Discussion 1A

For this discussion, Note 1 is helpful for background in transistors and RC circuits.

1. **NAND Circuit**

Let us consider a NAND logic gate. This circuit implements the boolean function $\overline{(A \cdot B)}$. The $\cdot$ stands for the AND operation, and the $\overline{\quad}$ stands for NOT; combining them, we get NAND!

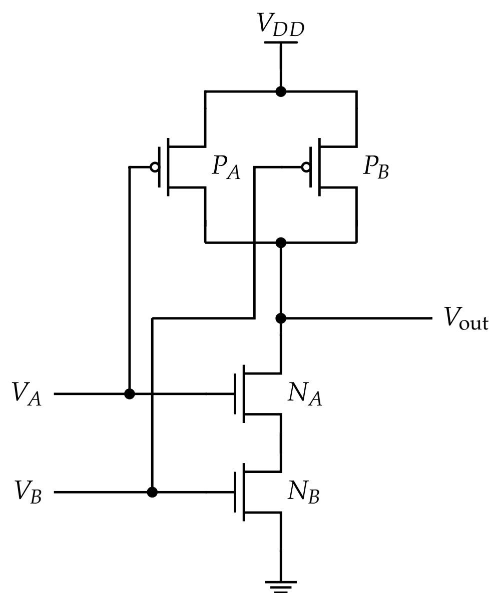

Figure 1: NAND gate transistor-level implementation.

V_{tn} and V_{tp} are the threshold voltages for the NMOS and PMOS transistors, respectively. Assume that $V_{DD} > V_{tn}, |V_{tp}| > 0$.

(a) **Label the gate, source, and drain nodes for the NMOS and PMOS transistors above.**

Solution: As a convention throughout the course, we will draw NMOS transistors with their source at the bottom (and drain at the top). On the other hand, PMOS transistors will have their source at the top. Therefore, the drains are at the top of N_A (connected to V_{out}) and the top of N_B (connected to N_A). The sources are at the bottom of N_A (connected to N_B) and the bottom of N_B (connected to ground). The gate terminal of N_A is connected to V_A; the gate of N_B is connected to V_B.

For the PMOS transistors, the source is at the top of P_A and P_B (connected to V_{DD}). The drain is at the bottom of P_A and P_B (connected to V_{out}). The gate terminal of P_A is connected to V_A; the gate of P_B is connected to V_B.

(b) If $V_A = V_{DD}$ and $V_B = V_{DD}$, **which transistors act like open switches? Which transistors act like closed switches? What is V_{out}?**

Solution: P_A and P_B are off (open switches). N_B and N_A are on (closed switches). $V_{\text{out}} = 0\text{V}$ because it is connected to ground through a closed circuit consisting of N_A and N_B (and detached from V_{DD}).

(c) **If $V_A = 0V$ and $V_B = V_{DD}$, what is V_{out}?**

Solution: P_B and N_A are off (open switches). P_A and N_B are on (closed switches). $V_{\text{out}} = V_{DD}$ because it is connected to V_{DD} through a closed circuit consisting of P_A (and detached from ground, since *both* N_A and N_B must be closed for V_{out} to be connected to ground).

(d) **If $V_A = V_{DD}$ and $V_B = 0V$, what is V_{out}?**

Solution: P_A and N_B are off (open switches), P_B is on (closed switch). So, $V_{\text{out}} = V_{DD}$ because it is connected to V_{DD} through a closed switch.

(e) **If $V_A = 0V$ and $V_B = 0V$, what is V_{out}?**

Solution: N_B is off, creating an open circuit. P_A and P_B are on, creating a closed circuit. $V_{\text{out}} = V_{DD}$ because it is connected by closed circuit to V_{DD}.

(f) **Write out the truth table for this circuit.**

V_A	V_B	V_{out}
0	0	
0	V_{DD}	
V_{DD}	0	
V_{DD}	V_{DD}	

Solution:

V_A	V_B	V_{out}
0	0	V_{DD}
0	V_{DD}	V_{DD}
V_{DD}	0	V_{DD}
V_{DD}	V_{DD}	0

2. RC Circuits - Part I

In this problem, we will find the voltage across a capacitor over time in an RC circuit. We set up our problem by first defining four functions over time: $I(t)$ is the current at time t, $V(t)$ is the voltage across the circuit at time t, $V_R(t)$ is the voltage across the resistor at time t, and $V_C(t)$ is the voltage across the capacitor at time t.

Recall from 16A that the voltage across a resistor is defined as $V_R = RI_R$ where I_R is the current across the resistor, and the voltage across a capacitor is defined as $V_C = \frac{Q}{C}$ where Q is the charge across the capacitor.

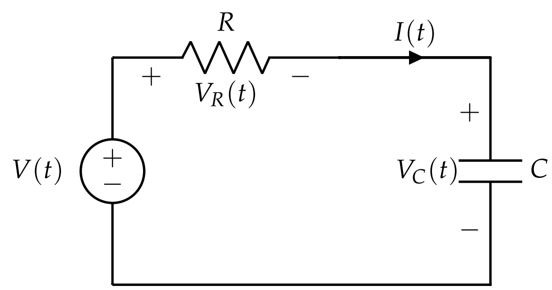

Figure 2: Example Circuit

(a) Starting from the given charge-voltage relation for a capacitor, **find an equation that relates the current across the capacitor $I(t)$ with the voltage across the capacitor $V_C(t)$.**

Solution: As noted in the problem statement, we start from the Q-V relationship of the capacitor:

$$Q(t) = CV_C(t) \tag{1}$$

Differentiating $V_C(t) = \frac{Q(t)}{C}$ in terms of t, we get

$$\frac{\mathrm{d}V_C(t)}{\mathrm{d}t} = \frac{\mathrm{d}Q(t)}{\mathrm{d}t}\frac{1}{C}. \tag{2}$$

By definition, the change in charge is the current across the capacitor, so

$$\frac{\mathrm{d}V_C(t)}{\mathrm{d}t} = I(t)\frac{1}{C} \tag{3}$$

(b) Analyzing the circuit, **write an equation that relates the functions $I(t)$, $V_C(t)$, and $V(t)$.**

Solution: From KCL, we have

$$\frac{V(t) - V_C(t)}{R} - I(t) = 0 \tag{4}$$

$$RI(t) + V_C(t) = V(t) \tag{5}$$

(c) So far, we have an equation that involves both $I(t)$ and $V_C(t)$. To solve this equation, we can remove $I(t)$ (one of the unknowns) using what we found in part 2.a. **Rewrite the previous equation in part 2.b in the form of a differential equation.** You will pick up with this in the next discussion.

Solution: From part (a), we have

$$I(t) = \frac{\mathrm{d}V_C(t)}{\mathrm{d}t}C \tag{6}$$

Substituting this into eq. (5) gives us

$$RC\frac{\mathrm{d}V_C(t)}{\mathrm{d}t} + V_C(t) = V(t) \tag{7}$$

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