1 KCL

Consider the circuit shown below:

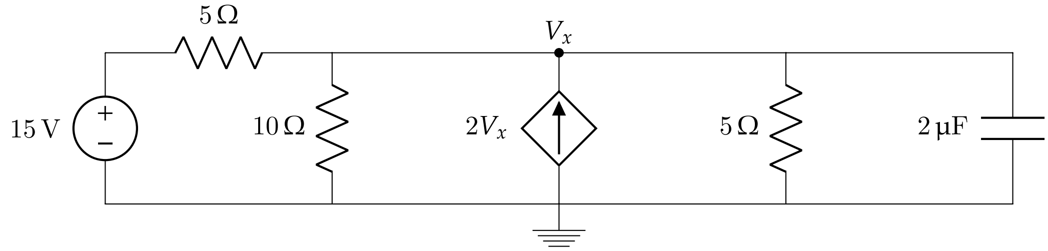


Figure 1: Adapted from Ulaby, Maharbiz, Furse. *Circuits*. Third Edition

Determine the voltage V_x at steady state.

Answer

At steady state, there is no current flowing through the capacitor.

In terms of the node voltage V_x, KCL gives

$$\frac{V_x - 15}{5} + \frac{V_x}{10} - 2V_x + \frac{V_x}{5} = 0,$$

whose solution leads to

$$V_x = -2\,\text{V}.$$

2 KVL

Consider the circuit shown below:

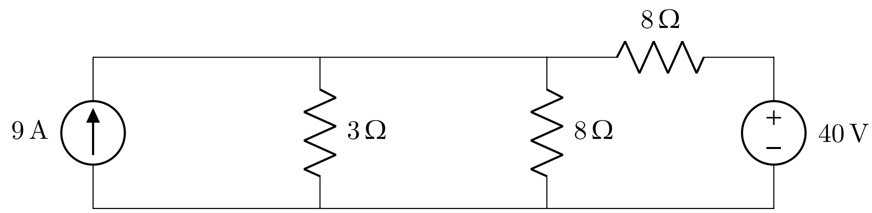
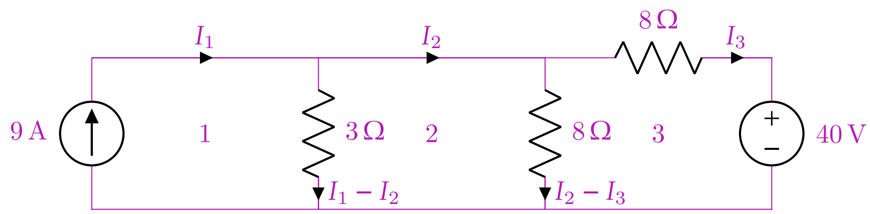


Figure 2: Adapted from Ulaby, Maharbiz, Furse. *Circuits*. Third Edition.

Using KVL, determine the amount of power supplied by the voltage source. Do not use superposition.

Answer

We will label the currents I_1, I_2, and I_3 as shown in the following diagram.



$$\begin{cases} \text{Loop 1:} & I_1 = 9\,\text{A} \\ \text{Loop 2:} & 3(I_2 - I_1) + 8(I_2 - I_3) = 0 \\ \text{Loop 3:} & 8(I_3 - I_2) + 8I_3 + 40 = 0 \end{cases}$$

Simplification leads to:

$$\begin{cases} 11I_2 - 8I_3 = 27 \\ -8I_2 + 16I_3 = -40 \end{cases}$$

Solving this system of equations gives:

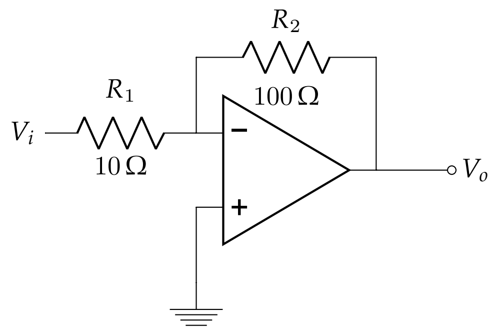
$$I_2 = 1\,\text{A} \qquad I_3 = -2\,\text{A}$$

The power supplied by the voltage source is:

$$P = VI = 40 \cdot (-2) = -80\,\text{W}$$

3 Op-Amp Review

Consider the circuit below:

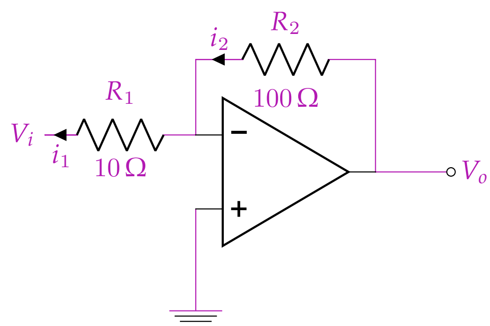


a) Calculate V_o if V_i if $V_i = 0.5\,\mathrm{V}$.

Answer

For an op-amp in negative feedback, the Golden Rules are (1) the voltage difference between the two inputs is zero ($V^+ = V^-$), and (2) no current goes into the inputs of an op-amp.

Let's label the branch currents as I_1 and I_2. Note that we have applied the second Golden Rule to have all input currents of op-amps be 0. See the following figure:



According to the first Golden Rule, we can write down:

$$V^+ = V^- = 0\,\mathrm{V}$$

Then we write down the node current equations based on Kirchhoff's Current Law (KCL) (the sum of total currents flowing into one node is the same as the sum of currents flowing out of that same node.):

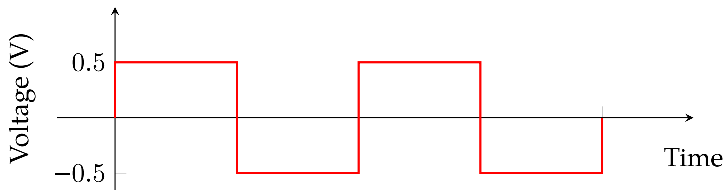
$$I_1 = I_2 \implies \frac{V^- - V_i}{10\,\Omega} = \frac{V_o - V^-}{100\,\Omega}$$

You can define currents in whichever directions you like, but in the end you should arrive at the same result:

$$V_o = -\frac{100}{10}V_i = -10V_i$$
$$V_o = -5\,\mathrm{V}$$

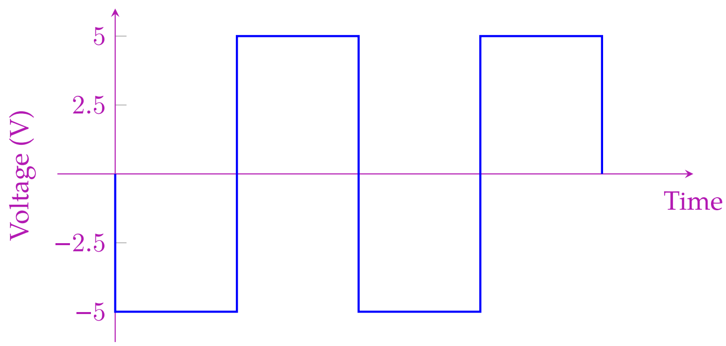
This takes the familiar form of the output of an inverting amplifier, $V_o = -(R_2/R_1)V_i$.

b) Sketch V_o if V_i is a square wave with $V_{pp} = 1\,\text{V}$.

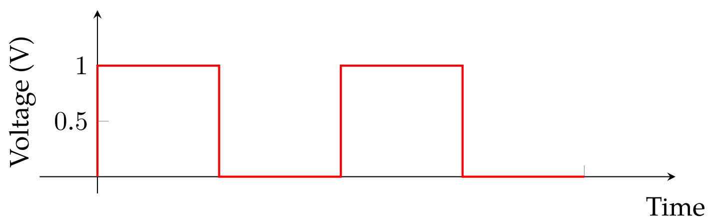


Answer

See the following figure:



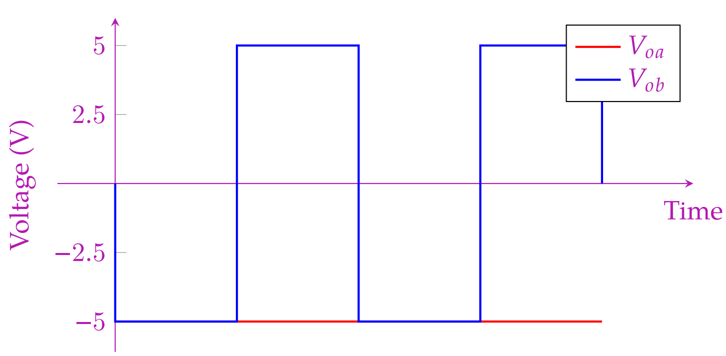
c) Use **superposition** to sketch V_o if V_i is a $1\,\text{V}_{\text{pp}}$ square wave with a $0.5\,\text{V}$ DC offset.

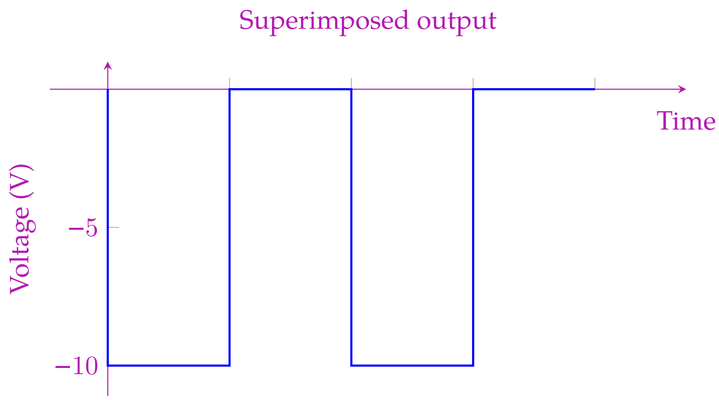


Answer

We can view the input as a superposition of the inputs from parts (a) and (b). Thus, we can construct the output by superimposing the outputs we calculated in the preceding parts.

Individual outputs

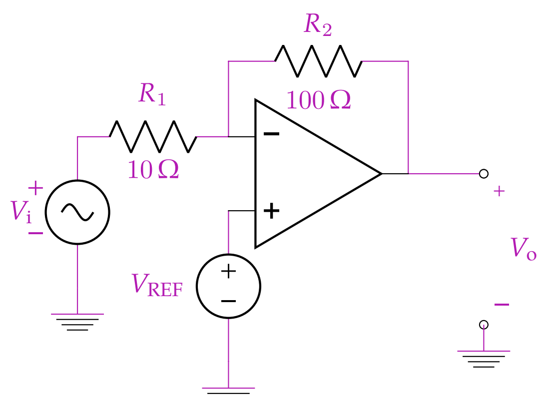




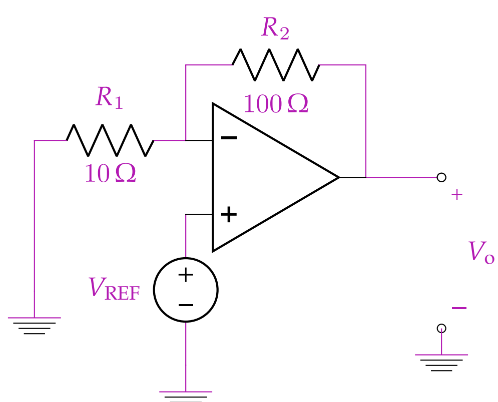
d) Consider the non-inverting input. What value could we replace ground with to make the output from part (c) centered around $0\,\mathrm{V}$?

Answer

In essence, we are considering the following circuit, where V_i is the input from part (c).



We can approach this problem using superposition once again. With V_{REF} shorted, we have the same circuit as in preceding parts – that is, an inverting amplifier. However, if we short V_i, the circuit takes on this form:



We may recognize this as a noninverting amplifier, with gain $1 + \frac{R_2}{R_1} = 11$. To center the output around $0\,\text{V}$ then, we require:

$$11V_{REF} + (-5\,\text{V}) = 0$$

The desired reference voltage is:

$$V_{REF} = \frac{5}{11}\text{V}$$

Alternatively, we can use nodal analysis.

$$\frac{V^- - V_i}{10\,\Omega} = \frac{V_o - V^-}{100\,\Omega}$$
$$\frac{V_{REF} - V_i}{10\,\Omega} = \frac{V_o - V_{REF}}{100\,\Omega}$$

In the second line we used the second golden rule to substitute $V^+ = V^- = V_{REF}$. The output is thus

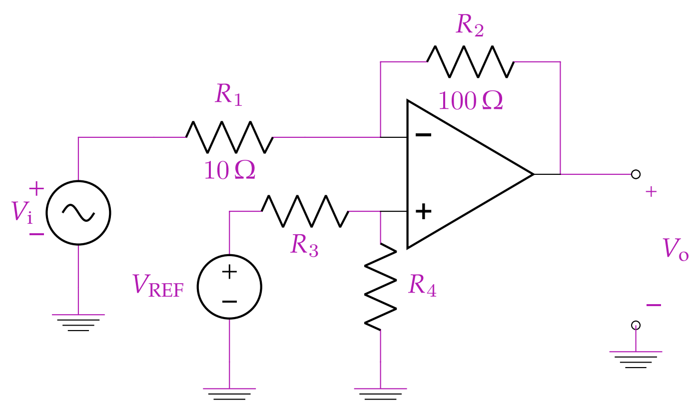
$$V_o = -10V_i + 11V_{REF}$$

which leads to the same result, $V_{REF} = \frac{5}{11}\text{V}$.

e) Suppose we only have a $1\,\text{V}$ source, but still wish to center the output from (c) about $0\,\text{V}$. What circuit block should we place at the noninverting input to accomplish this goal?

Answer

Use a voltage divider (see figure).



Since we still want the voltage at the noninverting input to be $V^+ = \frac{5}{11}\text{V}$, choose R_3 and R_4 such that $\frac{R_4}{R_3+R_4} = \frac{5}{11}$. For example, we could pick $R_3 = 60\,\Omega$ and $R_4 = 50\,\Omega$.