1 Transfer Function

When we write the transfer function of an arbitrary circuit, it always takes the following form. This is called a "rational transfer function." We also like to factor the numerator and denominator, so that they become easier to work with and plot:

$$\begin{aligned} H(\omega) = \frac{z(\omega)}{p(\omega)} &= \frac{(j\omega)^{N_{z0}}}{(j\omega)^{N_{p0}}} \left(\frac{(j\omega)^n \alpha_n + (j\omega)^{n-1}\alpha_{n-1} + \cdots + j\omega\alpha_1 + \alpha_0}{(j\omega)^m \beta_m + (j\omega)^{m-1}\beta_{m-1} + \cdots + j\omega\beta_1 + \beta_0} \right) \\ &= K \frac{(j\omega)^{N_{z0}} \left(1 + j\frac{\omega}{\omega_{z1}}\right)\left(1 + j\frac{\omega}{\omega_{z2}}\right)\cdots\left(1 + j\frac{\omega}{\omega_{zn}}\right)}{(j\omega)^{N_{p0}} \left(1 + j\frac{\omega}{\omega_{p1}}\right)\left(1 + j\frac{\omega}{\omega_{p2}}\right)\cdots\left(1 + j\frac{\omega}{\omega_{pm}}\right)} \end{aligned}$$

Here, we define the constants ω_z as "zeros" and ω_p as "poles", and N_{z0}, N_{p0} are the number of zeros and poles at $\omega = 0$

2 Low-pass Filter

You have a $1\,\mathrm{k\Omega}$ resistor and a $1\,\mathrm{\mu F}$ capacitor wired up as a low-pass filter.

a) **Draw the filter circuit, labeling the input node, output node, and ground.**

b) **Write down the transfer function of the filter, $H(j\omega)$ that relates the output voltage phasor to the input voltage phasor.** Be sure to use the given values for the components.

c) **Write an exact expression for the *magnitude* of $H(j\omega = j10^2)$, and give an approximate numerical answer.**

d) **Write an exact expression for the *magnitude* of $H(j\omega = j10^6)$, and give an approximate numerical answer.**

e) **Write an exact expression for the *phase* of $H(j\omega = j1)$, and give an approximate numerical answer.**

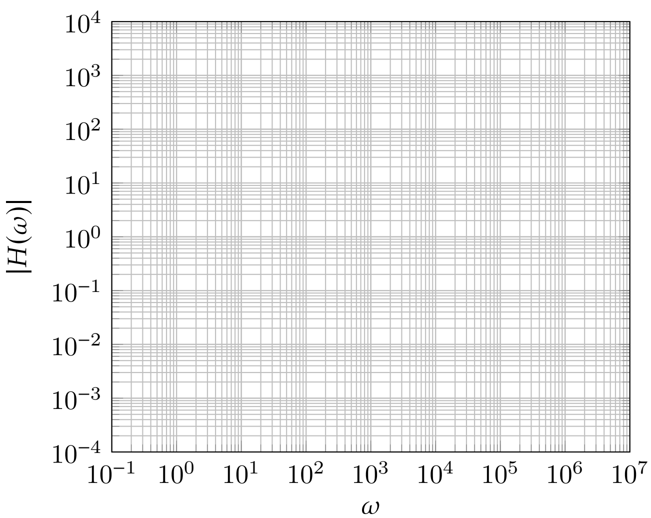
f) **Write an exact expression for the *phase* of $H(j\omega = j10^6)$, and give an approximate numerical answer.**

g) **Write down an expression for the *corner frequency* ω_c of this circuit. Evaluate the *magnitude* and *phase* of $H(j\omega = j\omega_c)$.**

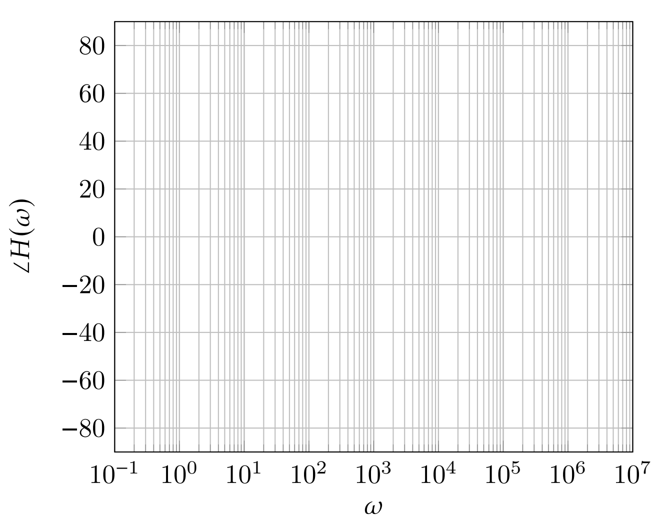
h) **Write down an expression for the time-domain output waveform $V_{out}(t)$ of this filter if the input voltage is $V(t) = 1\sin(1000t)$ V.** You can assume that any transients have died out — we are interested in the steady-state waveform.

i) **Based off the values calculated in the previous parts, predict what the Bode plots (both magnitude and phase) of the filter will look like and sketch them on the graph paper below. You may use a straight line approximation.**

Log-log plot of transfer function magnitude



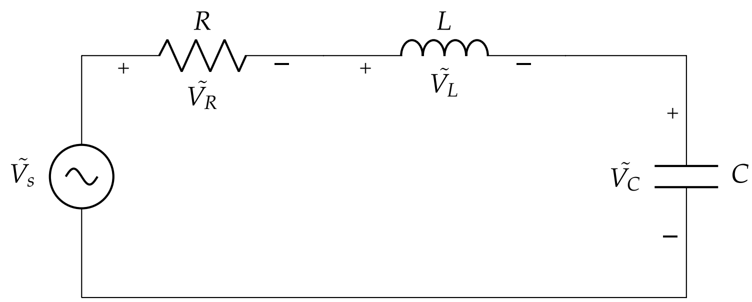
Semi-log plot of transfer function phase



j) **Use a computer to draw the Bode plots (both magnitude and phase) of the filter. Compare them to your sketch from before.**

3 RLC Circuit

In this question, we will take a look at an electrical systems described by second-order differential equations and analyze it in the phasor domain. Consider the circuit below where $\tilde{V}_s$ is a sinusoidal signal, $L = 1\,\mathrm{mH}$, and $C = 1\,\mathrm{nF}$:



a) **Transform the circuit into the phasor domain.**

b) **Solve for the transfer function** $H_C(\omega) = \frac{\tilde{V}_C}{\tilde{V}_s}$ **in terms of** R**,** L**, and** C**.**

c) **Solve for the transfer function** $H_L(\omega) = \frac{\widetilde{V}_L}{\widetilde{V}_\mathbf{s}}$ **in terms of** R**,** L**, and** C**.**

d) **Solve for the transfer function** $H_R(\omega) = \frac{\widetilde{V}_R}{\widetilde{V}_\mathbf{s}}$ **in terms of** R**,** L**, and** C**.**

e) **Use a computer to draw the magnitude Bode plots of $H_C(\omega)$, $H_L(\omega)$, and $H_R(\omega)$ when $R = 2\,\mathrm{k}\Omega$.**