This homework is due on Thursday, December 3, 2020, at 10:59PM.
Self-grades are due on Thursday, December 10, 2020, at 10:59PM.

1 Sampling Theorem

Consider the following signal, $x(t)$ defined as,

$$x(t) = \cos(2\pi t) + \sin(4\pi t)$$

a) Find the maximum frequency, ω_{max}, of $x(t)$ in radians per second.

b) If I sample every T seconds, what is the sampling frequency in radians per second?

c) What is the smallest sampling period T that may result in an imperfect reconstruction?

2 Aliasing

Watch the following video: `https://www.youtube.com/watch?v=jQDjJRYmeWg`.

Assume the video camera running at 30 frames per second. That is to say, the camera takes 30 photos within a second, with the time between photos being constant.

a) Given that the main rotor has 5 blades, list *all* the possible rates at which the main rotor is spinning in revolutions per second assuming no physical limitations.
Hint: Your answer should depend on k where k can be any integer.

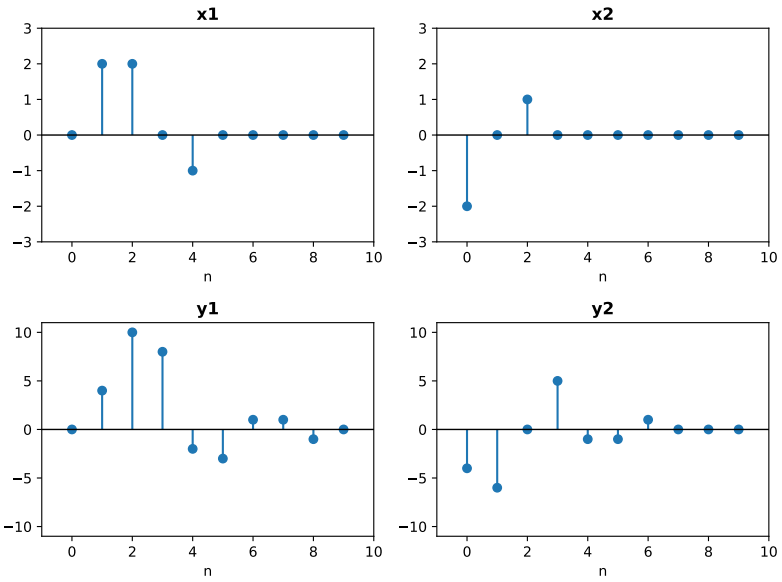
b) Given that the back rotor has 3 blades and completes 2 revolutions in 1 second **in the video**, list *all* the possible rates at which the back rotor is spinning in revolutions per second assuming no physical limitations.
Hint: Your answer should depend on k where k can be any integer.

3 LTI Inputs

We have an LTI system whose exact characteristics we do not know. However, we know that it has a finite impulse response that is not longer than 5 samples. We also observed two sequences, x_1 and x_2, pass through the system and observed the system's responses y_1 and y_2.

x_1	0	2	2	0	−1	0	0	0	0	0
y_1	0	4	10	8	−2	−3	1	1	−1	0

x_2	−2	0	1	0	0	0	0	0	0	0
y_2	−4	−6	0	5	−1	−1	1	0	0	0



a) Given the above sequences, what would be the output for the input?

x_3	0	0	−2	0	1	0	0	0	0	0

b) What is the output of the system for the input $x_1 - x_2$?

c) Given the above information, how could you find the impulse response of this system? What is the impulse response?

d) What is the output of this system for the following input:

x_4	1	1	1	0	0	0	0	0	0	0

4 LTI Low Pass Filters

Given a sequence of discrete samples with high frequency noise, we can de-noise our signal with a discrete low-pass filter. Two examples are given below:

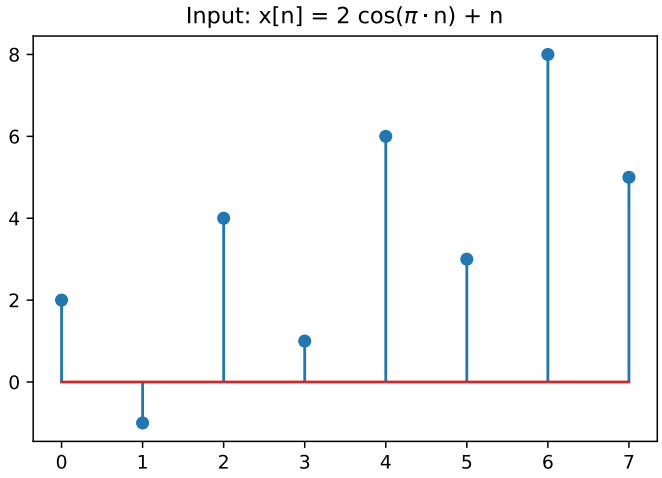
$$y[n] = 0.5y[n-1] + x[n] \tag{1}$$
$$y[n] = 0.25x[n] + 0.25x[n-1] + 0.25x[n-2] + 0.25x[n-3] \tag{2}$$

a) Show that both systems (1) and (2) are LTI.

b) Write the impulse responses $h[n]$ for (1) and (2). You may assume that $h[n] = 0$ for $n < 0$.

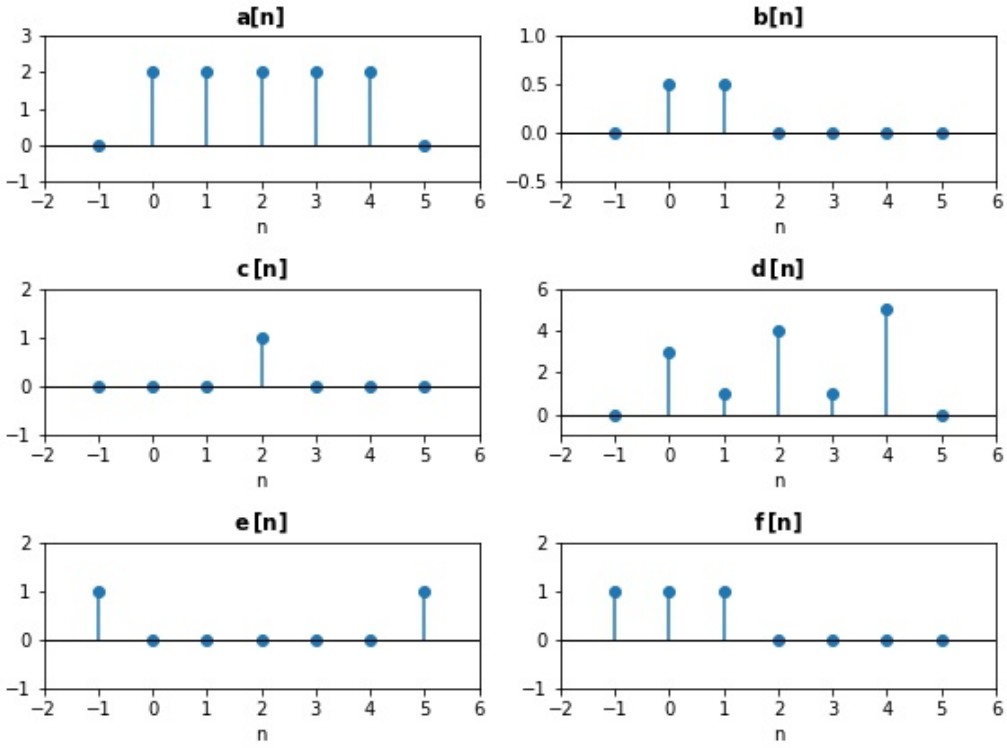
c) Are either of these systems causal? Are either of these systems stable?

d) Given the input sequence $x[n] = 2\cos(\pi n) + n$ from $n = 0$ to $n = 7$, find the output y for each system from $n = 0$ to $n = 7$. Assume that $y[n] = 0$ for $n < 0$.

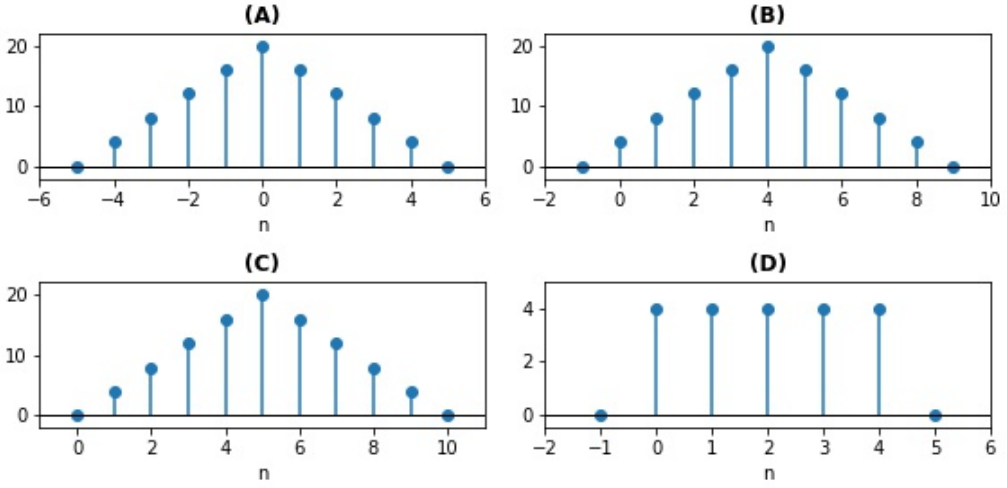


5 Convolution Matching

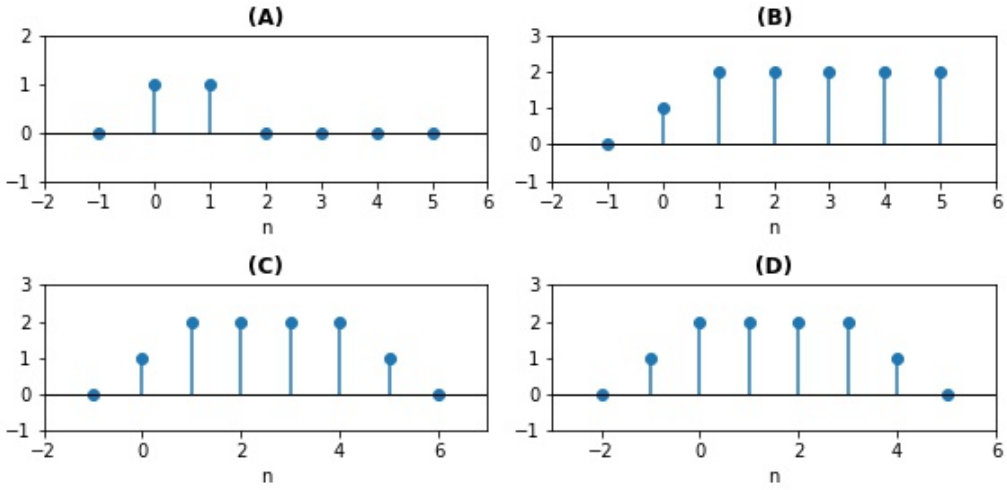
Consider the following discrete time signals:



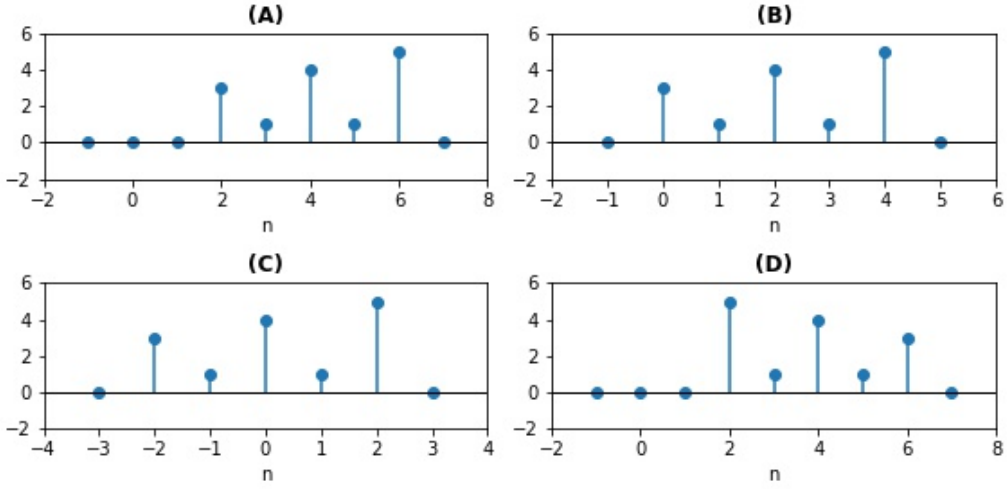
a) Which of the options below shows the correct plot for the convolution $a[n] * a[n]$?



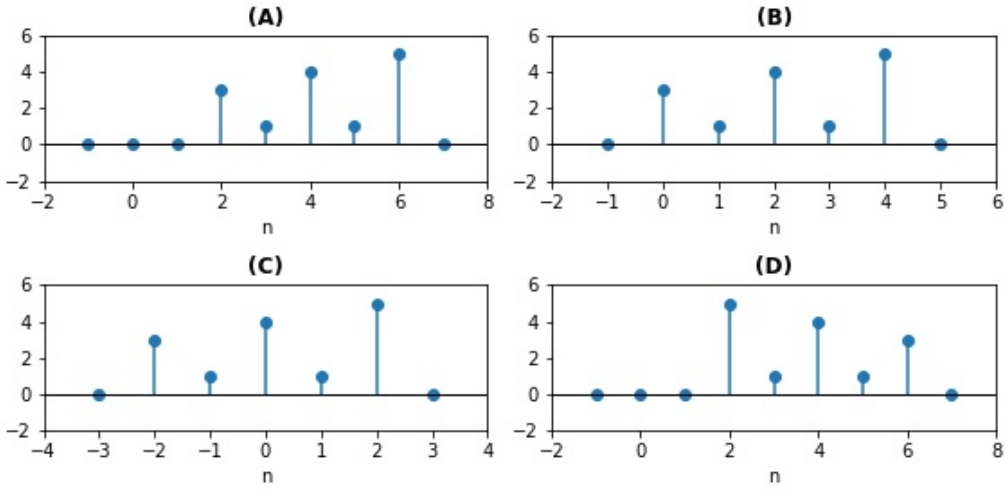
b) Which of the options below shows the correct plot for the convolution $a[n] * b[n]$?



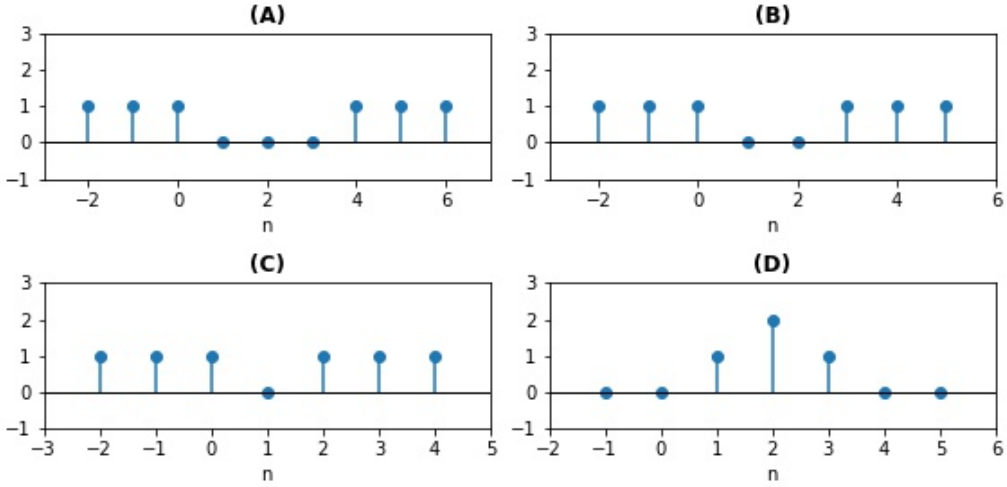
c) Which of the options below shows the correct plot for the convolution $c[n] * d[n]$?



d) Which of the options below shows the correct plot for the convolution $d[n] * c[n]$?



e) Which of the options below shows the correct plot for the convolution $e[n] * f[n]$?



6 Homework Process and Study Group

Citing sources and collaborators are an important part of life, including being a student!
We also want to understand what resources you find helpful and how much time homework is taking, so we can change things in the future if possible.

a) **What sources (if any) did you use as you worked through the homework?**

b) **If you worked with someone on this homework, who did you work with?**
List names and student ID's. (In case of homework party, you can also just describe the group.)

c) **Roughly how many total hours did you work on this homework?**

d) **Do you have any feedback on this homework assignment?**