Agenda

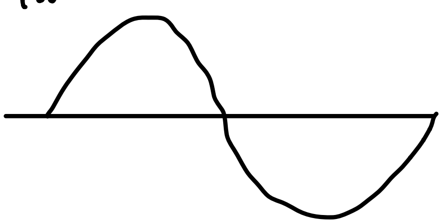
1. Mini-Review (Recorded)
2. Problems in Breakouts
3. Q + A (10 minutes)

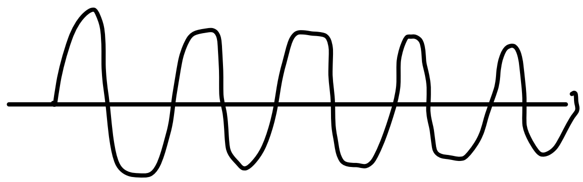
The DFT

How fast does a signal change

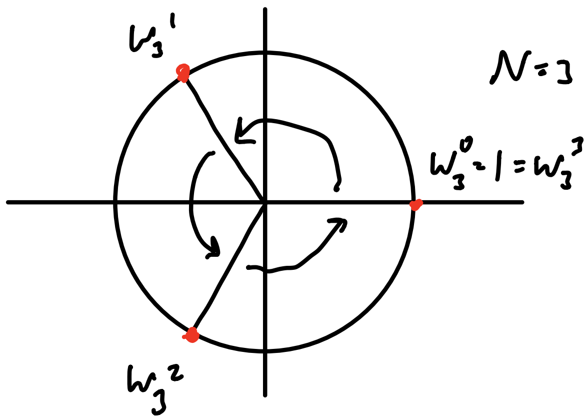
$\sin(\omega x)$ $\quad \cos(\omega x)$ $\qquad A e^{j\phi}$

low ω





$$W_N^K = e^{j\frac{2\pi}{N}K}$$



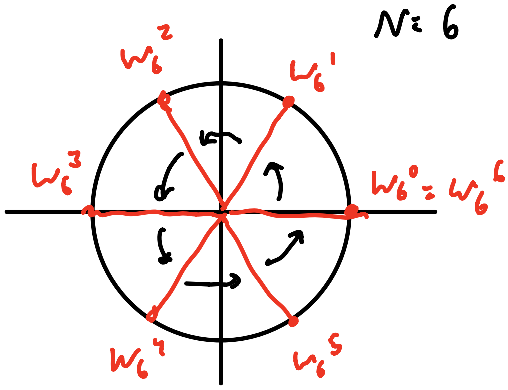
$$W_N^N = 1$$

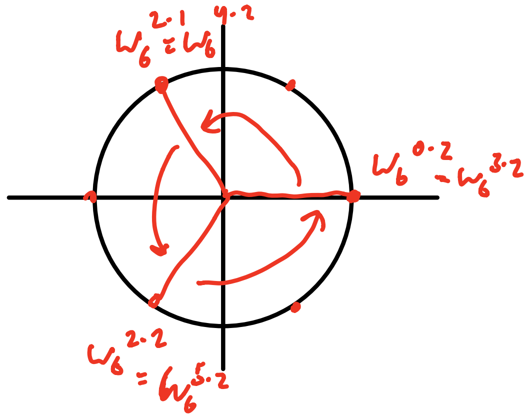
$$z^n = 1 \equiv z^n - 1 = 0$$

Properties of Roots of Unity

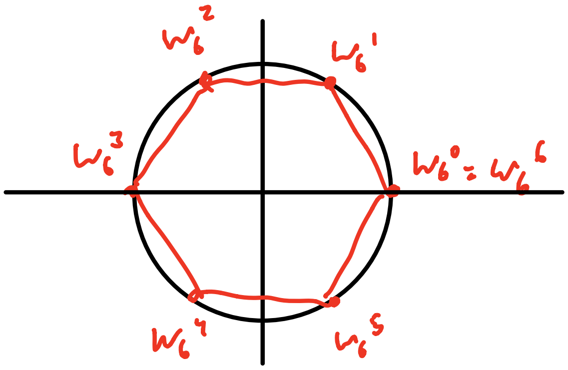
1. periodicity ⟶ $W_N^K = W_N^{K+N}$
2. $$\sum_{K=0}^{N-1} W_N^K = 0$$
3. Complex conjugacy : $W_N^K = \overline{W_N^{-K}}$

$N = 6$

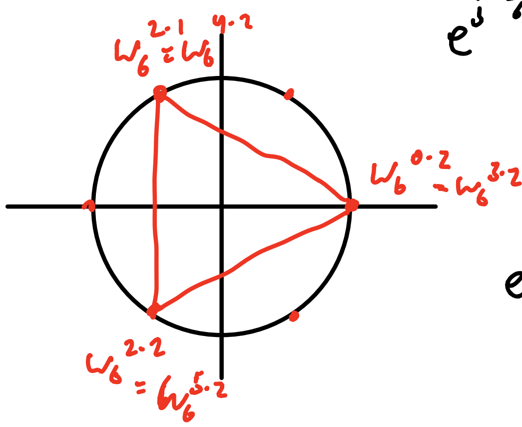




$$u_k = \left[W_N^{k\cdot 0} \quad W_N^{k\cdot 1} \quad W_N^{k\cdot 2} \quad \cdots \quad W_N^{k\cdot (N-1)} \right]^T$$



u_1



u_2

$e^{j\frac{2\pi}{6}\cdot 2\cdot 1} = e^{j\frac{2\pi}{3}}$

$e^{j\frac{2\pi}{6}\cdot 2\cdot 2} = e^{j\frac{4\pi}{3}}$

$e^{j\frac{2\pi}{6}\cdot 3\cdot 2} = e^{j2\pi}$

Properties of DFT basis vectors

1. periodicity $u_k = u_{k+N}$
2. $\|u_k\| = \sqrt{N}$
3. $u_k = \overline{u_{N-k}}$
4. $\langle u_k, u_i \rangle = 0$ if $k \neq i$

How to determine how much 1 basis vectors explain x

$$\langle x, \tfrac{1}{\sqrt{N}} \cdot u_k \rangle = \frac{1}{\sqrt{N}} u_k^* x$$

$$\frac{1}{\sqrt{N}} \begin{bmatrix} - & u_0^* & - \\ - & u_1^* & - \\ & \vdots & \\ - & u_{N-1}^* & - \end{bmatrix} x = X$$

$$0 \rightarrow \frac{2\pi}{N}(N-1)$$

1. It is a change of basis
2. Understand what the basis vectors represent
3. Be comfortable with the roots of unity