# Lab 4: Sensing Part 1

## Part 0: Introduction

In this lab, you will design four filters to pass different sections of the audible frequency spectrum to separate LEDs so they will appear to flash in time to music — in other words, you'll have made your very own color organ!

To do this, you will select your desired cutoff frequencies and calculate the appropriate resistor and capacitor values to build filters with said cutoff frequencies.

The audible range is actually a somewhat small spectrum of frequencies, as demonstrated below:

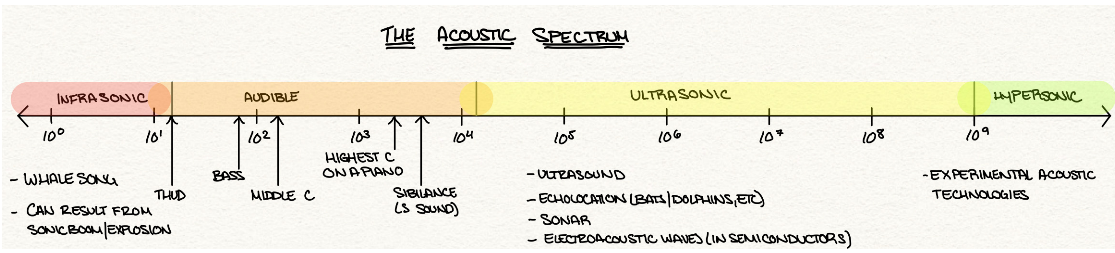



Figure 1: Sketch of the acoustic spectrum.

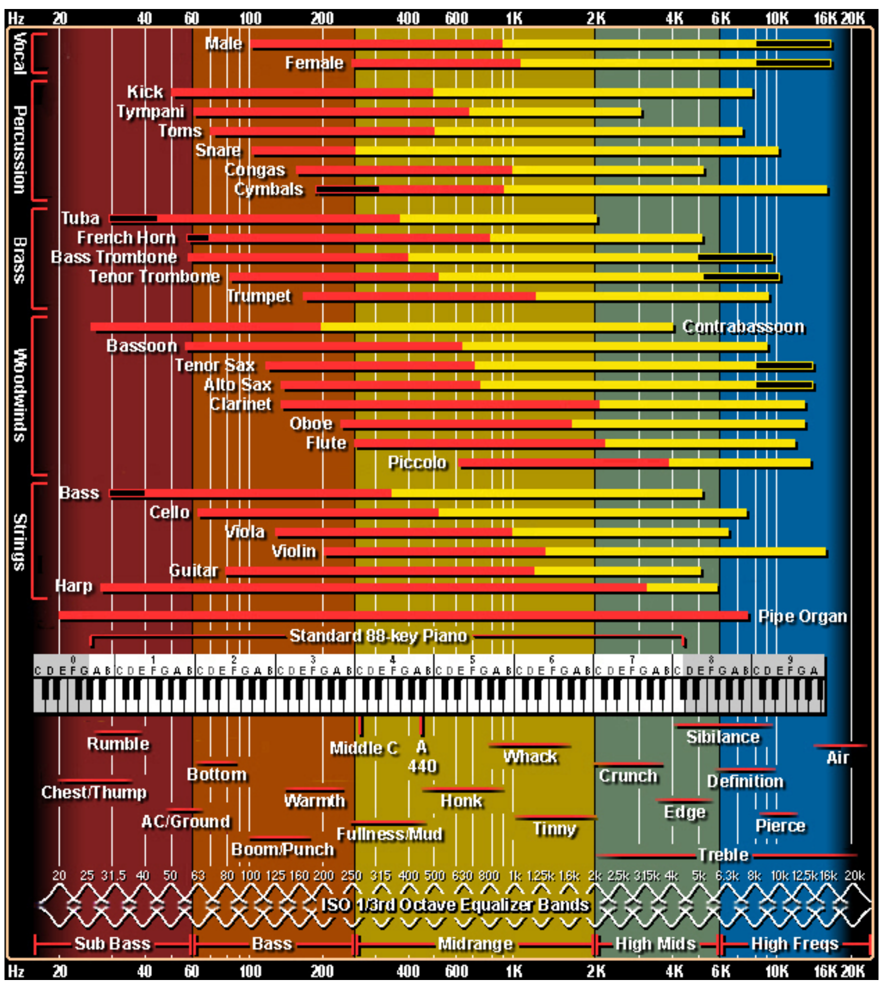



Figure 2: Expansion of the audible range of the acoustic spectrum.

> ***Sanity check question:*** *What challenge does the relatively small size of the audible spectrum create? Remember, the word "cutoff" in the phrase "cutoff frequency" is somewhat of a misnomer; the cutoff frequency indicates the point at which the signal power is attenuated by half, not the point at which it is fully eliminated.* ***Hint****: Think about separation in frequency domain.*

Note: Acoustic waves are *not* electromagnetic (EM) waves: sound waves are mechanical and therefore need a medium through which to propagate. While EM waves do not need a medium[1]: they can propagate through the vacuum of space.

You will be targeting the bass, midrange, and treble (aka "high mids"-"high freqs") sections depicted in Figure 2 above, which we define as follows:

| | |
|---|---|
| Bass | 0-500 Hz |
| Midrange | 1000-5,000 Hz |
| Treble | 6,000-20,000 Hz |

Ultimately, these frequency ranges are **guidelines**: the goal of this lab is to independently light up your LEDs (with little to no overlap — two LEDs should ideally not light up at the same pure frequency, and "dead zones" where none of the LEDs light up should be minimal/imperceptible) and generate sound with the piezo speaker on TinkerCAD.

## Part 1: Piezoelectric speaker

A piezo or piezoelectric speaker is a loudspeaker that uses the piezoelectric effect for generating sound. The mechanical motion is created by applying a voltage to a piezoelectric material, and this motion is typically converted into audible sound using diaphragms and resonators.

As shown in Figure 3(a), the heart of each piezoelectric speaker is a ceramic disc that interacts when it feels a certain voltage difference. An increase of the peak-to-peak voltage $V_{pp}$, will result in a larger piezo deformation and larger sound output. Piezoelectric speakers have a complex electronic equivalent circuit [Figure 3(b)] but mainly they can be seen as a capacitive load with values between $nF$ to $\mu F$(e.g. $1.5\mu F$). For simplicity, we will treat the piezoelectric speaker as a resistor during calculation in this lab.

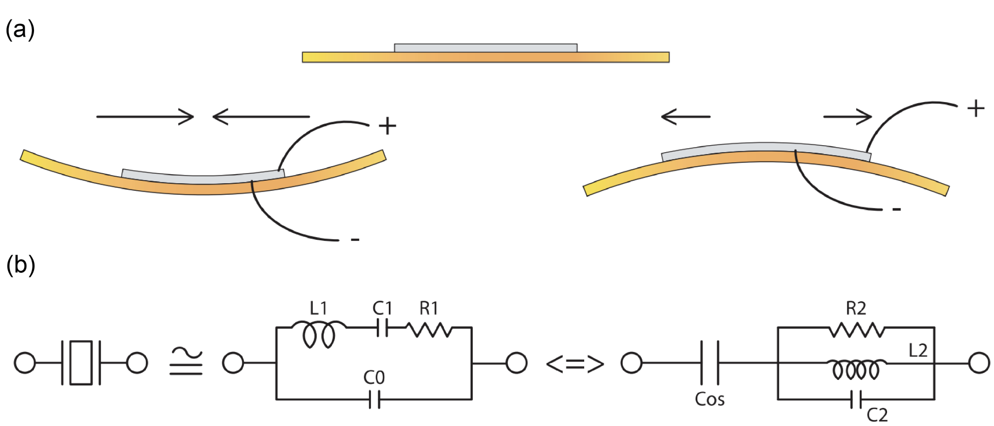



Figure 3: (a) Piezo interacting with voltage variations, (b) Equivalent circuit of Piezo.

[1]This bothered early scientists, so they came up with the concept of the aether (subsequently decommissioned in 1897).

## Part 2: A *Bass*-ic Color Organ

Now we are ready to begin building the color organ circuit! The finished product will look something like this:

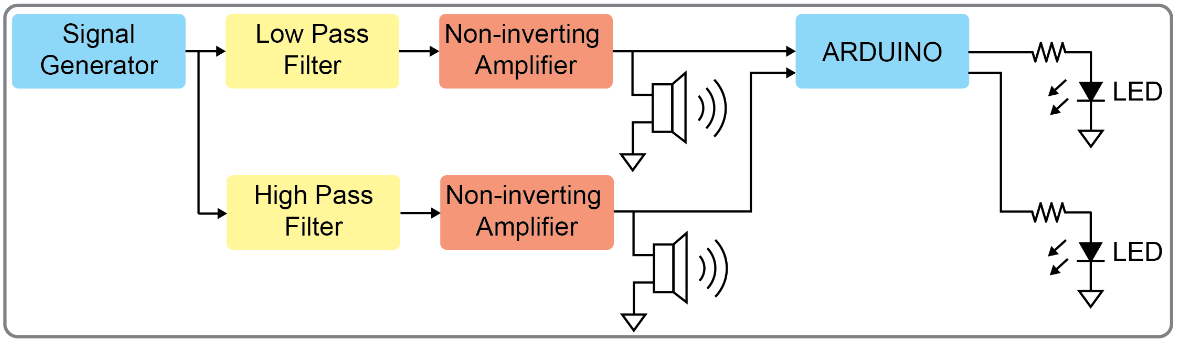



Figure 4: High-level overview of completed color organ.

This is a significantly larger circuit than the circuits you have built in previous labs, so **be sure to plan ahead when constructing your circuit, and keep your circuit clean!**

**Generalizing the first-order filter**

The general first-order (or **bilinear**, since it is linear in both the numerator and denominator) transfer function is as follows (recall, $s = j\omega$):

$$H(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

*Think: What is the gain at $s = \infty$? What about at $s = 0$?* The gain at $s = 0$ is the DC gain, and the gain at $s = \infty$ is the high frequency gain. In this case, the high-frequency gain approaches $a_1$, and the DC gain is $a_0/\omega_0$. The coefficients $a_0$ and $a_1$ determine what kind of filter we have. As an exercise, think of the relationships among the numerator coefficients that realize the different kinds of filters.

The first filter you will build in this lab will be a first-order low-pass RC filter to isolate the bass frequencies, as detailed in the lab ipynb. It would be also helpful to take a look at Part 4 of this note to familiarize yourself with the derivation of low-pass RC filters.

Now, you're ready to build your filter! Go to the ipython notebook and complete Part 1.

## Part 3: A *Treble*-some Color Organ

Next, we will build a first-order high-pass RC filter to isolate the treble frequencies. The derivation of high-pass RC filters can also be found in Part 4.

***Sanity check question:*** *Does placing the filters in parallel affect their respective cutoff frequencies? Think: is the signal you're trying to process a current signal or a voltage signal?*

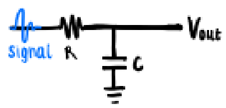
Go to the ipython notebook and complete Part 2.
***Sanity check question:*** *Why do we use sinusoid waveform with 0V DC offset? Hint: The input to our filters, which is generated by the signal generator, has both a sinusoidal/fluctuating component with a frequency and a DC offset component. What does a low-pass filter do to each component? What about a high-pass filter?*

**Part 4: Derivation of first-order RC filters**

## Building Filters

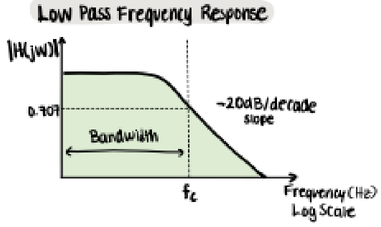
### Lowpass Filter

Circuit Schematic




Think: the "gate" C is lower.

Low Pass Frequency Response




$$\tilde{V}_{out} = \tilde{V}_{in} \cdot \frac{Z_C}{Z_R + Z_C} = \tilde{V}_{in} \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \tilde{V}_{in} \frac{1}{j\omega RC + 1}$$

$\frac{\tilde{V}_{out}}{\tilde{V}_{in}} = H(j\omega)$ and cutoff frequency is at half power, where $\frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|} = \frac{1}{\sqrt{2}} = 0.707$.

$$|H(j\omega)| = \frac{1}{\sqrt{2}} = \frac{\sqrt{1}}{\sqrt{(\omega RC)^2 + (1)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$2 = 1 + (\omega RC)^2$$

$$1 = \omega RC$$

Conceptually: as $\omega \to \infty$, $|H(j\omega)| \to 0$
as $\omega \to 0$, $|H(j\omega)| \to 1$

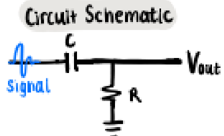
$\omega = \frac{1}{RC}$ angular cutoff frequency

$f_c = \frac{1}{2\pi RC}$ cutoff frequency

Everything that is less than $f_c$ gets through. Note that our cutoff isn't clean & perfect because the attenuation is gradual.

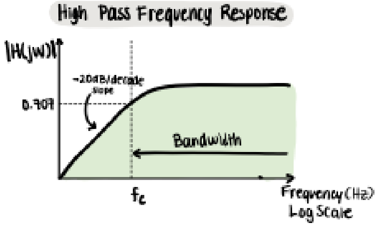
### High Pass Filter

Circuit Schematic




Think: the "gate" C is higher.

High Pass Frequency Response




$$\tilde{V}_{out} = \tilde{V}_{in} \cdot \frac{Z_R}{Z_R + Z_C} = \tilde{V}_{in} \frac{R}{\frac{1}{j\omega C} + R}$$

$$|H(j\omega)| = \frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{R^2}}{\sqrt{(\frac{1}{\omega C})^2 + R^2}}$$

$$\frac{1}{2} = \frac{R^2}{(\frac{1}{\omega C})^2 + R^2}$$

$$\left(\frac{1}{\omega C}\right)^2 + R^2 = 2R^2$$

$$\left(\frac{1}{\omega C}\right)^2 = R^2$$

Conceptually: as $\omega \to \infty$, $|H(j\omega)| \to 1$
as $\omega \to 0$, $|H(j\omega)| \to 0$

$\omega = \frac{1}{RC}$ angular cutoff frequency

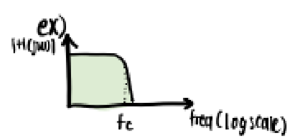
$f_c = \frac{1}{2\pi RC}$ cutoff frequency

Everything higher that $f_c$ gets through.

**Thought:** What happens to DC Voltage in a high pass filter?
↳ It get's destroyed, $\omega = 0$!

**Thought:** How can we make attenuation faster?
↳ multiple filters cascaded. Our transfer functions multiply, making the drop-off faster.
↳ make sure to place a unity gain buffer in between to prevent loading

ex)



## References

Horowitz, P. and Hill, W. (2015). *The Art of Electronics*. 3rd ed. Cambridge: Cambridge University Press, ch 1.
Sedra, A. and Smith, K. (2015). *Microelectronic Circuits.* 7th ed. New York: Oxford University Press, ch 17.

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