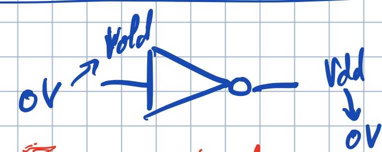
Lecture 3 EECS 16B

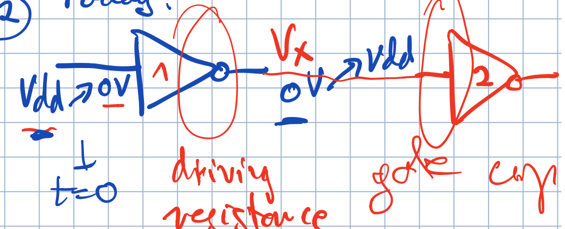
* RC Transients
- nonhomogenous diff eqns (with piecewise & const. input)

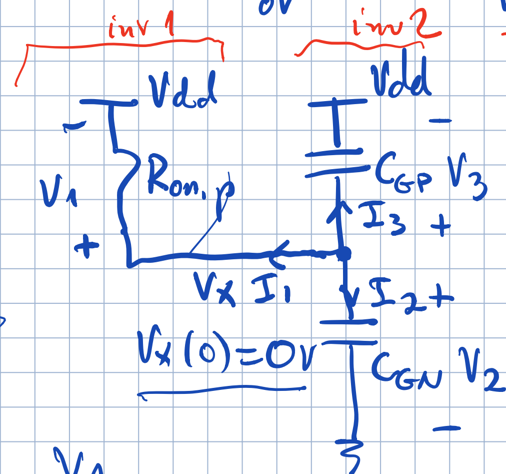
* Intro to filters

① Lecture 2



② Today:





KCL: $I_1 + I_2 + I_3 = 0$

$$\Downarrow$$

$$\frac{V_1}{R_{on,p}} + C_{GN} \cdot \frac{dV_2}{dt} + C_{GP} \frac{dV_3}{dt} = 0$$

$$\frac{\overbrace{V_x - V_{dd}}^{V_1}}{R_{on,p}} + C_{G_n} \cdot \frac{d\overset{V_2}{\overset{\parallel}{V_x}}}{dt} + C_{GP} \cdot \frac{d(V_x - V_{dd})}{dt} = 0$$

$$\frac{V_x - V_{dd}}{R_{on,p}} + (C_{Gn} + C_{Gp}) \cdot \frac{dV_x}{dt} = 0 \qquad (1)$$

$$\frac{dV_x}{dt} = \underbrace{-\frac{V_x}{R_{on,p} \cdot (C_{Gn} + C_{Gp})}}_{\text{homogeneous diff.-eq.}} + \underbrace{\frac{V_{dd}}{R_{on,p} \cdot (C_{Gn} + C_{Gp})}}_{\text{non-homogeneous.}}$$

0

Form:

$$\frac{d}{dt}x(t) = \lambda x(t) + u$$

Non-homogeneous

$$\frac{d}{dt}x(t) = \lambda x(t) \quad \text{(homogeneous)}$$

From (1)

$$\frac{V_x - V_{dd}}{R_{on,p}} + (C_{Gn} + C_{Gp}) \cdot \frac{d}{dt}(V_x - V_{dd}) = 0$$

$$\hat{v}_x = V_x - V_{dd},$$

$$\frac{\hat{v}_x}{R_{on,p}} + (C_{Gn} + C_{Gp}) \cdot \frac{d\hat{v}_x}{dt} = 0$$

$$\frac{d\hat{v}_x}{dt} = -\frac{\hat{v}_x}{R_{on,p}(C_{Gn} + C_{Gp})}$$

(homogeneous diff. eq. from lecture 2)

$$\tau = R_{on,p} \cdot (C_{Gn} + C_{Gp})$$

$$\frac{d\hat{v}_x}{dt} = -\frac{\hat{v}_x}{\tau}$$

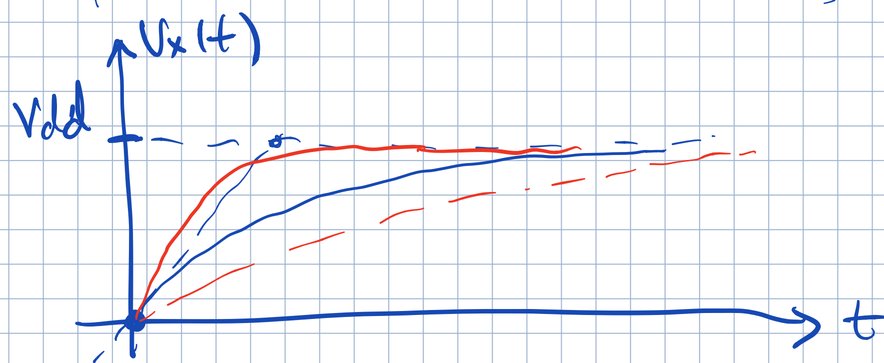
$$\hat{v}_x(t) = \hat{v}_x(0) \cdot e^{-\frac{t}{\tau}}$$

$$V_x(t) - V_{dd} = (\cancel{V_x(0)}^{0} - V_{dd}) \cdot e^{-\frac{t}{\tau}}$$

$$U_x(0) = 0V$$

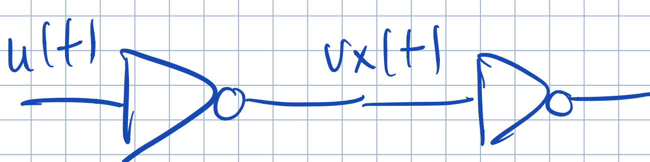
$$U_x(t) - V_{dd} = -V_{dd}\, e^{-\frac{t}{\tau}}$$

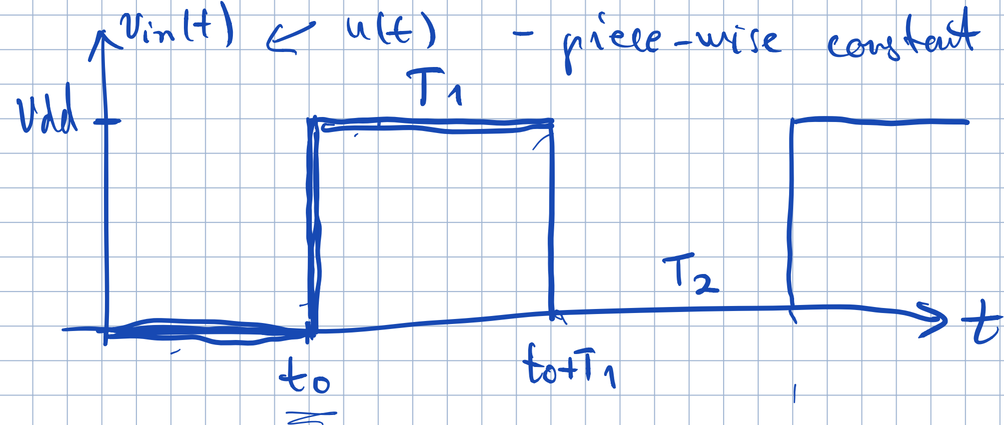
$$\boxed{U_x(t) = V_{dd}\left(1 - e^{-\frac{t}{\tau}}\right)}, \quad t \geqslant 0$$

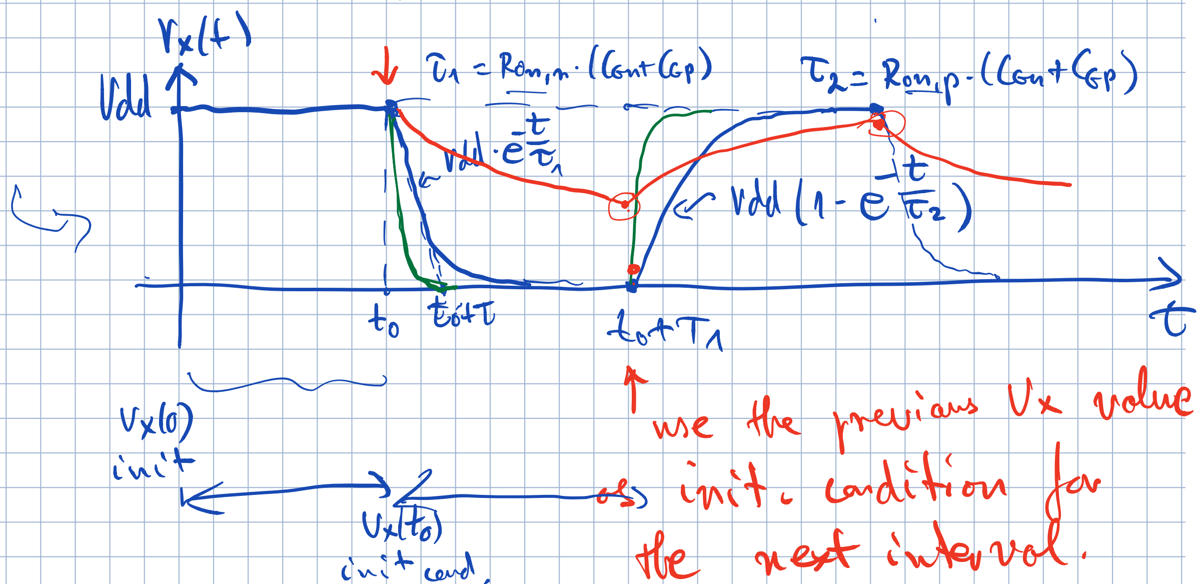


$$\frac{d}{dt}\left(U_x(t) - V_{dd}\right) = \frac{d}{dt}\left(U_x(t)\right) - \cancel{\frac{d}{dt}\left(V_{dd}\right)}^{\,0} =$$

$$= \underbrace{\frac{d}{dt}\left(U_x(t)\right)}$$



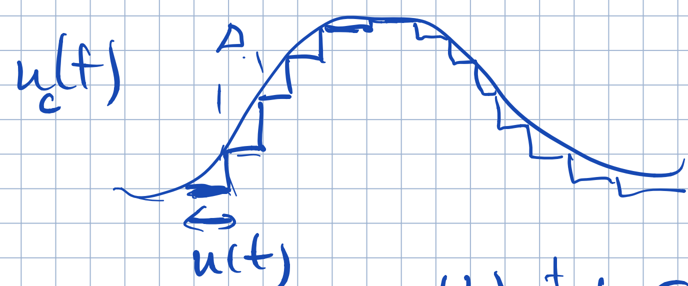




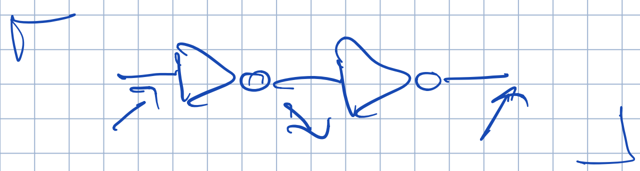
use the previous V_x value as init. condition for the next interval.

$$\frac{d}{dt} x(t) = \lambda x(t) + u(t) \Leftarrow$$ piece wise constant

then use the solution to solve for continuous $u(t)$ → will use limits



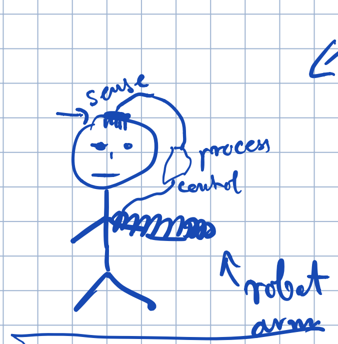
$$u(t) = c_i \cdot t \in [t_0, t_0+\delta]$$

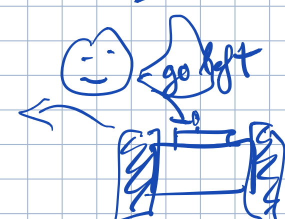


$$\underbrace{\tau_1 < \tau_1 < \tau_1}$$

16A/B Pipeline

Analog world → Sense → Process → Control/Actuate → Analog World





voice-controlled car

Module 1: Get signals from the brain (sensing)

Module 2: Controling the arm (control/actuation)

Module 3: Figure out the intention from brain (processing)

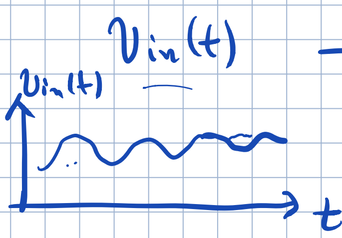
Sensor electrodes pick-up the

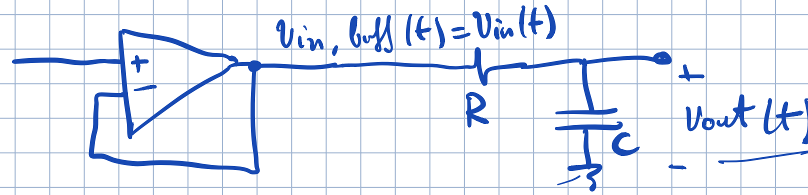
brain signal + other unwanted signals

$\underbrace{\text{☺}}_{\checkmark}$ $\underbrace{\text{☹ (AC power, RF, FM radio, (wifi/cellular))}}_{\otimes}$

Goal: Filter the electrical signals (with circuits)

How can a circuit process/filter the signal?





"buffer" or unity-gain amplifier

$$V_{in}(t) = V_{out}(t) + RC\frac{dV_{out}(t)}{dt}$$

$$\frac{dV_{out}(t)}{dt} = -\frac{V_{out}(t)}{RC} + \frac{V_{in}(t)}{RC}$$

"λ"

$$\left[\frac{d}{dt}x(t) = \lambda x(t) + u(t)\right]$$

will see in discussion:

$$V_{out}(t) = \underbrace{V_{in}(0)\,e^{-\frac{t}{RC}}}_{\text{homogeneous solution}} + \underbrace{\frac{1}{RC}\int_0^t v_{in}(\theta)\,e^{-\frac{1}{RC}(t-\theta)}\,d\theta}_{\text{response to input in time}}$$

The circuit "computes" this

Try different examples for Vin(t) to see what the circuit does.

Example 1: $V_{in}(t) = e^{st}$

(1) $$\frac{d}{dt}x(t) = \lambda x + u(t) \qquad \boxed{u(t) = e^{st}} \quad t \geq 0 \quad s \neq \lambda$$

can solve: $$x(t) = x_0 e^{\lambda t} + \int_0^t e^{s\theta} \cdot e^{\lambda(t-\theta)} d\theta \leftarrow$$

Guess: $x(t) = K e^{st}$

From (1)

$$Ks \cancel{e^{st}} = \lambda K \cancel{e^{st}} + \cancel{e^{st}}$$

$$Ks = \lambda K + 1$$

$$\boxed{K = \frac{1}{s-\lambda}} \Rightarrow x(t) = \frac{e^{st}}{s-\lambda}$$

$$x(t) = \frac{e^{st}}{s-\lambda} + \underbrace{K_2 \cdot e^{\lambda t}}_{\text{homogeneous solution}}$$

(or solution to no input)

Example 2: $u(t) = \cos(\omega t)$ - mimics AC power interference