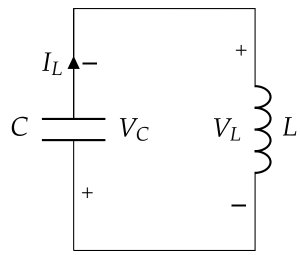
1 LC Tank

Consider the following circuit like you saw in lecture:



This is sometimes called an LC tank and we will derive its response in this problem. Assume at $t = 0$ we have $V_C(0) = V_S = 1$ V and $\frac{dV_C}{dt}(t = 0) = 0$. Also suppose $L = 9\,\text{nH}$ and $C = 1\,\text{nF}$.

a) **Write the system of differential equations in terms of state variables $x_1(t) = I_L(t)$ and $x_2(t) = V_C(t)$ that describes this circuit for $t \geq 0$. Leave the system symbolic in terms of V_s, L, and C.**

b) **Write the system of equations in vector/matrix form with the vector state variable** $\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$**.** This should be in the form $\frac{d}{dt}\vec{x}(t) = A\vec{x}(t)$ with a 2×2 matrix A.

Find the initial conditions $\vec{x}(0)$.

c) **Find the eigenvalues of the** A **matrix symbolically.**

d) Recall from yesterday's discussion that solutions for $x_i(t)$ will all be of the form

$$x_i(t) = \sum_k c_k e^{\lambda_k t}$$

where λ_k is an eigenvalue of our differential equation relation matrix A. Thus, we make the following guess for $\vec{x}(t)$:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \\ c_3 e^{\lambda_1 t} + c_4 e^{\lambda_2 t} \end{bmatrix}$$

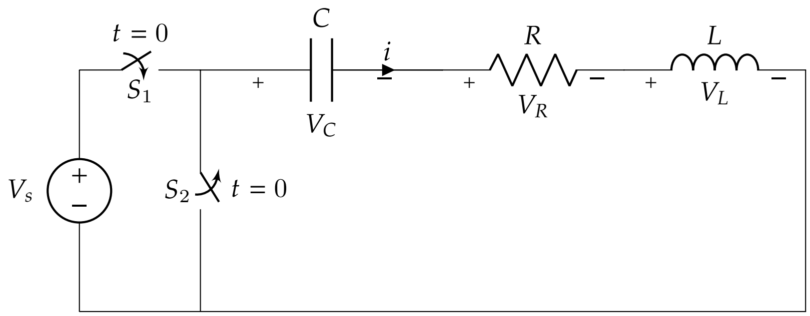
where $c_1,\ c_2,\ c_3,\ c_4$ are all constants.

Evaluate $\vec{x}(t)$ and $\frac{d\vec{x}}{dt}(t)$ at time $t = 0$ in order to obtain four equations in four unknowns.

e) **Solve those equations for c_1, c_2, c_3, c_4 and plug them into your guess for $\vec{x}(t)$.** What do you notice about the solutions? Are they complex functions? HINT: Remember $e^{j\theta} = \cos(\theta) + j\sin(\theta)$.

2 Charging RLC Circuit

Consider the following circuit. Before $t = 0$, switch S_1 is off while S_2 is on. At $t = 0$, both switches flip state (S_1 turns on and S_2 turns off):



a) Write out the differential equation describing this circuit for $t \geq 0$ in the form:

$$\frac{d^2 V_c}{dt^2} + a_1 \frac{dV_c}{dt} + a_0 V_c = b$$

b) Find a $\tilde{V}_c$ and substitute it to the previous equation such that

$$\frac{d^2\tilde{V}_c}{dt^2} + a_1\frac{d\tilde{V}_c}{dt} + a_0\tilde{V}_c = 0$$

c) Solve for $V_c(t)$ for $t \geq 0$. Use component values $V_s = 4\text{V}$, $C = 2\text{fF}$, $R = 60\text{k}\Omega$, and $L = 1\mu\text{H}$.