To do : Phasors!

1. Motivation behind phasors
2. Defining a phasor [1(a)]
3. Why the phasor domain is useful? [1(b), (d)]
4. Converting back and forth between domains [1(e), (f)]

Complex #s

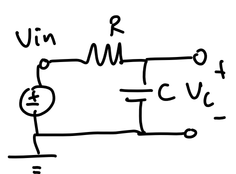
- $z = a + bj \iff z = |z| e^{j\theta}$

$\hookrightarrow |z| = \sqrt{a^2+b^2}$, $\theta = \text{atan}\,2(b, a)$

- $z_1 \cdot z_2 = |z_1| \cdot |z_2| e^{j(\theta_1+\theta_2)}$
- $\dfrac{z_1}{z_2} = \dfrac{|z_1|}{|z_2|} e^{j(\theta_1-\theta_2)}$

Phasor Motivation

Recall:



$\dfrac{dV_c(t)}{dt} = \dfrac{V_{in} - V_c}{RC} \longrightarrow$ solve via change of variables

Key Assumption: Vin is a constant

Today: What if $V_{in}(t)$ is a sinusoidal wave? e.g. $V_{in}(t) = 5\cos(2t)$

Phasor Domain: I can view the circuit in a different world

We define any time-varying $x(t)$ [$V_{in}(t)$] can be expressed as

$$\boxed{x(t) = \tilde{X} e^{j\omega t} + \bar{\tilde{X}} e^{-j\omega t}} \quad (1)$$

$\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

$\tilde{X}$: "phasor" of $x(t)$

$\underline{\text{Step 1}}$ [1(a)] $V_s(t) = 12\sin(\omega t - \frac{\pi}{4})$

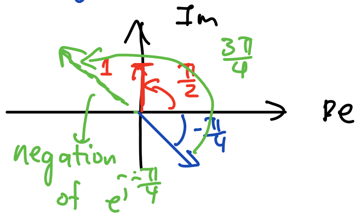
$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$\sin(\theta) = \cos(\theta - \frac{\pi}{2})$

$$V_s(t) = 12\left(\frac{e^{j(\omega t - \frac{\pi}{4})} - e^{-j(\omega t - \frac{\pi}{4})}}{2j}\right)$$

$$= 12\left(\frac{e^{j\omega t}e^{-j\frac{\pi}{4}}}{2j} - \frac{e^{-j\omega t}e^{j\frac{\pi}{4}}}{2j}\right)$$

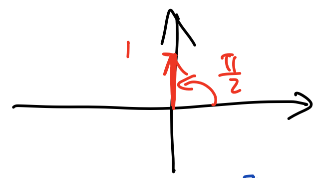
Convert j : $j = e^{j\frac{\pi}{2}}$



$$= 6e^{j\omega t}e^{-j\frac{3\pi}{4}} - 6e^{-j\omega t}e^{-j\frac{\pi}{4}}$$

$$= 6e^{-j\frac{3\pi}{4}}e^{j\omega t} + 6e^{j\frac{3\pi}{4}}e^{-j\omega t}$$

$$= \underbrace{6e^{-j\frac{3\pi}{4}}}_{\tilde{X}}e^{j\omega t} + 6e^{-j\frac{3\pi}{4}}e^{-j\omega t}$$



$\therefore$ phasor of $V_s(t) = 6e^{-j\frac{3\pi}{4}}$

$-e^{-j\frac{\pi}{4}} = ??$ + −

$\underline{\text{Alternate Way}}$: $V_s(t) = 12\sin(\omega t - \frac{\pi}{4})$

$$= 12\cos(\omega t - \frac{\pi}{4} - \frac{\pi}{2}) = 12\cos(\omega t - \frac{3\pi}{4})$$

$$= 12\left(\frac{e^{j(\omega t - \frac{3\pi}{4})} + e^{-j(\omega t - \frac{3\pi}{4})}}{2}\right)$$

$\times\frac{1}{2}$

$$= 6e^{j\omega t}e^{-j\frac{3\pi}{4}} + 6e^{-j\omega t}e^{\frac{3\pi}{4}j}$$

$\tilde{X} = 6e^{-j\frac{3\pi}{4}}$ ① polar form — magnitude
— phase

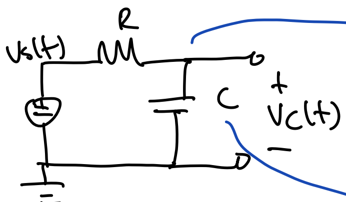
$$\boxed{V(t) = V_0\cos(\omega t + \phi) \Longleftrightarrow \tilde{V} = \frac{V_0}{2}e^{j\phi}}$$

★★★ Conversion between time domain & phasor domain

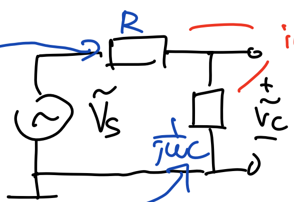
$\underline{\text{Step 2}}$: Convert circuit

KCL, KVL, NVA holds in the phasor domain!

Time



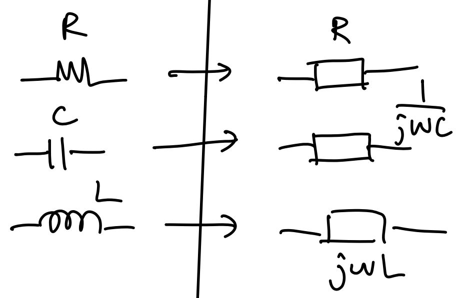
Phasor



impedance!
→ what is the "resistor" value of the component
→ everything here is treated like

a resistor

→ problem becomes voltage divider!



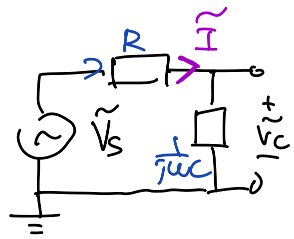
(b) $Z_R = R = \sqrt{3}\cdot 10^3\,\Omega$

$Z_C = \frac{1}{j\omega C}$

$= \frac{1}{j\cdot 10^3\cdot 10^{-6}} = -10^3 j$

$\therefore\ \tilde{V}_C = \left(\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}\right)\cdot \tilde{V}_s$

(d)



$\tilde{I} = \frac{\tilde{V}_s}{Z_R + Z_C} = \frac{\tilde{V}_s}{\sqrt{3}\cdot 10^3 - 10^3 j} = \frac{6e^{-j\frac{3\pi}{4}}}{\sqrt{3}\cdot 10^3 - 10^3 j}$

(e), (f) Convert our "solution" back to time domain

$$\boxed{V(t) = V_0\cos(\omega t + \phi) \iff \tilde{V} = \frac{V_0}{2} e^{j\phi}}$$

Complex #s

- $z = a + bj \iff z = |z| e^{j\theta}$

↳ $|z| = \sqrt{a^2 + b^2}$, $\theta = \text{atan}\,2(b, a)$

- $z_1 \cdot z_2 = |z_1|\cdot|z_2|\, e^{j(\theta_1 + \theta_2)}$
- $\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|} e^{j(\theta_1 - \theta_2)}$

$z_1 = |z_1| e^{j\theta_1}$

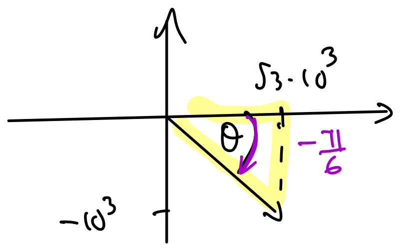
$z_2 = |z_2| e^{j\theta_2}$

$\tilde{I} = \frac{6e^{-j\frac{3\pi}{4}}}{\sqrt{3}\cdot 10^3 - 10^3 j}$ from (d)

$$\therefore \ |\tilde{I}| = \left|\frac{6e^{-j\frac{3\pi}{4}}}{\sqrt{3}\cdot 10^3 - 10^3 j}\right| = \frac{|6e^{-j\frac{3\pi}{4}}|}{|\sqrt{3}\cdot 10^3 - 10^3 j|} = \frac{6}{\sqrt{(\sqrt{3}\cdot 10^3)^2 + (10^3)^2}} = (3\times 10^{-3})$$

$$\therefore \ \measuredangle \tilde{I} = \measuredangle 6e^{-j\frac{3\pi}{4}} - \measuredangle(\sqrt{3}\cdot 10^3 - 10^3 j)$$

$$= -\frac{3\pi}{4} - \left(-\frac{\pi}{6} \right) = -\frac{7\pi}{12}$$



$$\tan\theta = \frac{\cancel{10^3}}{\sqrt{3}\cdot \cancel{10^3}}$$

$$\theta = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$\tilde{I} = (3\times 10^{-3})\, e^{j\left(-\frac{7\pi}{12}\right)}$$

$$\boxed{V(t) = V_0 \cos(\omega t + \phi) \Longleftrightarrow \tilde{V} = \frac{V_0}{2} e^{j\phi}}$$

$$\updownarrow$$

$$I(t) = 2\cdot(3\times 10^{-3}) \cos\left(\omega t - \frac{7\pi}{12}\right)$$

<u>Extra time</u>

$$\therefore \ \tilde{V_C} = \left(\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}\right)\cdot \tilde{V_S}$$

$$\tilde{V_C} = \frac{1}{1 + j\omega RC}\cdot \tilde{V_S}$$

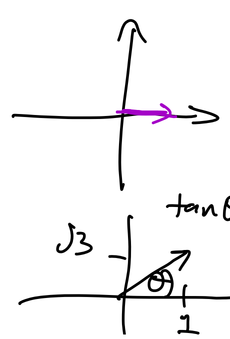
$$= \frac{1}{1+\sqrt{3}j}\tilde{V_S} = \underline{\frac{1}{1+\sqrt{3}j}}\cdot 6e^{-j\frac{3\pi}{4}}$$

convert to polar form

$$\cdot \ \left|\frac{1}{1+\sqrt{3}j}\right| = \frac{|1|}{|1+\sqrt{3}j|} = \frac{1}{2}$$

$$\cdot \ \measuredangle\left(\frac{1}{1+\sqrt{3}j}\right) = \measuredangle 1 - \measuredangle(1+\sqrt{3}j)$$

$$= 0 - \frac{\pi}{3} = -\frac{\pi}{3}$$



$$\tan\theta = \frac{\sqrt{3}}{1}$$

$$\theta = \frac{\pi}{3}$$

$$\tilde{V}_c = \frac{1}{2} e^{j\left(-\frac{\pi}{3}\right)} \cdot 6 e^{-j\frac{3\pi}{4}}$$

$$= 3 e^{-j\frac{13\pi}{12}}$$

$$\boxed{v(t) = V_0 \cos(\omega t + \phi) \Longleftrightarrow \tilde{V} = \frac{V_0}{2} e^{j\phi}}$$

$$\downarrow$$

$$V_c(t) = 6 \cos\left(\omega t - \frac{13\pi}{12}\right)$$

$$\frac{z_1}{z_2} = \frac{|z_1| e^{j\theta_1}}{|z_2| e^{j\theta_2}} = \frac{|z_1|}{|z_2|} e^{j\underline{(\theta_1 - \theta_2)}}$$