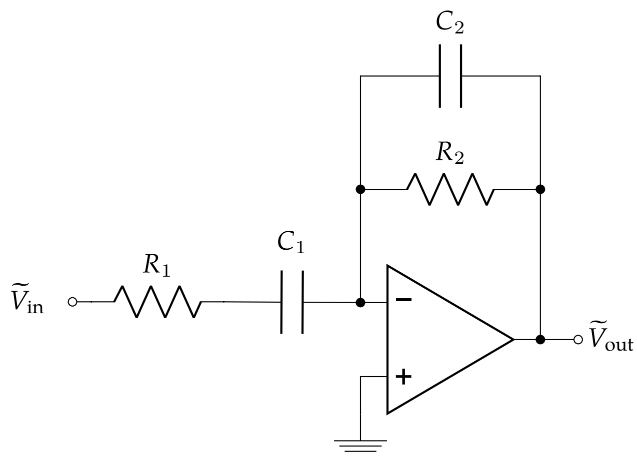
# 1 Differentiator Circuit

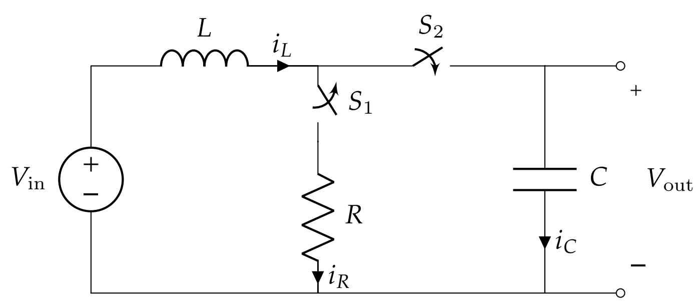
Consider the following circuit




1. What is the transfer function $H(j\omega)$?

# 2 Parallel RLC

Consider the circuit shown below.




At $t < 0$, $S_1$ is on (short-circuited), and $S_2$ is off (open-circuited).
At $t \geq 0$, $S_1$ is off (open-circuited), and $S_2$ is on (short-circuited).

1. Right after the switches change state (i.e., at $t = 0$), what is the value of $i_L$?

2. Choosing the state variables as $\vec{x}(t) = \begin{bmatrix} V_{\text{out}}(t) \\ i_L(t) \end{bmatrix}$, derive the $\mathbf{A}$ matrix that captures the behavior of this circuit for $t \geq 0$ with the matrix differential equation $\frac{d\vec{x}(t)}{dt} = \mathbf{A}\vec{x}(t) + \vec{b}$, where $\vec{b}$ is a vector of constants.

3. Assuming that $V_{\text{out}}(0) = 0\,\text{V}$, derive an expression for $V_{\text{out}}(t)$ for $t \geq 0$.

## 3 Diagonalizability and Invertibility

1. Given an example of a matrix $A$, or prove that no such example can exist.
   - Can be diagonalized and is invertible.
   - Cannot be diagonalized but is invertible.
   - Can be diagonalized but is non-invertible.
   - Cannot be diagonalized and is non-invertible.

## 4 Eigenvalue Decomposition and Singular Value Decomposition

We define Eigenvalue Decomposition as follows:

If a matrix $A \in \mathbb{R}^{n\times n}$ has $n$ linearly independent eigenvectors $\vec{p}_1, \dots, \vec{p}_n$ with eigenvalues $\lambda_i, \dots, \lambda_n$, then we can write:

$$A = P\Lambda P^{-1}$$

Where columns of $P$ consist of $\vec{p}_1, \dots, \vec{p}_n$, and $\Lambda$ is a diagonal matrix with diagonal entries $\lambda_i, \dots, \lambda_n$.

Consider a matrix $A \in \mathbb{S}^n$, that is, $A = A^T \in \mathbb{R}^{n\times n}$. This is a symmetric matrix and has orthorgonal eigenvectors. Therefore its eigenvalue decomposition can be written as,

$$A = P\Lambda P^T$$

1. First, assume $\lambda_i \geq 0, \forall i$. Find the SVD of $A$.
2. Let one particular eigenvalue $\lambda_j$ be negative, with the associated eigenvector being $p_j$. Succinctly,

   $$Ap_j = \lambda_j p_j \text{ with } \lambda_j < 0$$

   We are still assuming that,

   $$A = P\Lambda P^T$$

   a) What is the singular value $\sigma_j$ associated to $\lambda_j$?

   b) What is the relationship between the left singular vector $u_j$, the right singular vector $v_j$ and the eigenvector $p_j$?