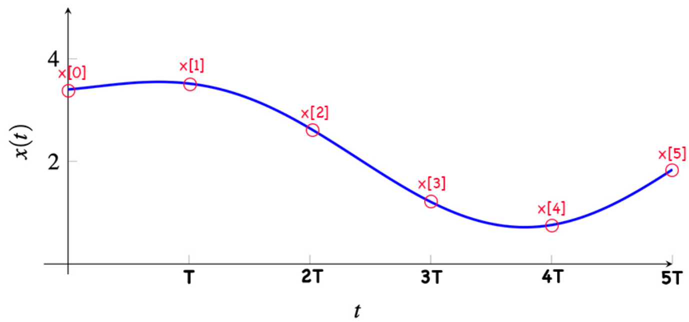
Sampling theorem

Let x be continuous signal bandlimited by frequency ω_{max}. We sample x with a period of T_s.



Given the discrete samples, we can try reconstructing the original signal f through sinc-interpolation where $\Phi(t) = \text{sinc}\left(\frac{t}{T_s}\right)$

$$\hat{x}(t) = \sum_{n=-\infty}^{\infty} x[n]\Phi(t - nT_s)$$

We define the **sampling frequency** as $\omega_s = \frac{2\pi}{T_s}$. The Sampling Theorem says if $\omega_{max} < \frac{\pi}{T_s}$, or $\omega_s > 2\omega_{max}$, then we are able to recover the original signal, i.e. $x = \hat{x}$.

1 Sampling Theorem basics

Consider the following signal, $x(t)$ defined as,

$$x(t) = \cos(2\pi t). \tag{1}$$

a) Sketch the signal $x(t)$, for $t \in [0, 4]$s.

b) Sketch discrete samples of $x(t)$ is the signal is sampled at a period of

i) $\frac{1}{4}$s

ii) $\frac{1}{2}$s

iii) 1s

iv) 2s

How would you reconstruct a continuous signal $\hat{x}(t)$ if you only had the discrete samples for reconstruction?

c) What is the maximum frequency, ω_{max}, in radians per second? In Hertz?

d) If I sample every T seconds, what is the sampling frequency?

e) What is the smallest sampling period T that would result in an imperfect reconstruction?

f) Repeat part (b), for the signal

$$y(t) = \sin(2\pi t) \tag{2}$$

2 Aliasing

Consider the signal $x(t) = \sin(0.2\pi t)$.

a) At what period T should we sample so that sinc interpolation recovers a function that is identically zero?

b) At what period T can we sample at so that sinc interpolation recovers the function $f(t) = -\sin\left(\frac{\pi}{15}t\right)$?