EECS 16B Designing Information Devices and Systems II
Spring 2021 Discussion Worksheet Discussion 11B

1. Understanding the SVD

We can compute the SVD for a wide matrix A with dimension $m \times n$ where $n > m$ using $A^\top A$ with the method covered in lecture. However, when doing so, you may realize that $A^\top A$ is much larger than $AA^\top$ for such wide matrices. This makes it more efficient to find the eigenvalues for $AA^\top$. In this question, we will explore how to compute the SVD using $AA^\top$ instead of $A^\top A$.

(a) What are the dimensions of $AA^\top$ and $A^\top A$?

(b) Given that the SVD of A is $A = U\Sigma V^\top$, find a symbolic expression for $AA^\top$.

(c) Using the solution to the previous part, how can we find U and Σ from $AA^\top$?

(d) Now that we have found the singular values σ_i and the corresponding vectors $\vec{u}_i$ in the matrix U, can you find the corresponding vectors $\vec{v}_i$ in V?

(e) Now we have a way to find the vectors $\vec{v}_i$ in matrix V! Verify that these vectors are orthonormal.

(f) Now that we have found $\vec{v}_i$, you may notice that we only have $m < n$ vectors of dimension n. This is not enough for a basis. How would you complete the m vectors to form an orthonormal basis?

(g) (Practice.) Given that $A = U\Sigma V^\top$ verify that the vectors you found to extend the $\vec{v}_i$ into a basis are in the nullspace of A.

(h) Using the previous parts of this question and what you learned from lecture, write out a procedure on how to find the SVD for *any* matrix.

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