Lab 5: Sensing Part 2

Part 0: Introduction

In this lab, you will design a band-pass filter to partition the audible frequency spectrum of the mic board. To do this, you will select your desired cutoff frequencies and calculate the appropriate resistor and capacitor values to build filters with said cutoff frequencies.

The audible range is actually a somewhat small spectrum of frequencies, as demonstrated below:

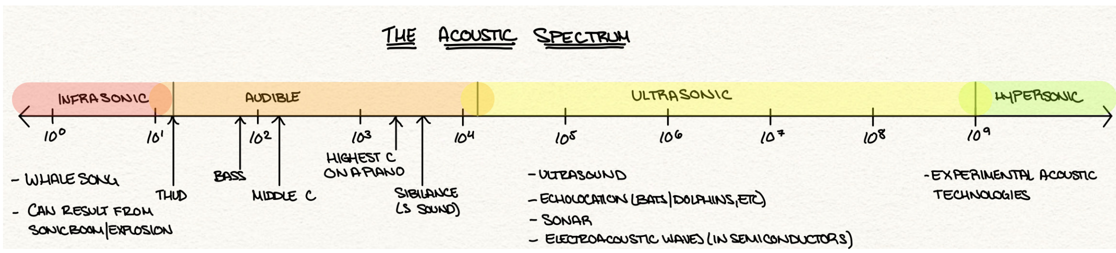


Figure 1: Sketch of the acoustic spectrum.

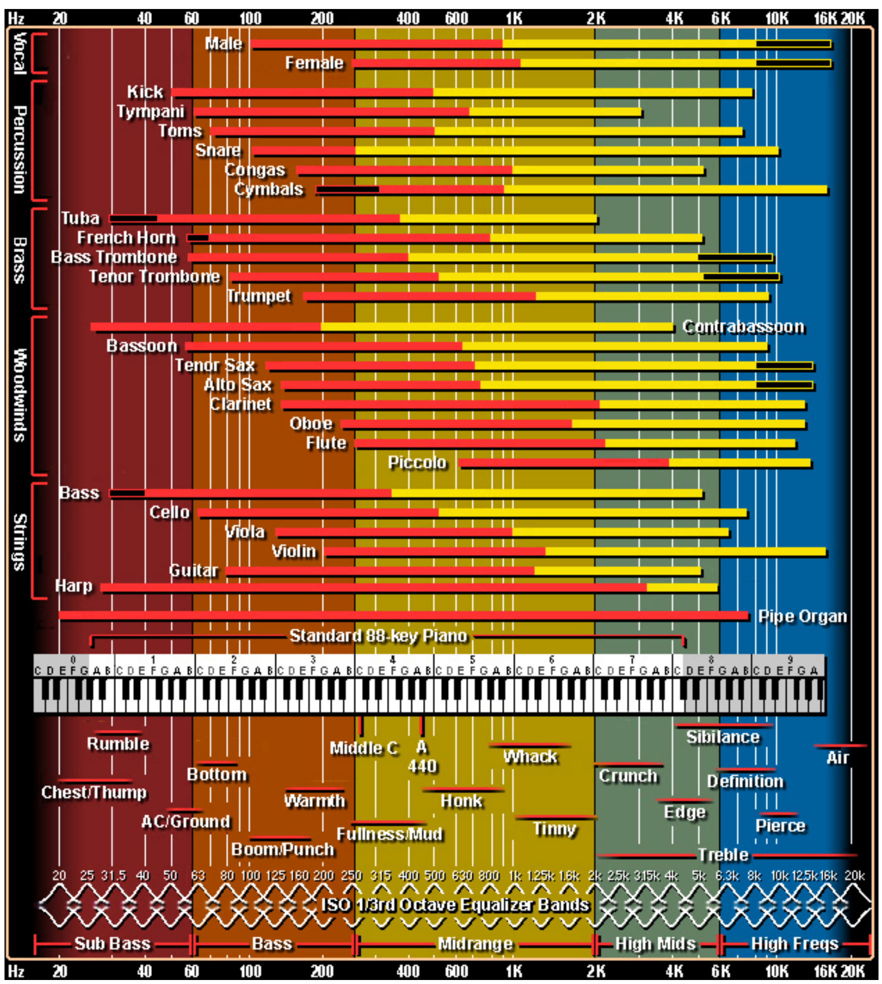


Figure 2: Expansion of the audible range of the acoustic spectrum.

> ***Sanity check question:*** *What challenge does the relatively small size of the audible spectrum create? Remember, the word "cutoff" in the phrase "cutoff frequency" is somewhat of a misnomer; the cutoff frequency indicates the point at which the signal power is attenuated by half or the voltage gain is attenuated by* $\frac{1}{\sqrt{2}}$*, not the point at which the signal is fully eliminated.* ***Hint****: Think about separation in frequency domain.*

Note: Acoustic waves are *not* electromagnetic waves: sound waves are mechanical and therefore need a medium through which to propagate, whereas EM waves do not need a medium[1]: they can propagate through the vacuum of space.

You will be targeting part of the midrange section (about 1000-2500 Hz) depicted in Figure 2 above, which we define as follows:

Bass	0-500 Hz
Midrange	1000-5,000 Hz
Treble	6,000-20,000 Hz

Now we are ready to begin building the filters! We will first build the low-pass filter then the high-pass filter, and finally cascade the two filters to create a band-pass filter. The finished product will look something like this:

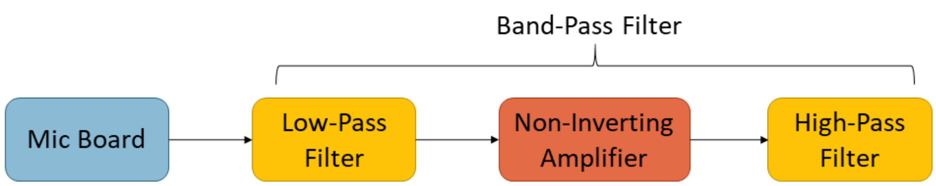


Figure 3: High-level overview of completed micboard band-pass system.

Part 1: Low-Pass Filter

Generalizing the first-order filter

The general first-order (or **bilinear**, since it is linear in both the numerator and denominator) transfer function is as follows (recall, $s = j\omega$):

$$H(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

Think: What is the gain at $s = \infty$? What about at $s = 0$? The gain at $s = 0$ is the DC gain, and the gain at $s = \infty$ is the high frequency gain. In this case, the high-frequency gain approaches a_1, and the DC gain is a_0/ω_0. The coefficients a_0 and a_1 determine what kind of filter we have. As an exercise, think of the relationships among the numerator coefficients that result in different kinds of filters.

The first filter you will build in this lab will be a first-order low-pass RC filter to isolate the frequencies below the desired cutoff frequency, as detailed in the lab ipynb. Please refer to Appendix A of this note to familiarize yourself with the derivation of low-pass RC filters. In addition to serving as part of the band-pass filter in this lab, this filter will eventually be used to better sample voice commands in later phases of the project. This is because human speech is typically below 2500 Hz, so the filter would pass through frequencies corresponding to human speech while attenuating higher frequency noise.

[1]This bothered early scientists, so they came up with the concept of the aether (subsequently decommissioned in 1897).

Part 2A: High-Pass Filter

As part of our band-pass filter, we will also build a first-order high-pass filter to isolate the frequencies above the desired cutoff frequency.

In the lab ipynb, there is a schematic of how to make a simple RC high-pass filter. Note that in the schematic, we ask you to connect the other end of the resistor to OS2, instead of gnd. This is different from the low-pass filter, where we asked you to connect the other end of the capacitor to gnd, not OS2.

> ***Sanity check question:*** *Why do we have you connect the second component of the filter to OS2 instead of gnd for the high-pass filter but not the low-pass filter? Hint: The input to our filters, which is the output of the micboard, has both a sinusoidal/fluctuating component with a frequency and a DC offset component. What does a low-pass filter do to each component? What about a high-pass filter?*

Please refer to Appendix A of this note to familiarize yourself with the derivation of high-pass RC filters.

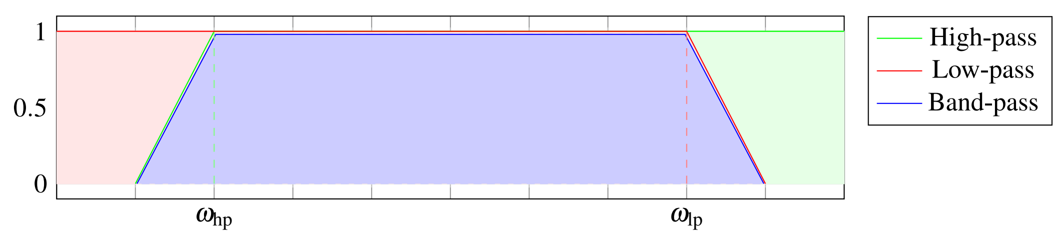
Part 2B: Caught in the Midrange

In the final part of this lab, we will be building a band-pass filter to isolate part of the midrange frequencies. The band-pass filter can be made simply by chaining/cascading together the low-pass filter and high-pass filter we built previously.

The band-pass filter has two cutoff frequencies: ω_{hp} and ω_{lp}, which correspond to the high-pass and low-pass cutoff frequencies respectively. Which of the two cutoff frequencies should be higher?

The frequency response of the band-pass will look something like this:

Band-pass Frequency Response (Bode Approximation)



> ***Sanity check question:*** *Does the order of the filters matter? I.e. does it matter whether we chain low-pass into high-pass or vice versa?*

In order to accomplish this, we want the transfer functions of our two filters to multiply. However, we need to be careful: we cannot simply plug the output of one of the filters into the input of the other directly; doing so would "load" the first filter and affect its cutoff frequency because the second filter ends up drawing current from the first. If we want to make sure the transfer functions to multiply and give us the desired band-pass frequency response without affecting each other, we need to somehow isolate the first filter from the second while still passing the output of the first to the input of the second. Here is where buffers come into play.

Buffers

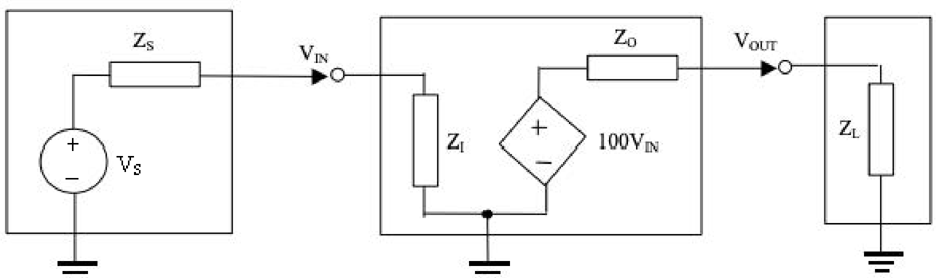
You can think of a buffer as providing an impedance transformation between two *cascaded* circuits. When you observe an undesired loading effect between two circuits, placing a buffer between them changes the load impedance of the first circuit to a very high value and the source impedance of the second to a very low value in accordance with (approximately) ideal op-amp characteristics. As the op-amp does not allow any current to flow into its input terminals, this prevents the second filter from drawing current from the first filter and affecting the frequency response. Instead, the second filter draws its current from the output of the op-amp, which is a replica of the first filter's output due to this being a buffer circuit. This allows you to build very modular circuits easily, without having to do lots of ugly algebra.

By placing a buffer in between our two filters that make up the band-pass filter, *cascading them does not change the transfer functions of the individual circuits* and the overall transfer function of the cascade is simply the product of the transfer functions of the individual circuits. This is why buffers are so useful in filter design. However, op-amps (and therefore these buffer circuits) do have their limits; they are not quite as perfect as the golden rules describe them to be.

Amplifier Loading

Recall the impedance characteristics of the ideal op-amp: its input impedance is infinite (no current flows into its inputs) and its output impedance is 0. This allows it to act like an ideal voltmeter at the input and supply infinite current at its output. But what happens if we don't have those characteristics?

Let's assume we have a noninverting amplifier with a gain of 100, with a finite input impedance Z_I and non-zero output impedance Z_o, as in the schematic below. The middle box represents the amplifier.



We now see that V_{IN} depends on the source output impedance Z_S and the amplifier input impedance Z_I (because of the voltage divider formed by the two), and V_{out} depends on the amplifier output impedance Z_O and the load impedance Z_L. Recalling the voltage divider equation,

$$V_{IN} = V_S \frac{Z_I}{Z_I + Z_S}$$

$$V_{out} = (100V_{IN}) \frac{Z_L}{Z_L + Z_O}$$

But $100V_{IN}$ is our desired V_{out}! To keep that approximately correct and avoid "loading" the output and reducing the voltage noticeably from what we expect, we need Z_L to be considerably larger than Z_O to keep $\frac{Z_L}{Z_L+Z_O}$ as close to 1 as possible.

This is why you set the "output load" on the signal generator to "High-Z": by doing so, you are telling it to expect a high-impedance load. The function generator has a 50-ohm output impedance, while the oscilloscope probes are high-impedance, so when the function generator is set to "High-Z," you can probe it with the oscilloscope and see the output voltage you expect (the one you explicitly set). If you set the function generator to "50 Ohm", it expects a 50 Ohm load. Since this is equal to its output impedance, V_{out} would be halved, so the function generator compensates for this by doubling its output voltage in 50 Ohm mode.

Note that in addition to the output loading effect, the amplifier itself can also load its input (e.g. the output of a preceding circuit) if it has a finite input impedance, reducing the V_{IN} from the desired value of V_S. We thus also need Z_I to be much larger than Z_S as well so that the input is not affected (keep $\frac{Z_I}{Z_I+Z_S}$ as close to 1 as possible).

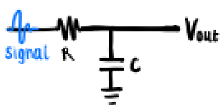
For our circuits, including the ones in this lab, you may assume that the load impedance is always sufficiently large enough and the op-amps' output impedance is small enough so that the loading effect is negligible. You may also assume that the input impedance of our op-amps is sufficiently large enough to prevent the op-amp from loading its own input. Thus, we can safely connect the op-amp without worrying about it loading the output of the previous circuit, and the outputs of our op-amp circuits, like the buffers we're using in this lab, will remain unaffected by whatever we connect it to, allowing us to safely use a buffer to build our band-pass filter.

Appendix A: Derivation of first-order RC filters

Building Filters

Lowpass Filter

Circuit Schematic



Think: the "gate" C is lower.

$$\tilde{V}_{out} = \tilde{V}_{in} \cdot \frac{Z_C}{Z_R + Z_C} = \tilde{V}_{in} \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \tilde{V}_{in} \frac{1}{j\omega RC + 1}$$

$\frac{\tilde{V}_{out}}{\tilde{V}_{in}} = H(j\omega)$ and cutoff frequency is at half power, where $\frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|} = \frac{1}{\sqrt{2}} = 0.707$.

$$|H(j\omega)| = \frac{1}{\sqrt{2}} = \frac{\sqrt{1}}{\sqrt{(\omega RC)^2 + (1)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$2 = 1 + (\omega RC)^2$$

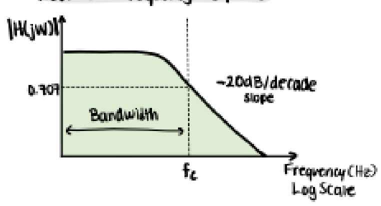
$$1 = \omega RC$$

$\omega = \frac{1}{RC}$ angular cutoff frequency

$f_c = \frac{1}{2\pi RC}$ cutoff frequency

Conceptually: as $\omega \to \infty$, $|H(j\omega)| \to 0$
as $\omega \to 0$, $|H(j\omega)| \to 1$

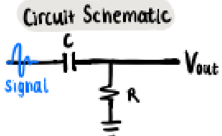
Low Pass Frequency Response



Everything that is less than f_c gets through. Note that our cutoff isn't clean & perfect because the attenuation is gradual.

High Pass Filter

Circuit Schematic



Think: the "gate" C is higher.

$$\tilde{V}_{out} = \tilde{V}_{in} \cdot \frac{Z_R}{Z_R + Z_C} = \tilde{V}_{in} \frac{R}{\frac{1}{j\omega C} + R}$$

$$|H(j\omega)| = \frac{|\tilde{V}_{out}|}{|\tilde{V}_{in}|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{R^2}}{\sqrt{(\frac{1}{\omega C})^2 + R^2}}$$

$$\frac{1}{2} = \frac{R^2}{(\frac{1}{\omega C})^2 + R^2}$$

$$\left(\frac{1}{\omega C}\right)^2 + R^2 = 2R^2$$

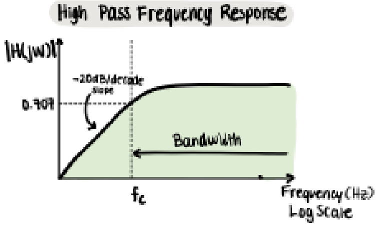
$$\left(\frac{1}{\omega C}\right)^2 = R^2$$

$\omega = \frac{1}{RC}$ angular cutoff frequency

$f_c = \frac{1}{2\pi RC}$ cutoff frequency

Conceptually: as $\omega \to \infty$, $|H(j\omega)| \to 1$
as $\omega \to 0$, $|H(j\omega)| \to 0$

High Pass Frequency Response

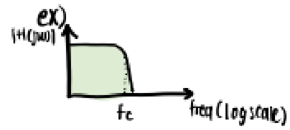


Everything higher that f_c gets through.

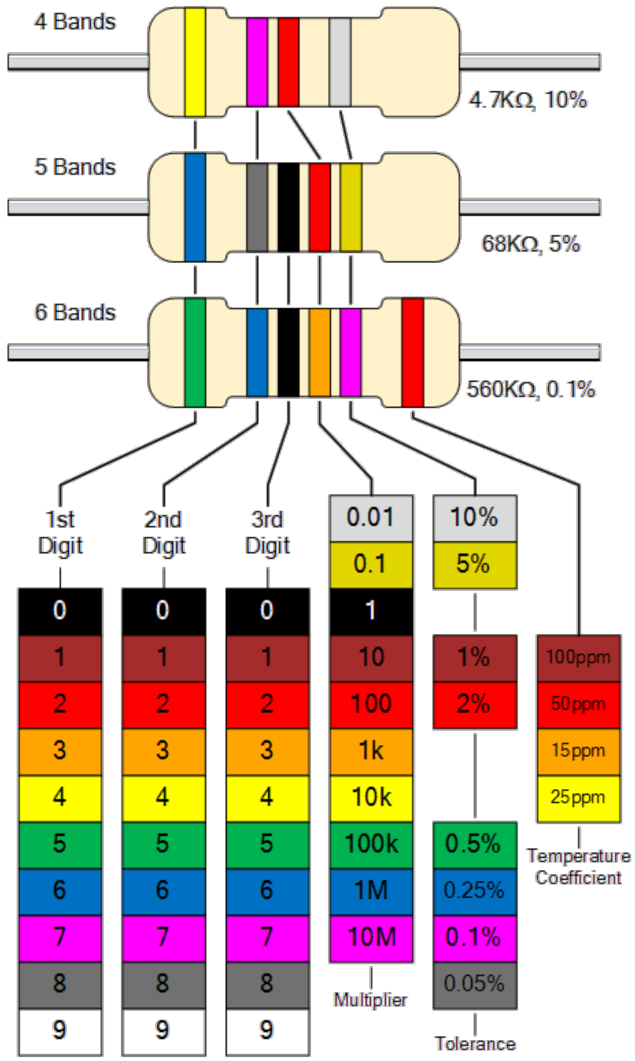
Thought: What happens to DC Voltage in a high pass filter?
↳ It gets destroyed, $\omega = 0$!

Thought: How can we make attenuation faster?
↳ multiple filters cascaded. Our transfer functions multiply, making the drop-off faster.
↳ make sure to place a unity gain buffer in between to prevent loading

ex)

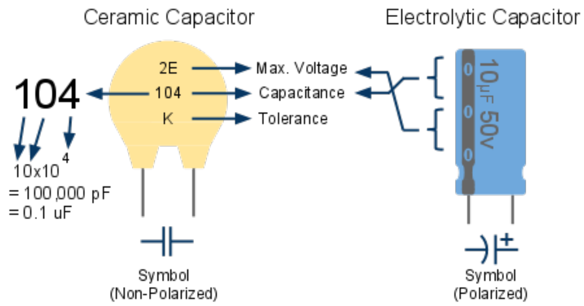


Appendix B: Resistor Color Code



Appendix C: Capacitor Codes

Capacitors



Max. Operating Voltage	
Code	**Max. Voltage**
1H	50V
2A	100V
2T	150V
2D	200V
2E	250V
2G	400V
2J	630V

Capacitance Conversion Values		
Microfarads (µF)	**Nanofarads (nF)**	**Picofarads (pF)**
0.000001 µF	0.001 nF	1 pF
0.00001 µF	0.01 nF	10 pF
0.0001 µF	0.1 nF	100 pF
0.001 µF	1 nF	1,000 pF
0.01 µF	10 nf	10,000 pF
0.1 µF	100 nF	100,000 pF
1 µF	1,000 nF	1,000,000 pF
10 µF	10,000 nF	10,000,000 pF
100 µF	100,000 nF	100,000,000 pF

Tolerance	
Code	**Percentage**
B	± 0.1 pF
C	±0.25 pF
D	±0.5 pF
F	±1%
G	±2%
H	±3%
J	±5%
K	±10%
M	±20%
Z	+80%, -20%

References

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