1 RC Circuits

In this problem, we will be using differential equations to find the voltage across a capacitor over time in an RC circuit. We set up our problem by first defining three functions over time: $I(t)$ is the current at time t, $V(t)$ is the voltage across the circuit at time t, and $V_o(t)$ is the voltage across the capacitor at time t.

Recall from 16A that the voltage across a resistor is defined as $V_R = RI_R$ where I_R is the current across the resistor. Also, recall that the voltage across a capacitor is defined as $V_o = \frac{Q}{C}$ where Q is the charge across the capacitor.

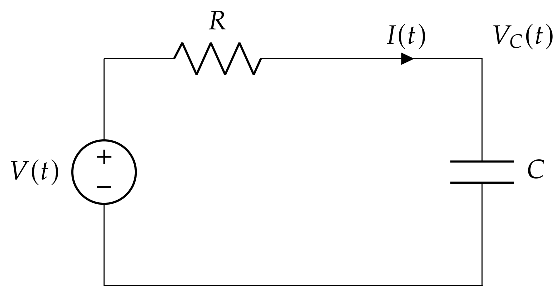


Figure 1: Example Circuit

a) First, find an equation that relates the current across the capacitor $I(t)$ with the voltage across the capacitor $V_C(t)$.

b) Write a system of equations that relates the functions $I(t)$, $V_C(t)$, and $V(t)$.

c) Rewrite the previous equation in part (b) in the form of a differential equation involving only $V_C(t)$ and $V(t)$.

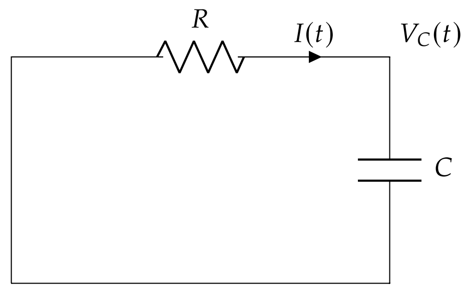


Figure 2: Circuit for part (d)

d) Let's suppose that at $t = 0$, the capacitor is charged to a voltage V_{DD} ($V_C(0) = V_{DD}$). Let's also assume that $V(t) = 0$ for all $t \geq 0$. Solve the differential equation for $V_C(t)$ for $t \geq 0$.

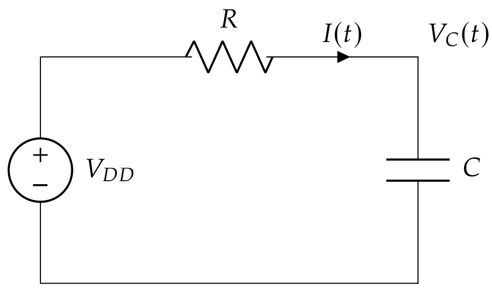


Figure 3: Circuit for part (e)

e) Now, let's suppose that we start with an uncharged capacitor $V_C(0) = 0$. We apply some constant voltage $V(t) = V_{DD}$ across the circuit. Solve the differential equation for $V_C(t)$ for $t \geq 0$.

2 Graphing RC Responses

Consider the following RC Circuit with a single resistor R, capacitor C, and voltage source $V(t)$.

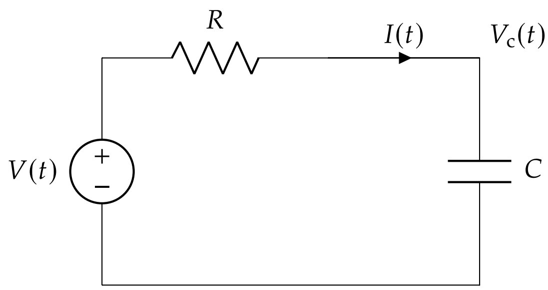


Figure 4: Example Circuit

a) Let's suppose that at $t = 0$, the capacitor is charged to a voltage V_{DD} ($V_c(0) = V_{DD}$) and that $V(t) = 0$ for all $t \geq 0$. Plot the response $V_c(t)$.

b) Now let's suppose that at $t = 0$, the capacitor is uncharged ($V_c(0) = 0$) and that $V(t) = V_{DD}$ for all $t \geq 0$. Plot the response $V_c(t)$.

To better understand our responses, we now define a **time constant** which is a measure of how long it takes for the capacitor to charge or discharge. Mathematically, we define τ as the time at which $V_C(\tau)$ is $\frac{1}{e} = 36.8\%$ away from its steady state value.

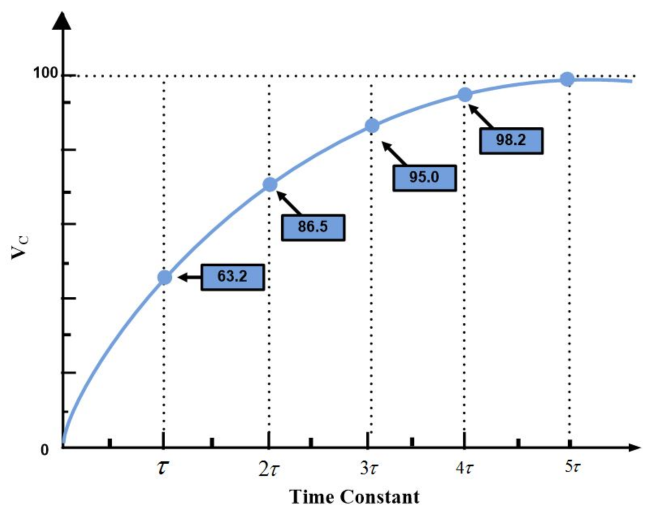


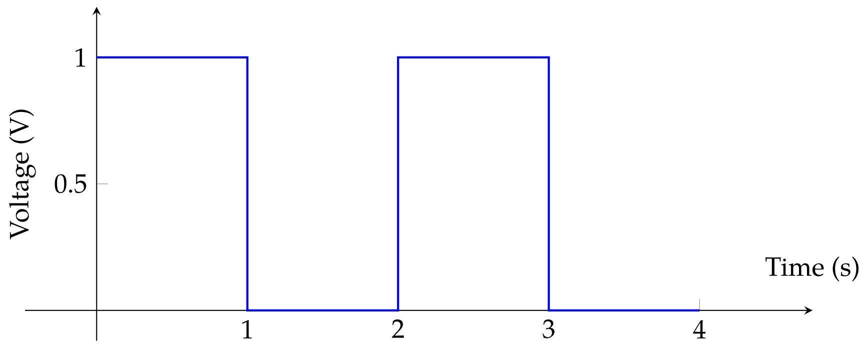
Figure 5: Different values of capacitor voltage at different times, relative to τ.

c) Suppose that $V_{DD} = 5\,\text{V}$, $R = 100\,\Omega$, and $C = 10\,\mu\text{F}$. What is the time constant τ for this circuit?

d) Going back to part (b), on what order of magnitude of time (nanoseconds, milliseconds, 10's of seconds, etc.) does this circuit settle (V_c is > 95% of its value as $t \to \infty$)?

e) Give 2 ways to reduce the settling time of the circuit if we are allowed to change one component in the circuit.

f) Suppose we have a source $V(t)$ that alternates between 0 and $V_{DD} = 1\,\text{V}$. Given $RC = 0.1\,\text{s}$, plot the response V_c if $V_c(0) = 0$.



g) Now suppose we have the same source $V(t)$ but $RC = 1\,\text{s}$, plot the response V_c if $V_c(0) = 0$.