Phasors, Filters, Bode Plots

16B Review Session

12-1 PM

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Phasors [not a core topic by itself; unlocks filters + circuit analysis/design]

Motivation:

→ often, $V_{in}(t) = A\cos(\omega t + \phi)$ for circuits [sinusoidal inputs]

→ linear circuits (RLC): frequencies don't change.

→ use phasor (tilde, ~) to analyze circuit; retain non-freq info. [amplitude, phase] ⟶ Bode Plots: Magnitude, Phase

DON'T BE SCARED OF COMPLEX NUMBERS!
→ they make computations a lot easier!

Formulas:

Time Domain ⟶ Phasor Domain: $\boxed{\underbrace{A\cos(\omega t + \phi)}_{V_{in}(t)} \iff \underbrace{\frac{A}{2}e^{j\phi}}_{\tilde{V}_{in}}}$

not changing!

$$\left(A\cos(\omega t+\phi) = \frac{A}{2}\left(e^{j(\omega t+\phi)} + e^{-j(\omega t+\phi)}\right) = \left(\underbrace{\frac{A}{2}e^{j\phi}}_{\text{phasor}} e^{j\omega t} + \underbrace{\frac{A}{2}e^{-j\phi}e^{-j\omega t}}_{\text{complex conjugate}}\right)\right)$$

Phasor Domain ⟶ Time Domain: $\boxed{Be^{j\psi} \iff 2B\cos(\omega t + \psi)}$

Impedances

R:	$Z_R = R$
L:	$Z_L = j\omega L$
C:	$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

Euler's Formulas

$$\sin(x) = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$$

$$\cos(x) = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$$

References:

→ Note 4: Sections 5/6/7

→ Lecture 4B: page 9 onwards

→ Lecture 5A

Problems:

→ Midterm 1, Q3b)

→ HW 6 Q3, 5, 6 (5, 6: extends beyond phasors.)

→ Dis 5B

$$e^{j\pi/2} = j$$

Examples

1) $V_{in}(t) = \cos(t) \implies \tilde{V}_{in} = 0.5$

2) $V_{in}(t) = 4\cos(5t + \pi/3) \implies \tilde{V}_{in} = 2e^{j\pi/3}$

3) $V_{in}(t) = 4\cos(20t + \pi/3) \implies \tilde{V}_{in} = 2e^{j\pi/3}$

} need to remember ω! plug into circuits component impedances.

4) $V_{in}(t) = 3\sin(6t - \pi/6) = 3\cos(6t + 4\pi/3) \implies \tilde{V}_{in} = \frac{3}{2}e^{j4\pi/3}$

5) $V_{in}(t) = 9\cos(\pi/3) \implies \tilde{V}_{in} = \frac{9}{2}e^{j\pi/3}$

Dis 6B notes

Filters

Motivation:

→ signals exist in real world: not "pure sinusoids" (contain noise)

→ want to cancel noise but keep signal!

Considerations:

→ is signal freq. > or < noise freq?

→ what components are available?

RC ⟺ LR but maybe no inductors in lab!

→ cutoff frequency: what frequency do we begin to attenuate (decrease amplitude) by more than $1/\sqrt{2}$?

Motivation

→ see recording for Interactive Matlab demo

→ Analog signals aren't clean! noisy.

→ Filtering: keep what we want, attenuate what we don't.

→ Later courses:

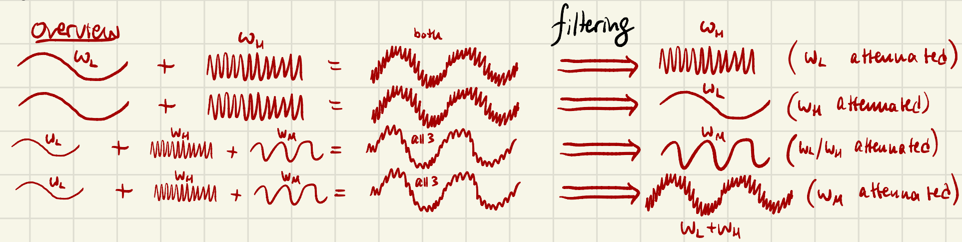
$x[n] \rightarrow \boxed{H(\omega)} \rightarrow y[n]$

↳ some system! Maybe a circuit, could be something else.

→ concept of a transfer function (that depends on frequency) is general.

Overview / Categories

Type	Topology
High-Pass	CR, RL
Low-Pass	RC, LR
Band-Pass	RLC
Band-Stop	RLC



KEY Formula: if circuits TF is

Dis 6B notes

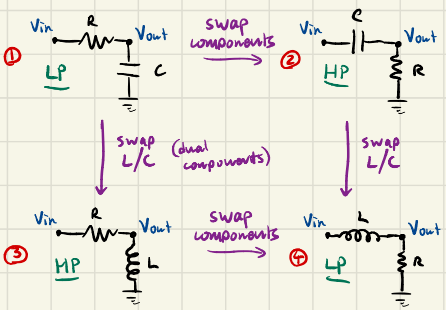
$$H(\omega) = \underbrace{|H(\omega)|}_{\text{magnitude}} e^{j\overbrace{\measuredangle H(\omega)}^{\text{phase}}}$$

$$\boxed{V_{in}(t) = A\cos(\omega_s t + \phi) \xRightarrow[\text{filtering}]{\text{circuit}} A \cdot |H(\omega_s)| \cos(\omega_s t + \phi + \measuredangle H(\omega_s)) = V_{out}(t)}$$

→ Midterm 1 (Q3c)

Remember: RC is low-pass (all others follow)

→ swap components OR swap L ↔ C = LP ⟺ HP



References

→ Note 5

→ Lectures 5A/5B/6A

Problems:

→ Midterm 1 Q4/8

→ Dis 6A/6B

→ HW 6, Q7 [HW 5 is entirely about RLC variants]

→ HW 7, Q 2/3/4

→ Note 5: worked example at start / Note 4: worked example at end

Bode Plots

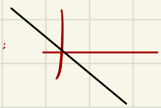
Motivation:

- → exact transfer functions : need graphing calc to plot
- → want an approximation method
- → $\boxed{\text{Bode plots summarize effect of a circuit on input at any frequency.}}$
- → given input signal, use bode plot to near-instantly get output signal (MT1 Q3c)

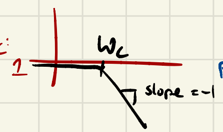
Main Concepts

→ $\boxed{\text{Bode Plot Summary : Note 6}}$

pole at $w=0$:

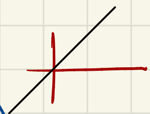


pole at w_c:

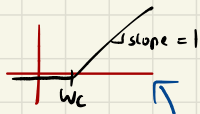


gain K

zero at $w=0$



zero at w_c:



N_{z_0} zeros at origin

$$H(jw) = K \frac{(jw)^{N_{z_0}}}{(jw)^{N_{p_0}}} \frac{\left(1+j\frac{w}{w_{z1}}\right) \cdots \left(1+j\frac{w}{w_{zn}}\right)}{\left(1+j\frac{w}{w_{p1}}\right) \cdots \left(1+j\frac{w}{w_{pm}}\right)} \quad \begin{matrix} \}\, n \text{ "simple" zeros} \\ \}\, m \text{ "simple" poles} \end{matrix}$$

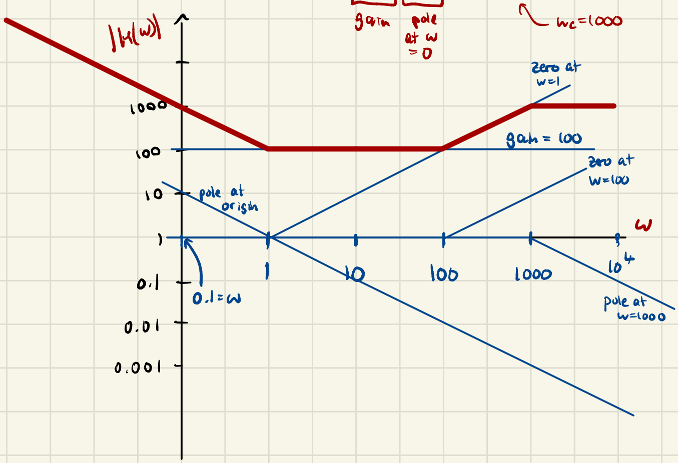
gain

N_{p_0} poles at origin

Example : $H(w) = \underbrace{100}_{\text{gain}} \underbrace{\frac{1}{jw}}_{\substack{\text{pole}\\ \text{at } w=0}} \cdot \frac{\left(1+j\frac{w}{100}\right)\left(1+j\frac{w}{1}\right)}{\left(1+j\frac{w}{1000}\right)} \begin{matrix} \}\, 2 \text{ zeros} \\ \}\, 1 \text{ pole} \end{matrix}$

$w_c=100$ $w_c=1$

$w_c=1000$



References

- → Note 6
- → Lecture 5B/6A

Problems:

- → Midterm 1, Q3c / 8
- → Dis 6A/6B/9A (Q1)
- → HW 6/7

HW 6, Problem 6 [like Note 5, first example; Note 4, last example]

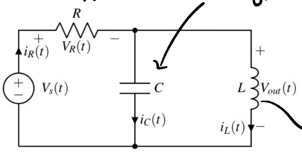
6. Phasor-Domain Circuit Analysis

The analysis techniques you learned previously in 16A for resistive circuits are equally applicable for analyzing circuits driven by sinusoidal inputs in the phasor domain. In this problem, we will walk you through the steps with a concrete example.

Consider the following circuit where the input voltage is sinusoidal. The end goal of our analysis is to find an equation for $V_{out}(t)$.

$Z_R = 5$

$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 100 \cdot 2e\text{-}3} = \frac{1}{5j}$



$Z_L = j\omega L$

$= j \cdot 100 \cdot 50 \times 10^{-3}$

$= 5j$

The components in this circuit are given by:

$$V_s(t) = 10\sqrt{2}\cos\left(100t - \frac{\pi}{4}\right)$$

$$R = 5\,\Omega$$

$$L = 50\,\text{mH}$$

$$C = 2\,\text{mF}$$

(a) **Give the amplitude V_0, oscillation frequency ω, and phase ϕ of the input voltage V_s.**

(b) Transform the circuit into the phasor domain. **What are the impedances of the resistor, capacitor, and inductor? What is the phasor $\tilde{V}_S$ of the input voltage $V_s(t)$?**

(c) Use the circuit equations to **solve for $\tilde{V}_{out}$**, the phasor representing the output voltage.

(d) **Convert the phasor $\tilde{V}_{out}$ back to get the time-domain signal $V_{out}(t)$.**

a) $V_s(t) = \underbrace{10\sqrt{2}}_{V_0 = 10\sqrt{2}} \cos(\underbrace{100}_{\omega = 100}t \underbrace{- \pi/4}_{\phi = -\pi/4})$

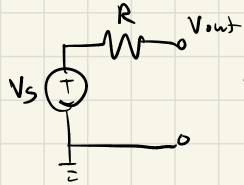
b) ✓

$\tilde{V}_S = ?$

$= \frac{10\sqrt{2}}{2} e^{j \cdot -\pi/4}$

$= 5\sqrt{2}e^{-j\pi/4}$

c)



$\Rightarrow \tilde{V}_{out} = \tilde{V}_s$

$V_{out}(t) = V_s(t)$

c) alt) $\frac{Z_C \| Z_L}{Z_C \| Z_L + Z_R} = \frac{\frac{1}{j\omega C} \| j\omega L}{\frac{1}{j\omega C} \| j\omega L + R} \Rightarrow \frac{j\omega L}{R - \omega^2 LC + j\omega L}$

$H(\omega) = 1$ @ $\omega = 100$

$V_{out}(t) = \underset{\uparrow\, V_{in}(t)}{A} \underbrace{|H(\omega)|}_{1} \cos(100t - \pi/4 + \underbrace{\angle H(\omega)}_{0})$

$\boxed{= 10\sqrt{2}\cos(100t - \pi/4)}$