EECS 16A Designing Information Devices and Systems I
Summer 2020 Homework 5A

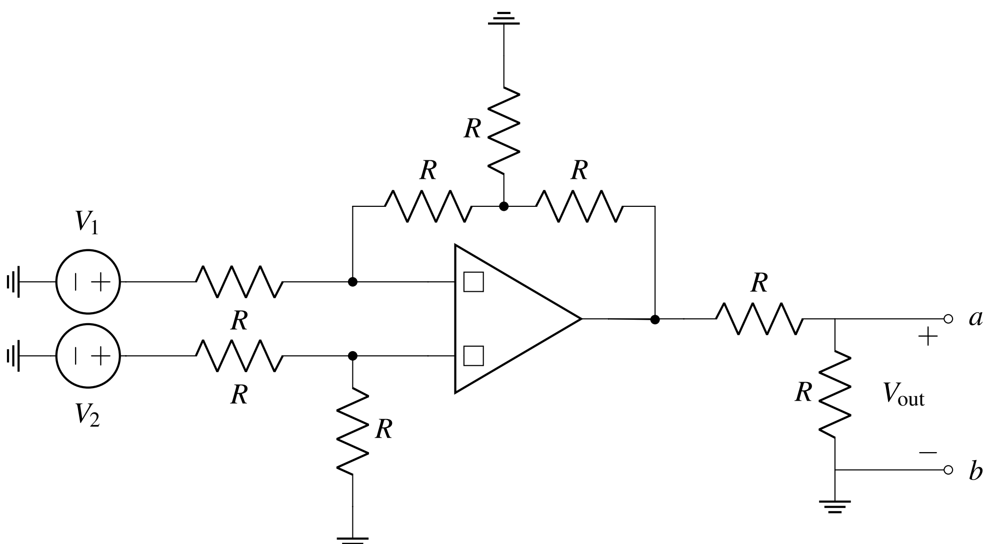
This homework is due Wednesday, July 29, 2020, at 23:59.
Self-grades are due Sunday, August 2, 2020, at 23:59.

Submission Format
Your homework submission should consist of a single PDF file that contains all of your answers (any handwritten answers should be scanned).

Homework Learning Goals: *The objective of this homework is to introduce Node Voltage Analysis for Op-Amps and reinforce your circuit modeling and design skills using Op-Amps in negative feedback.*

1. Op Amp Nodal Analysis

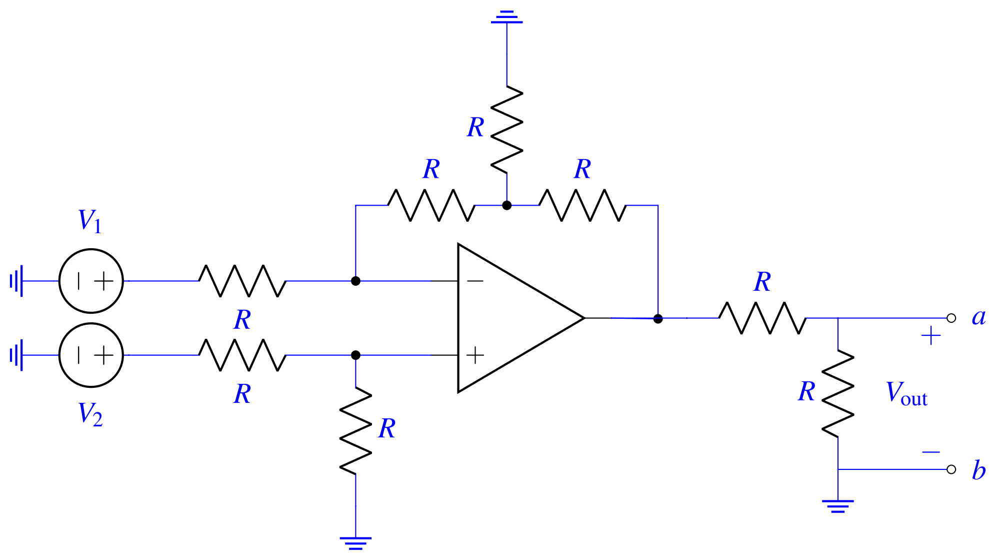
Consider this Op Amp circuit below:



We are interested in analyzing its input-output relationship.

(a) Redraw the circuit with a choice of $+$ and $-$ terminal labelings to guarantee that the circuit is in negative feedback.

Solution: To be in negative feedback, the top terminal must be labeled with a minus. So we have:

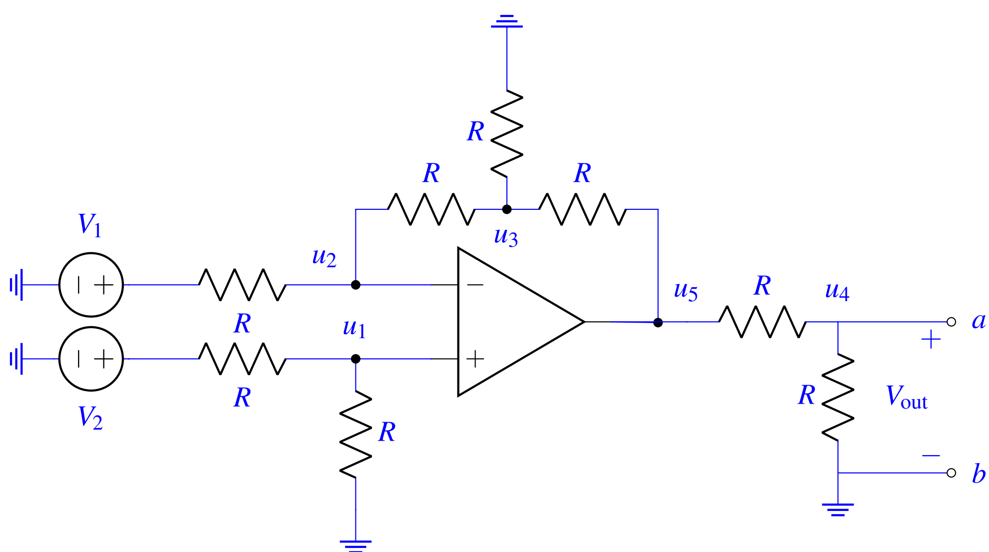


Our goal in the succeeding parts will be to find the Thevenin equivalent of this op amp circuit, and make some observations about the resulting equivalent.

(b) Find the open circuit output voltage, V_{out} as a function of the input voltages V_1 and V_2. This will be the Thevenin Voltage, V_{Th}.

Solution: Begin nodal analysis by considering the nodes at which we do not know the node voltages, and do not have a voltage source (independent or dependent) connected to it so that we can write our KCL expressions.

We will use the following node voltage labelings from u_1 to u_5, with all the KCL expressions written to have currents directed outward. So long as the right node voltage difference is taken, this will yield the correct equations.



At node u_1, if we apply the ideal op amp assumptions, there should be no current going into the positive terminal. This means we can interpret the resistor network attached to the positive terminal as a voltage divider.

$$u_1 = \frac{R}{R+R}V_2 = \frac{V_2}{2}$$

At node u_2, we can write the KCL equation considering that there will be no current going into the negative terminal.

$$\frac{u_2 - V_1}{R} + \frac{u_2 - u_3}{R} = 0$$

At node u_3, the KCL equation is:

$$\frac{u_3 - u_2}{R} + \frac{u_3 - 0V}{R} + \frac{u_3 - u_5}{R} = 0$$

At node u_5, there is a voltage divider, where u_4 is the node voltage that has the same value as the branch voltage of the resistor that bridges terminals a and b.

$$u_4 = \frac{R}{R+R} u_5 = \frac{u_5}{2}$$

Note that the output node with node voltage u_5 did not have a KCL equation written for it. This is because the output is determined by a dependent voltage source and the current coming out from the source will not be expressible until we've found all the node voltages.

Since the circuit is in negative feedback, the input terminal voltages should be the same: $u_1 = u_2$. Substituting into the KCL equations, we have:

$$\frac{u_1 - V_1}{R} + \frac{u_1 - u_3}{R} = 0$$

$$\frac{u_3 - u_1}{R} + \frac{u_3}{R} + \frac{u_3 - u_5}{R} = 0$$

Treating u_1 and V_1 as knowns, the first KCL equation tells us that u_3 is:

$$u_3 = 2u_1 - V_1 = V_2 - V_1$$

Treating u_3 and u_1 as knowns, the second KCL equation tells us that u_5 is:

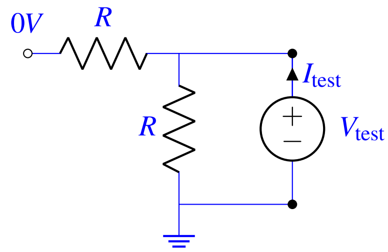
$$\begin{aligned} u_5 &= 3u_3 - u_1 \\ &= 3V_2 - 3V_1 - \frac{V_2}{2} = \frac{5}{2}V_2 - 3V_1 \end{aligned}$$

Lastly, our output voltage $u_4 = V_{\text{out}} = V_{Th}$ is:

$$u_4 = V_{\text{out}} = \frac{\frac{5}{2}V_2 - 3V_1}{2} = \frac{5}{4}V_2 - \frac{3}{2}V_1$$

(c) Turn off all independent sources ($V_1 = V_2 = 0V$). What is the equivalent resistance as seen between terminals a and b? This will be your Thevenin resistance, R_{Th}. (Hint: Consider what the voltage at the output of the op amp becomes and use a test source, or replace the op amp with its internal model where it has a dependent source.)

Solution: When both inputs V_1 and V_2 are off, the analysis in the previous subpart showed that u_5, the node voltage at the output of the op amp would be 0V. If this is the case, by applying a test source we have:



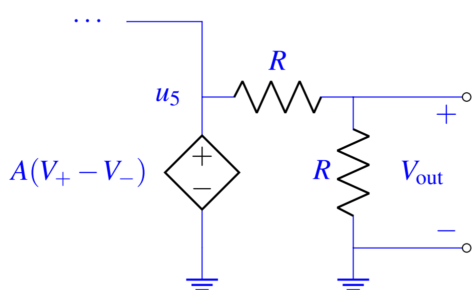
Since both resistors will have branch voltages of V_{test} over them, I_{test} can be written as:

$$I_{\text{test}} = \frac{V_{\text{test}}}{R} + \frac{V_{\text{test}}}{R} = \frac{2V_{\text{test}}}{R}$$

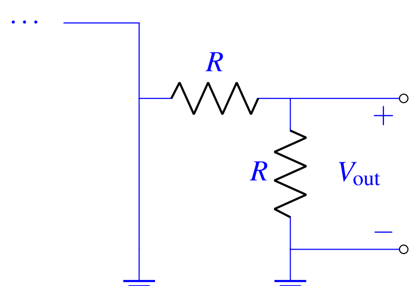
So the equivalent resistance as seen from the output (which is also the Thevenin resistance since we turned off all sources) is:

$$R_{eq} = \frac{V_{\text{test}}}{I_{\text{test}}} = \frac{R}{2}$$

Another way to approach the problem is to consider the internal model of the op amp:



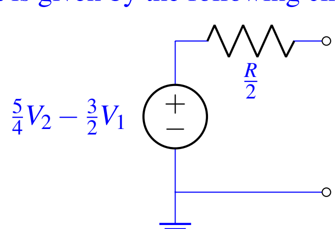
The output u_5 being zero when V_1 and V_2 are zero essentially means the dependent source is off and behaves like a short - we can turn off the dependent source in this case because it is directly influenced by the independent sources V_1 and V_2:



In this scenario, the equivalent resistance seen at the output is just $R||R = \frac{R}{2}$ since the resistors are in parallel.

(d) Use what you found in parts *b* and *c* to draw the Thevenin equivalent.

Solution: The Thevenin equivalent is given by the following circuit:



(e) **Practice (Optional)**: Does the Thevenin resistance depend on all the resistors or a strict subset? What might explain this?

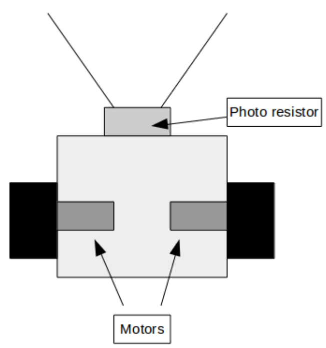
Solution: Since the output of an ideal op amp is a dependent voltage source, only the resistances that come after the output it will come into play, which is why we saw only the divider at the output influence the resistance.

This highlights the benefit of an op amp in allowing circuits to present with a uncomplicated or choosable Thevenin equivalent resistance (e.g. a buffer, 0 resistance) so that designed circuit modules can be connected together with predictable effect.

2. PetBot Design

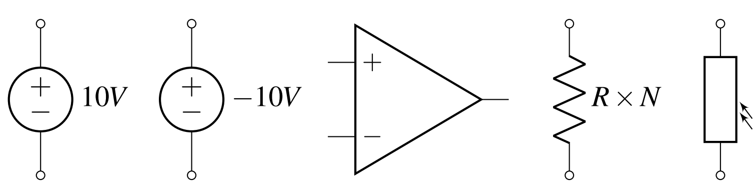
In this problem, you will design circuits to control PetBot, a simple robot designed to follow light. PetBot measures light using photoresistors. A photoresistor is a light-sensitive resistor. As it is exposed to more light, its resistance decreases. Given below is the circuit symbol for a photoresistor.

Below is the basic layout of the PetBot. It has one motor on each wheel. We will model each motor as a $1\,\Omega$ resistor. When motors have positive voltage across them, they drive forward; when they have negative voltage across them, they drive backward. At zero voltage across the motors, the PetBot stops. The speed of the motor is directly proportional to the magnitude of the motor voltage. The light sensor is mounted to the front of the robot.



(a) **Speed control** – Let us begin by first having PetBot decrease its speed as it drives toward the flashlight. Design a motor driver circuit that outputs a decreasing positive motor voltage as the PetBot drives toward the flashlight. The motor voltage should be at least $5\,\text{V}$ far away from the flashlight. When far away from the flashlight, the photoresistor value will be $10\,\text{k}\Omega$ and dropping toward $100\,\Omega$ as it gets closer to the flashlight.

In your design, you may use any number of resistors with any value and just 1 op-amp. You also have access to voltage sources of $10\,\text{V}$ and $-10\,\text{V}$. Based on your circuit, derive an expression for the motor voltage as a function of the circuit components that you used.

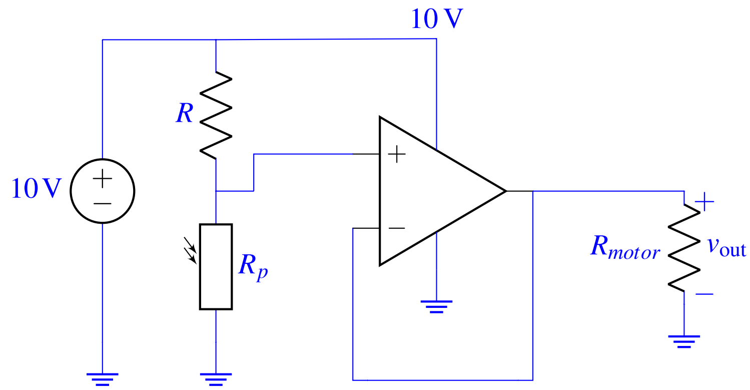


Hint 1: You should consider the loading effect of connecting this circuit to your motor, which has resistance. A buffer may help solve this problem.

Hint 2: If you're not sure where to start, try playing around with connecting the circuit elements in different ways, and think about circuits you have seen before.

Solution:

We can use a voltage divider circuit to adjust the output motor voltage as the PetBot drives towards the flashlight (and the photoresistor's resistance decreases).



The output of the above circuit is:

$$v_{\text{out}} = \frac{R_p}{R_p + R} \cdot 10\,\text{V}$$

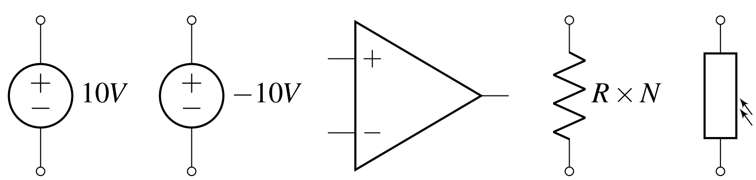
where R_p represents the photoresistor. Note that we use a voltage buffer to prevent loading effects when connecting the motor.

We set $R \leq 10\,\text{k}\Omega$ to achieve $v_{\text{out}} \geq 5\,\text{V}$ when the PetBot is far away from the flashlight (i.e. $R_p = 10\,\text{k}\Omega$). As the PetBot drives towards the flashlight, the resistance R_p drops, so that v_{out}, the motor voltage, decreases.

(b) **Distance control** – Let us now have PetBot drive up to a flashlight (or away from the flashlight) and stop at distance of 1 m away from the light. At the distance of 1 m from the flashlight, the photoresistor has a value 1 kΩ.

Design a circuit to output a motor voltage that is positive when the PetBot is at a distance greater than 1 m from the flashlight (making the PetBot move toward it), zero at 1 m from the flashlight (making the PetBot stop), and negative at a distance of less than 1 m from the flashlight (making the PetBot back away from the flashlight.)

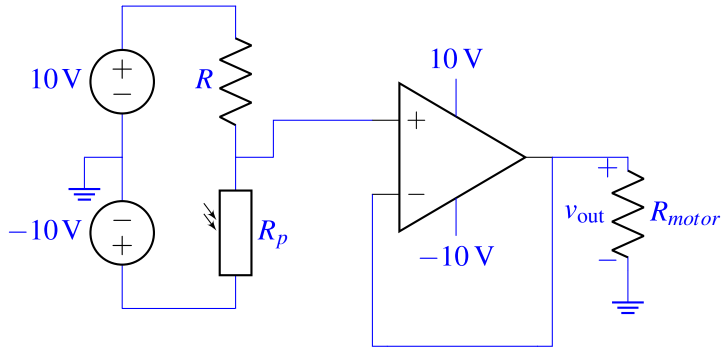
In your design, you may use any number of resistors of any value and just 1 op-amp. You also have access to voltage sources of 10 V and −10 V. Based on your circuit, derive an expression for the motor voltage as a function of the values of circuit components that you used.



Solution:

We outline two possible solutions here:

Method 1:



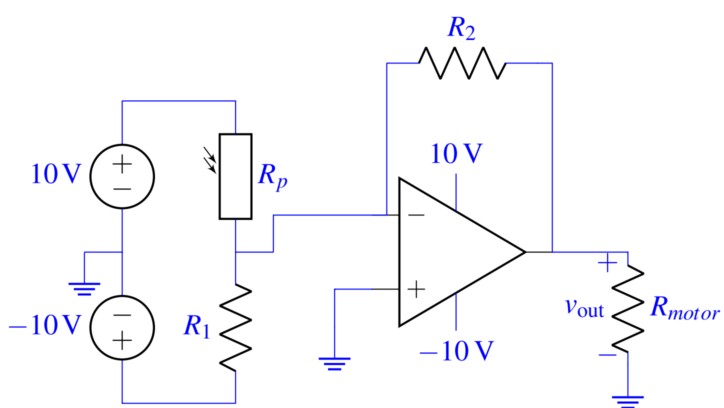
Here observe that $v_{\text{out}} = u_+$. Using superposition we find that

$$v_{\text{out}} = u_+ = 10\text{V}\frac{R_p}{R+R_p} - 10\text{V}\frac{R}{R+R_p} = 10\text{V}\frac{R_p - R}{R+R_p}$$

To satisfy the condition that $v_{\text{out}} = 0\text{V}$ when Petbot is 1 m away, we have that $R = 1\,\text{k}\Omega$. Similar to the previous design, we can do the analysis for when the Petbot is far away and close by. We will show how to do it when the Petbot is close by here.

$$R_p < R = 1\,\text{k}\Omega$$
$$\Rightarrow R_p - R < 0$$
$$\Rightarrow v_{\text{out}} = 10\text{V}\frac{R_p - R}{R+R_p} < 0\text{V}$$

Method 2:



Choosing $R_1 = R = 1\,\text{k}\Omega$ we observe that the voltage $v_{\text{out}} = 0\text{V}$ when Petbot is 1 m away as required. To find v_{out} more generally, observe that $u_- = u_+ = 0$, so we need to find the current i going through R_2 from node u_- to node u_{out} to get v_{out}.

$$v_{\text{out}} = -iR_2 = -\left(\frac{10\text{V}}{R_p} + \frac{-10\text{V}}{R_1}\right)R_2$$

From this we see that when PetBot is greater than 1 m away we have

$$R_p > R_1 = 1\,\text{k}\Omega$$

$$\frac{1}{R_p} - \frac{1}{R_1} < 0$$
$$\Rightarrow -\left(\frac{1}{R_p} - \frac{1}{R_1}\right) \cdot R_2 10\text{V} > 0$$

Similarly when the Petbot is less than 1 m away, we have that the motor voltage will be negative.

3. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.