equation we have is

$$\frac{d}{dt}x(t) = \sin\bigl(x(t)\bigr) + u(t). \tag{1}$$

a) Find the equilibrium points for $u^* = 0$. You can do this by sketching $\sin(x)$ for $-4\pi \leq x \leq 4\pi$ and intersecting it with the horizontal line at 0. This will give you the equilibrium points x^* where $sin(x^*) + u^* = 0$.

b) Linearize the system (1) around the equilibrium $(x_0^*, u^*) = (0, 0)$. **What is the resulting linearized scalar differential equation for $\tilde{x}(t) = x(t) - x_0^* = x(t) - 0$, involving $\tilde{u}(t) = u(t) - u^* = u(t) - 0$?**

2 Jacobian Warm-Up

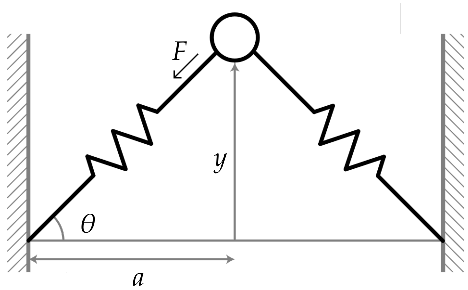
Consider the following function $f : \mathbb{R}^2 \mapsto \mathbb{R}^3$

$$f(x_1, x_2) = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \\ f_3(x_1, x_2) \end{bmatrix} = \begin{bmatrix} x_1^2 - x_2^2 \\ x_1^2 + x_1 x_2^2 \\ x_1 \end{bmatrix}$$

Calculate its Jacobian.

3 Linearization

Consider a mass attached to two springs:



We assume that each spring is linear with spring constant k and resting length X_0. We want to build a state space model that describes how the displacement y of the mass from the spring base evolves. The differential equation modeling this system is $\frac{d^2y}{dt^2} = -\frac{2k}{m}(y - X_0\frac{y}{\sqrt{y^2+a^2}})$.

a) Write this model in state space form $\dot{x} = f(x)$.

b) Find the equilibrium of the state-space model. You can assume $X_0 < a$.

c) Linearize your model about the equilibrium.