EECS 16A Designing Information Devices and Systems I
Summer 2020 Homework 3B

This homework is due Sunday, July 19, 2020, at 23:59.
Self-grades are due Wednesday, July 22, 2020, at 23:59.

Submission Format
Your homework submission should consist of a single PDF file that contains all of your answers (any handwritten answers should be scanned).

Homework Learning Goals: *The objective of this homework is to familiarize you with the use of resistive dividers in the design of useful, real-world applications, introduce the concepts of power dissipation, and voltage and current measurement. Finally, it will give you some practice on the superposition circuit analysis technique .*

1. 1-D Resistive Touchscreen

Figure 1 shows the top view of a resistive touchscreen consisting of a conductive layer with resistivity ρ_{t1}, thickness t, width W, and length L. At the top and bottom it is connected to good conductors ($\rho = 0$), represented in the figure by two rectangles. The touchscreen is wired to voltage source V_s.

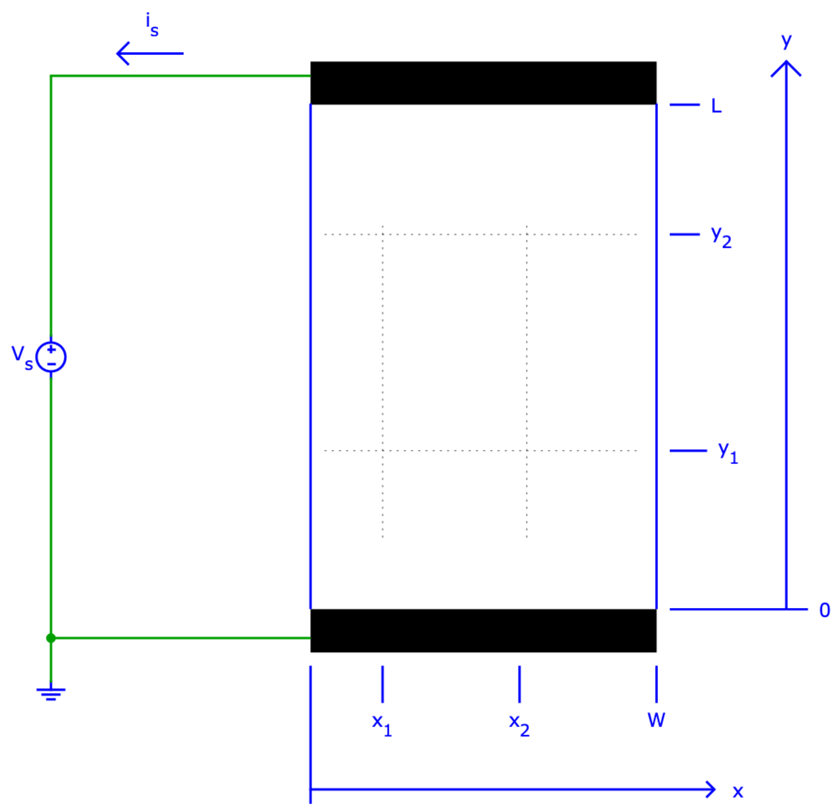
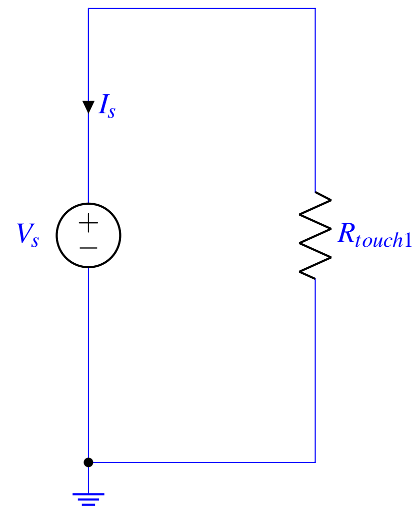


Figure 1: Top view of 1-D resistive touchscreen (not to scale).

Use the following numerical values in your calculations: $W = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_{t1} = 0.5\,\Omega\,\text{m}$, $V_s = 5\text{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm.

(a) Draw a circuit diagram representing the touchscreen shown in Figure 1. Remember that circuit diagrams consist of only circuit elements (resistors, current sources, etc) represented by symbols, connecting wires, and the reference symbol.

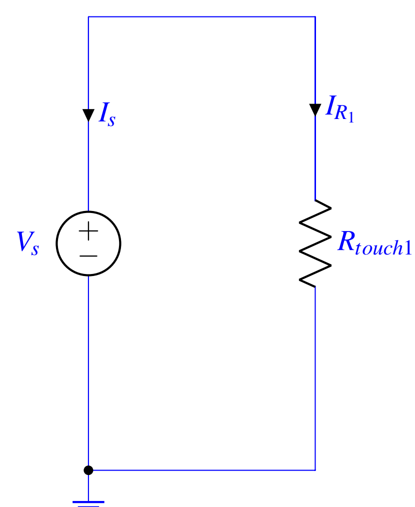
Solution:



(b) Calculate the value of current I_s. Do not forget to specify the correct unit as always.

Solution: The touchscreen resistance can be found from the following expression:

$$\begin{aligned}
R_{touch1} &= \rho_{t1} \cdot \frac{L}{A} \\
&= \rho_{t1} \cdot \frac{L}{W \cdot t} \\
&= 0.5\,\Omega\,\text{m}\left(\frac{80 \times 10^{-3}\,\text{m}}{50 \times 10^{-3}\,\text{m} \cdot 1 \times 10^{-3}\,\text{m}}\right) \\
R_{touch1} &= 800\,\Omega
\end{aligned}$$



From KCL, we can write:

$$I_s + I_{R_1} = 0 \quad (1)$$
$$I_s = -I_{R_1} \quad (2)$$

Therefore, the current I_{R_1} is equal to:

$$I_{R_1} = \frac{V_s}{R_{touch1}} = \frac{5}{800}A = 6.25mA$$

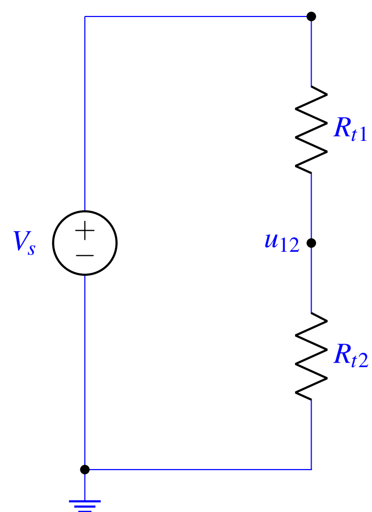
And the current I_s is equal to:

$$I_s = -I_{R_1} = -6.25mA$$

(c) What is the node voltage u_{12} (with respect to the reference node) of the touchscreen at coordinates (x_1, y_2)? Redraw the circuit diagram from part (a) to include node u_{12}. Specify all component values (resistances, ...) in the diagram. Hint: you need more than one resistor to represent this situation.

Solution:

We can represent this setup with the circuit shown below.



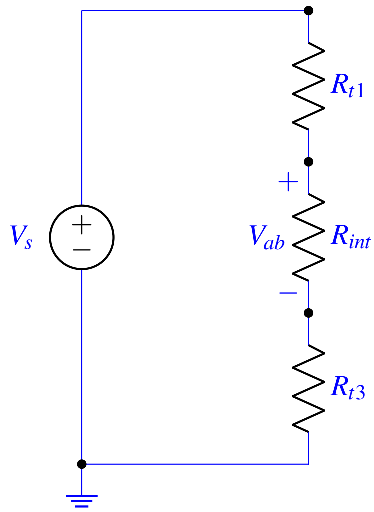
Using voltage division, u_{12} can be found from the following expression:

$$u_{12} = V_s \frac{R_{t2}}{R_{t1} + R_{t2}} = V_s \frac{y_2}{L} = 5 \cdot \frac{3}{4} V = 3.75V$$

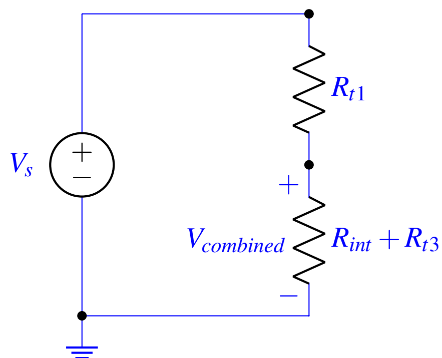
where $R_{t1} = \rho_{t1} \cdot \frac{L-y_2}{Wt}$ and $R_{t2} = \rho_{t1} \cdot \frac{y_2}{Wt}$.

(d) Calculate (absolute value of) voltage V_{ab} between touchscreen coordinates (x_1, y_1) and coordinates (x_1, y_2). Suggestion: Draw an augmented circuit diagram and calculate all component values.

Solution:



One method to find the voltage is by using node voltage analysis. Presented is an alternative approach by using resistor equivalence. To find the voltage, V_{ab}, first, find the voltage over R_{int} and R_{t3} together. We can represent them as an equivalent resistance as follows:



As this circuit is a voltage divider, we can find $V_{combined}$ by voltage division.

$$V_{combined} = \frac{R_{int}+R_{t3}}{R_{t1}+R_{int}+R_{t3}}V_s$$

Once we know $V_{combined}$, this will be the voltage over the two resistors, R_{int} and R_{t3}. We can apply voltage division again to get V_{ab}:

$$V_{ab} = \frac{R_{int}}{R_{int}+R_{t3}}V_{combined}$$

By substituting, we get that V_{ab} is:

$$\begin{aligned} V_{ab} &= \frac{R_{int}}{R_{int}+R_{t3}}\frac{R_{int}+R_{t3}}{R_{t1}+R_{int}+R_{t3}}V_s \\ &= \frac{R_{int}}{R_{t1}+R_{int}+R_{t3}}V_s \end{aligned}$$

Each of the resistances can be calculated as $R_{t1} = \rho_{t1} \cdot \frac{L-y_2}{Wt}$, $R_{int} = \rho_{t1} \cdot \frac{y_2-y_1}{Wt}$ and $R_{t3} = \rho_{t1} \cdot \frac{y_1}{Wt}$. This gives for V_{ab}:

$$V_{ab} = \frac{y_2 - y_1}{L} Vs = \cdot \frac{3}{8} 5V = 1.875V$$

(e) Calculate (absolute value of) the voltage between touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_1).

Solution:

The two points have the same y coordinate, therefore they have the same potential. Thus, $\Delta V = 0$

(f) Calculate (absolute value of) the voltage between touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_2).

Solution:

The two points have different x and y coordinates. However, the potential is the same across the x axis for a fixed y coordinate. Therefore, the problem is similar to part d, since the potential is only determined by the y coordinate of a point. Using the same equivalent circuit of part d we have:

$$\Delta V = V_s \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}} = 1.875V$$

(g) Figure 2 shows a new arrangement with two touchscreens. The second touchscreen is identical to the one shown in Figure 1, except for different width, W_2, and resistivity, ρ_{t2}. Use the following numerical values in your calculations: $W = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_{t1} = 0.5\,\Omega$m, $V_s = 5$V, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm, which are the same values as before, with $W_2 = 85$ mm, $\rho_{t2} = 0.6\,\Omega$m for the new touchscreen.

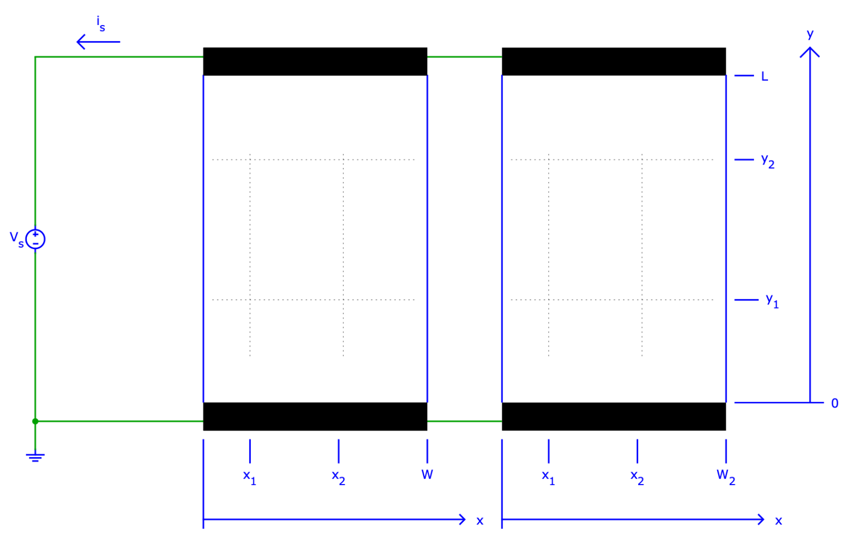
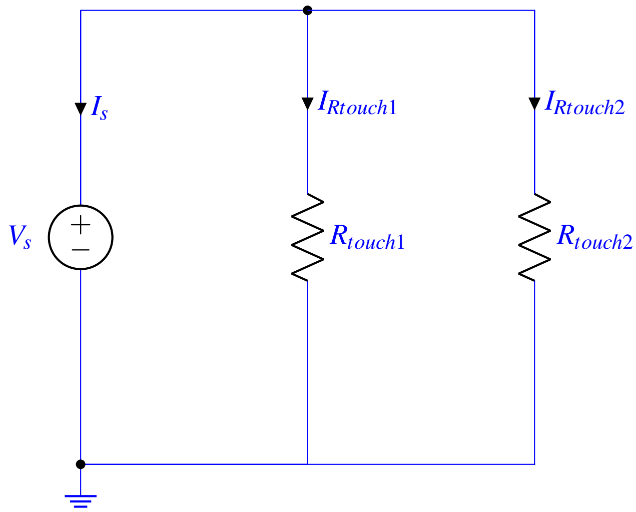


Figure 2: Top view of two touchscreens wired in parallel (not to scale).

Draw a circuit diagram representing the two touchscreens shown in Figure 2.

Solution:



(h) Calculate the value of current I_s for the two touchscreen arrangement.

Solution:

From KCL, we can write:

$$I_s + I_{Rtouch1} + I_{Rtouch2} = 0 \tag{3}$$
$$I_s = -(I_{Rtouch1} + I_{Rtouch2}) \tag{4}$$

Using Ohm's Law for each element:

$$I_s = -\left(\frac{V_s}{R_{touch1}} + \frac{V_s}{R_{touch2}}\right)$$

However, the resistance of the second touchscreen can be given by:

$$R_{touch2} = \rho_{t2} \cdot \frac{L}{W_2 t} = 0.6\,\Omega\,\text{m}\left(\frac{80 \times 10^{-3}\,\text{m}}{85 \times 10^{-3}\,\text{m} \cdot 1 \times 10^{-3}\,\text{m}}\right)$$
$$R_{touch2} = 564.705\,882\,4\,\Omega$$

Therefore, using the resistance values for the first and second touchscreen and the applied voltage source, we have:

$$I_s \approx -(6.25\text{mA} + 8.85\text{mA}) = -15.1\text{mA}$$

(i) Now assume a wire is connected between coordinates (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right. Calculate the current I_{12} flowing through this wire.

Solution:

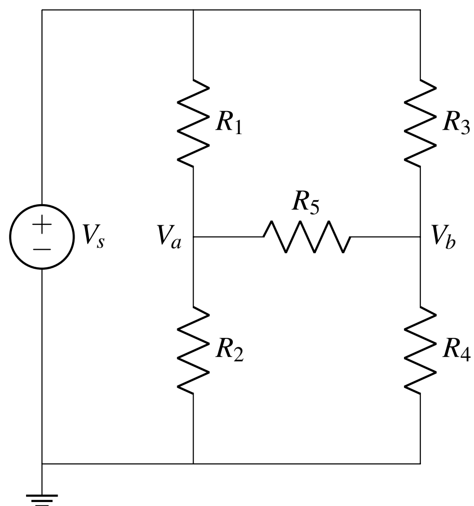
There is no current flowing through the wire, since the points in the two touchscreens have the same potential. Therefore,

$$I_{wire} = \frac{V12_{touch1} - V22_{touch2}}{R_{wire}} = 0$$

We should note that what causes current to flow is a voltage difference. Here, we have a voltage difference of zero, so the current flowing through the wire is zero. Generally, in a wire when current is flowing we have a very small voltage difference $V_{wire} = IR_{wire}$. However, since R_{wire} is super small we neglect V_{wire}.

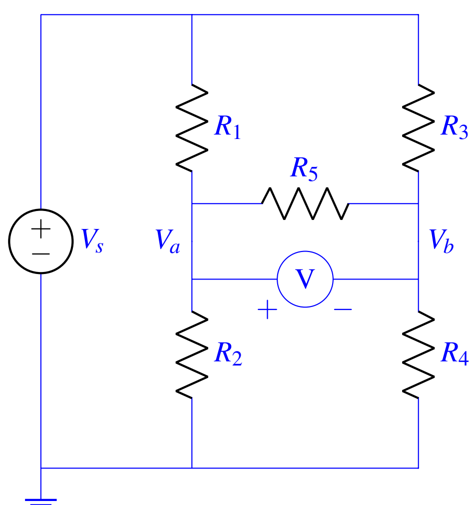
2. Volt and ammeter

Use the following numerical values in your calculations: $R_1 = 1\,\mathrm{k}\Omega$, $R_2 = 2\,\mathrm{k}\Omega$, $R_3 = 3\,\mathrm{k}\Omega$, $R_4 = 4\,\mathrm{k}\Omega$, $R_5 = 5\,\mathrm{k}\Omega$, $V_s = 10V$.



(a) Redraw the circuit diagram shown in Figure 1 by adding a voltmeter (letter V in a circle and plus and minus signs indicating direction) to measure voltage V_{ab} from node V_a (positive) to node V_b (negative). Calculate the value of V_{ab}.
Hint: You have analyzed a very similar circuit in a previous assignment, reuse the result.

Solution: Below is the redrawn circuit with the voltmeter. Note that it is also correct to have the voltmeter above R_5, as it will still be connected to the same nodes.



Using the same analysis with part b from Problem 1, we end up with the following matrix relation for the node voltages V_a and V_b:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_5} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ \frac{V_s}{R_3} \end{bmatrix}$$

Plugging in the values we were given into the matrix above and using Gaussian elimination we can find the vector of unknowns.

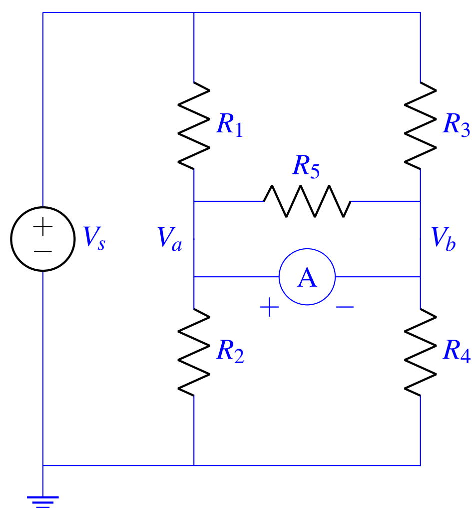
$$\begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 6.58V \\ 5.936V \end{bmatrix}$$

From these node voltages, the voltage V_{ab} can be calculated.

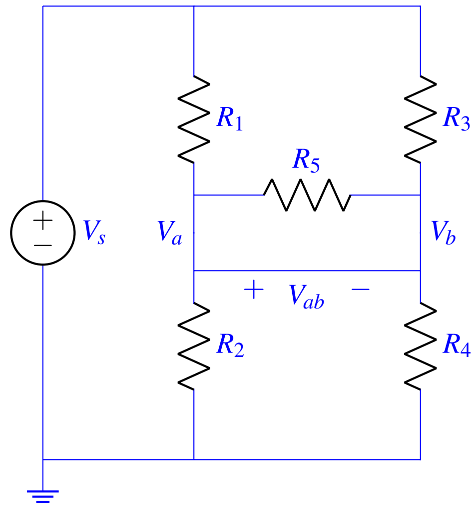
$$V_{ab} = V_a - V_b = 0.644V$$

(b) Suppose you inadvertently connected an ammeter in part (a) above, rather than a voltmeter (we all goof sometimes). Calculate the value of V_{ab} with the ammeter connected. Note: it differs from the value calculated in part (a).

Solution: While you did not have to redraw the circuit, it is depicted below.

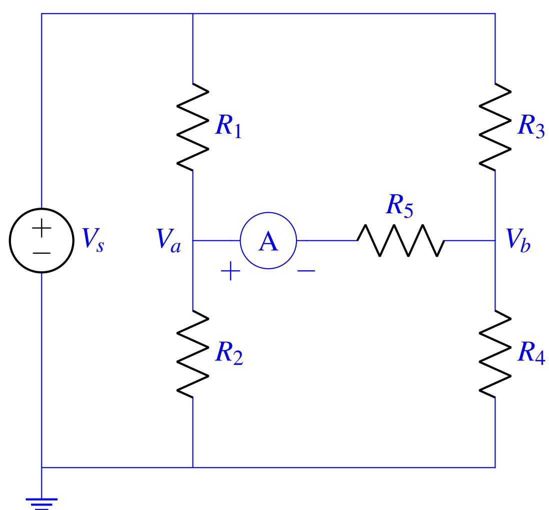


If we assume that the internal resistance of an ammeter is ideally zero, placing it across the nodes V_a and V_b will short them. So $V_a = V_b$. Thus $V_{ab} = 0$. The circuit below shows how the ammeter behaves as a short that unifies the previously separate nodes.



(c) Redraw the circuit diagram shown in Figure 1 by adding an ammeter (letter A in a circle and plus and minus signs indicating direction) to measure the current I_{R_5} through resistor R_5. Calculate the value of I_{R_5}.

Solution: The redrawn circuit with the ammeter measuring the current through R_5 is shown in the following circuit. It is also correct to draw the ammeter to the right of R_5 with the orientation of the meter remaining the same: the plus sign should be most proximal to the node labeled V_a, and the minus sign is most proximal to the node labeled V_b.

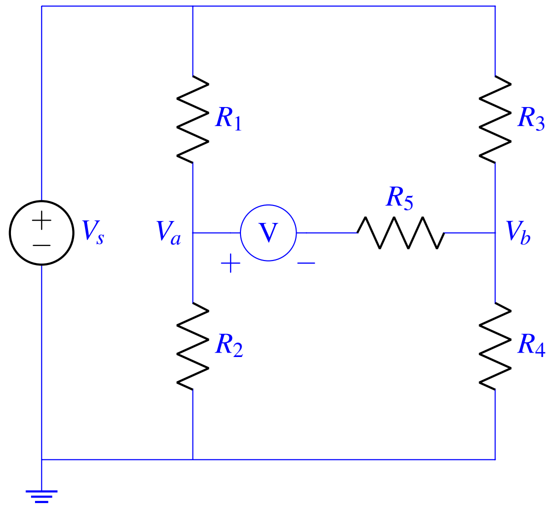


After calculating the node voltages V_a and V_b from part a, we can write:

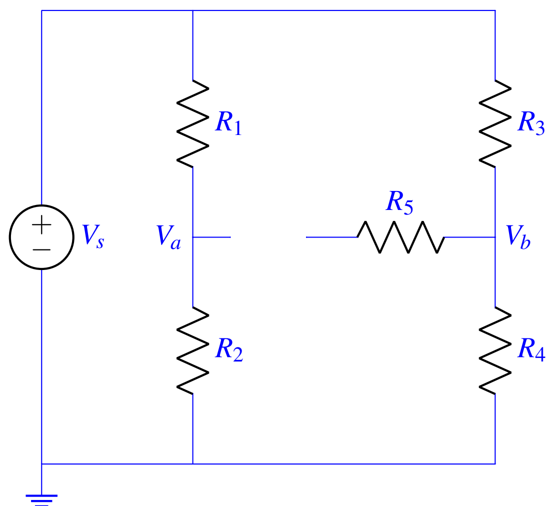
$$I_{R_5} = \frac{V_a - V_b}{R_5} = 128.8\mu A$$

(d) Your friend inadvertently connected a voltmeter in part (c) above, rather than an ammeter. Calculate the value of I_{R_5} with the voltmeter connected. Note: it differs from the value calculated in part (c).

Solution: While you were not required to redraw the new circuit, the circuit is shown bellow.



The resistance of a voltmeter is infinite and it behaves as an open circuit. There will be no current flowing through R_5. Therefore, $I_{R_5} = 0$. The circuit below depicts how the voltmeter behaves as an open that prevents any current through R_5.



3. Maximum Power Transfer

Smartphones use "bars" to indicate strength of the cellular signal. Few "bars" translate to slow or no connectivity.

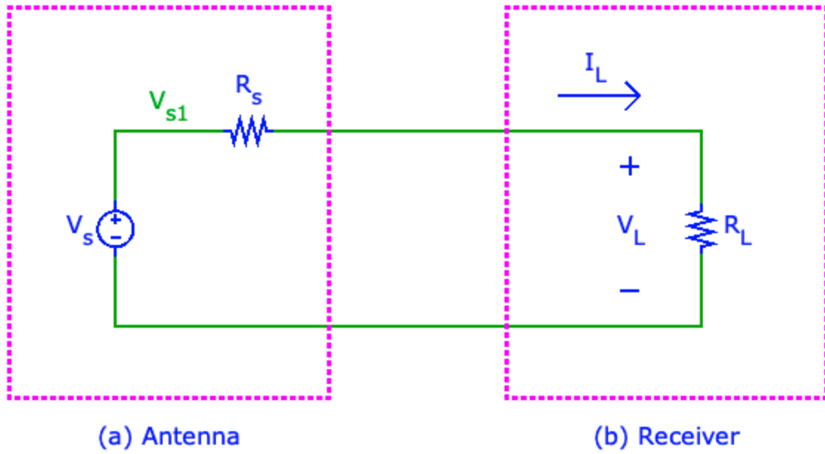
But what do these "bars" actually stand for? Voltage, current? Well, not quite. Good radio (and a cellular modem is nothing but a particular kind of radio) reception depends on the **power** received from the transmitter.

In this assignment we design a receiver that maximizes the power received, and hence connection speed.

The figure below shows electronic circuit models for the antenna (a) and the receiver (b). The antenna consists of source V_s, typically in the range of micro- or milli-Volts (10^{-6} and 10^{-3}, respectively) depending on transmitter strength and resistance R_s, usually 50Ω or 75Ω, depending on the particular antenna design.

The radio receiver is represented by resistor R_L and chosen carefully by the designer to maximize the received power (i.e. the "number of bars").

Figure 3: Electronic circuit models for radio antenna and receiver.



It is important to understand that these are ***models***. For example, the complete receiver circuit consists of many more elements. Likewise, the antenna consists of several appropriately shaped conductors. The resistor R_s is nowhere to be found, and neither is V_s physically present (i.e. you cannot connect a wire to it). However, these simple models ***act*** like the devices they represent. In other words, the voltages and currents at their terminal are identical to the voltages and currents at the terminals of the actual circuits.

Models are very important in engineering design for their ability to abstract away details when they are not needed and are the key to successful design of complex systems.

We will discuss the use and properties of electronic circuit models further in class.

Use the following component values for your calculations: $V_s = 100\mu V$, and $R_s = 50\Omega$.

(a) Find the value of R_L that maximizes the ***voltage*** V_L across resistor R_L. Calculate the values of V_L, I_L, and the power P_L delivered to (i.e. dissipated in) resistor R_L.

Solution:

Note that this circuit is a voltage divider, where the voltage across R_L can be written as $V_L = V_S\left(\frac{R_L}{R_S+R_L}\right)$. Taking the derivative of V_L with respect to R_L, we find $\frac{dV_L}{dR_L} = \frac{V_S R_S}{(R_S+R_L)^2}$. We note that as R_L increases, $\frac{dV_L}{dR_L}$ approaches 0. Therefore, making R_L as large as possible (ideally $R_L = \infty$) maximizes V_L.
If $R_L = \infty$ then $V_L = V_S = 100\mu V$.
We can use the result $V_L = V_S\left(\frac{R_L}{R_S+R_L}\right)$ to find $I_L = \frac{V_L}{R_L} = \frac{V_S}{R_S+R_L}$. If $R_L = \infty$ then $I_L = 0\text{A}$.
We know that $P_L = V_L I_L$. Therefore, $P_L = (100\mu\text{V})(0\text{A}) = 0\text{W}$.

(b) Find the value of R_L that maximizes the ***current*** I_L through resistor R_L. Calculate the values of V_L, I_L, and the power P_L delivered to resistor R_L.

Solution:

We can again use the result $V_L = V_S\left(\frac{R_L}{R_S+R_L}\right)$ to calculate I_L and find what R_L should be to maximize I_L. We know that $I_L = \frac{V_L}{R_L} = \frac{V_S}{R_S+R_L}$. We can then see R_L should be as small as possible (ideally $R_L = 0$) to maximize I_L. If $R_L = 0$ then it behaves like a wire, and there will be no voltage drop across it. Therefore, $I_L = \frac{V_S}{R_S} = \frac{100\mu\text{V}}{50\Omega} = 2\mu\text{A}$. We know that $P_L = V_L I_L$. Therefore, $P_L = (0\text{V})(2\mu\text{A}) = 0\text{W}$.

(c) Find the value of R_L that maximizes the ***power*** P_L delivered to resistor R_L. Calculate the values of V_L, I_L, and the power P_L delivered to resistor R_L. (*Hint*: The power optimization is best performed algebraically by setting the derivative of P_L with respect to R_L to zero. Alternatively you can do the optimization graphically. Plot P_L versus R_L and find the maximum.)

Solution: We can algebraically calculate the maximum of P_L by taking its derivative with respect to R_L. First, we write $P_L = I_L V_L = \left(\frac{V_S}{R_S+R_L}\right)\left(\frac{V_S R_L}{R_S+R_L}\right)$.

Next, we find that $\frac{dP_L}{dR_L} = \frac{{V_S}^2}{(R_S+R_L)^4}\left((R_S+R_L)^2 - 2R_L(R_S+R_L)\right)$.

We can then find when $\frac{dP_L}{dR_L} = 0$ by setting the expression $\left((R_S+R_L)^2 - 2R_L(R_S+R_L)\right)$ equal to 0.

We then find ${R_S}^2 + 2R_S R_L + {R_L}^2 - 2R_L R_S - 2{R_L}^2 = 0$. Further simplifying we find that ${R_S}^2 - {R_L}^2 = 0$. This leads us to the final result that $R_L = R_S$ for maximizing the power in R_L.

If $R_L = 50\Omega$, then $V_L = 100\mu\text{V}\left(\frac{50\Omega}{(50\Omega+50\Omega)}\right) = 50\mu\text{V}$.

Using Ohm's Law, we know that $I_L = \frac{V_L}{R_L} = \frac{50\mu\text{V}}{50\Omega} = 1\mu\text{A}$.

Finally, we know that $P_L = I_L V_L = (1\mu\text{A})(50\mu\text{V}) = 5\times10^{-11}\text{W}$.

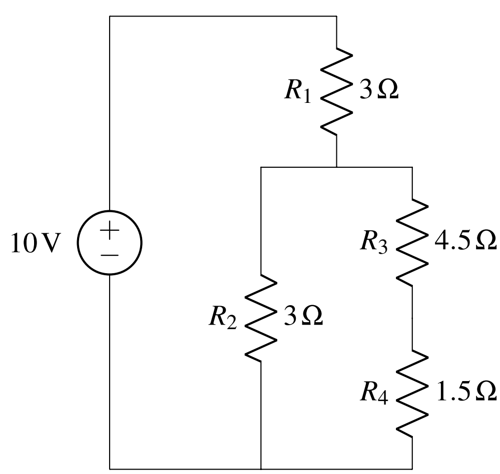
(d) What's the best value of R_L that optimizes cellular connectivity (i.e. provides the most amount of received power)?

Solution: From the problem description we know that more received power means more "bars" (i.e better reception). Therefore, the best R_L is the value that maximizes its power. Specifically, $R_L = R_S = 50\Omega$.

The next step is to design the receiver circuit such that it behaves like a resistor R_L and extracts the information sent. How to do this is taught in EE142, "Integrated Circuits for Communications."

4. Mechanical Circuits

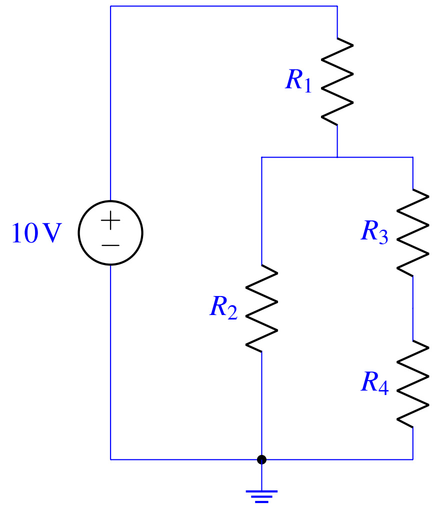
Find the voltages across and currents flowing through all of the resistors.



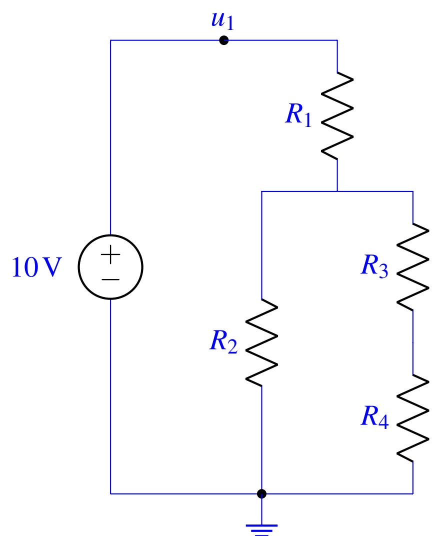
Solution:

Seven Step Method:

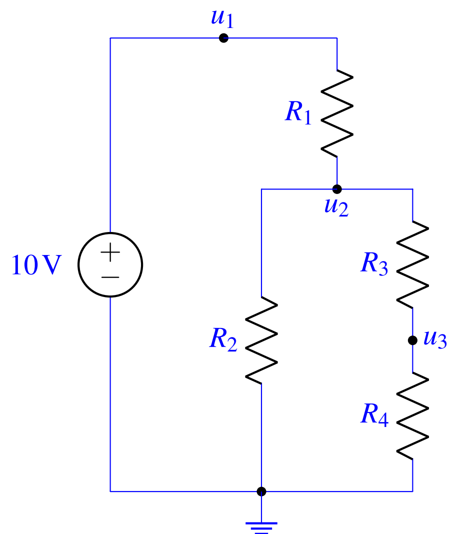
Step 1) Select a ground node. Any choice is valid, but we choose the bottom node:



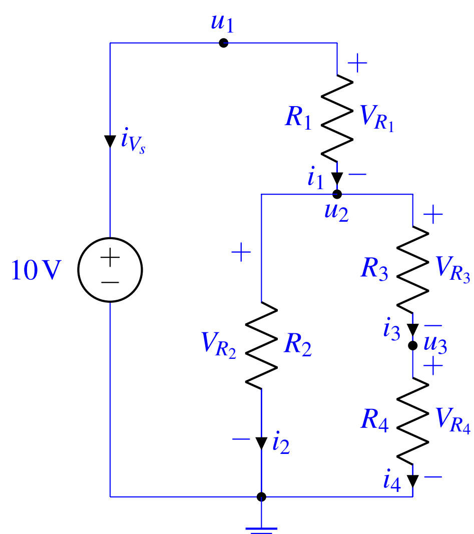
Step 2) Label nodes with voltage set by sources:



Step 3) Label all remaining nodes:



Step 4) Label element voltages and currents



Step 5) Write KCL equations for all nodes with unknowns (u_1, u_2, etc.). The definition of KCL is that the sum of all currents entering and leaving the node must equal 0. We will call current leaving positive, and current entering negative. Going node by node from u_1 to u_3 in order, we set up our KCL expressions:

$$-i_1 + i_2 + i_3 = 0$$
$$-i_3 + i_4 = 0$$

Step 6) Use IV relationships to fill in the remaining rows. Remember that only potential (voltage) *differences* make sense physically, thus we will look at the differences in node potentials and use the appropriate IV relationship to try and get useful equations.

The voltage source is connected to u_1 and ground. We know that the value of the voltage source $V_s = 10$, and that a voltage source forces the difference between its nodes to be V_s. Since we know the ground node is 0 by definition, we get

$$u_1 - 0 = V_s = 10$$

Continuing in a similar manner looking at the differences of node potentials. u_1 and u_2 are separated by a resistor, and Ohm's law relates the potential *difference* between each side of the resistor to the current through it, so we have:

$$u_1 - u_2 = i_1 R_1$$

Similarly for the other nodes

$$\begin{aligned} u_2 - 0 &= i_2 R_2 \\ u_2 - u_3 &= i_3 R_3 \\ u_3 - 0 &= i_4 R_4 \end{aligned}$$

We can thus substitute these node voltages into the KCL equations from Step 5).

$$\begin{aligned} -\left(\frac{u_1 - u_2}{R_1}\right) + \frac{u_2}{R_2} + \left(\frac{u_2 - u_3}{R_3}\right) &= 0 \\ -\left(\frac{u_2 - u_3}{R_3}\right) + \frac{u_3}{R_4} &= 0 \end{aligned}$$

We know that $u_1 = 10$, therefore we can solve for the two unknowns (u_2, u_3) with the system of two equations. We find that

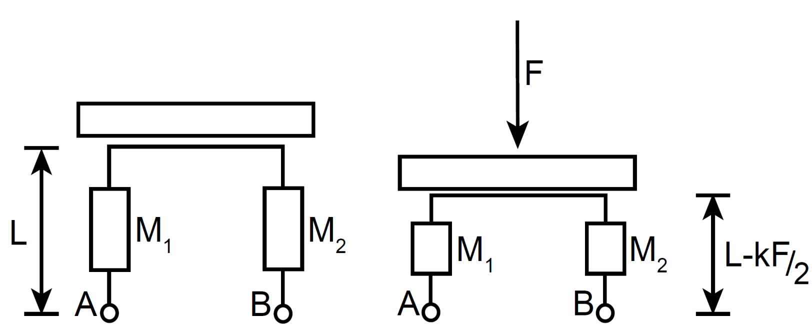
$$\begin{aligned} u_2 &= 4V \\ u_3 &= 1V \end{aligned}$$

5. Fruity Fred

Fruity Fred just got back from Berkeley Bowl with a bunch of mangoes, pineapples, and coconuts. He wants to sort his mangoes in order of weight, so he decides to use his knowledge from EECS16A to build a scale.

He finds two identical bars of material (M_1 and M_2) of length L (meters) and cross-sectional area A_c (meters^2), which are made of a material with resistivity ρ. He knows that the length of these bars decreases by k meters per Newton of force applied, while the cross-sectional area remains constant.

He builds his scale as shown below, where the top of the bars are connected with an ideal electrical wire. The left side of the diagram shows the scale at rest (with no object placed on it), and the right side shows it when the applied force is F (Newtons), causing the length of each bar to decrease by $kF/2$ meters. Fred's mangoes are not very heavy, so $L \gg kF/2$.



(a) Let R_{AB} be the resistance between nodes A and B. Write an expression for R_{AB} as a function of A_c, L, ρ, F, and k.

Solution:

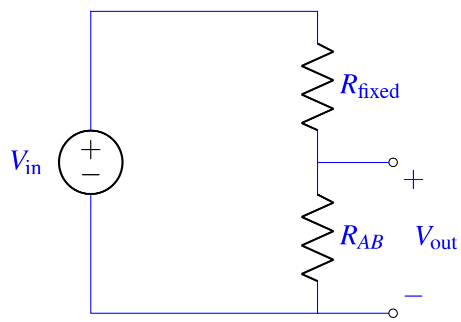
Note that, the length of each bar decreases by k meters per newton of force applied it, and since there are two bars supporting the scale, each bar can be thought to have half the force, $\frac{F}{2}$ on it. Because of this, each bar's length diminishes by $\frac{kF}{2}$, to have lengths of $L - \frac{kF}{2}$.

The combination of R_1 and R_2 has a resistance $R_{AB} = R_1 + R_2$, which is $R_{AB} = \frac{2\rho(L - \frac{kF}{2})}{A_c}$

(b) Fred's scale design is such that the resistance R_{AB} changes depending on how much weight is placed on it. However, he really wants to measure a voltage rather than a resistance.

Design a circuit for Fred that outputs a voltage that is some function of the weight. Your circuit should include R_{AB}, and you may use any number of voltage sources and resistors in your design. Be sure to label where the voltage should be measured in your circuit. Also provide an expression relating the output voltage of your circuit to the force applied on the scale.

Solution: If you utilized either of the resistances, R_{AB} indicated in the previous subpart, that is acceptable. One possible solution: use a voltage divider.

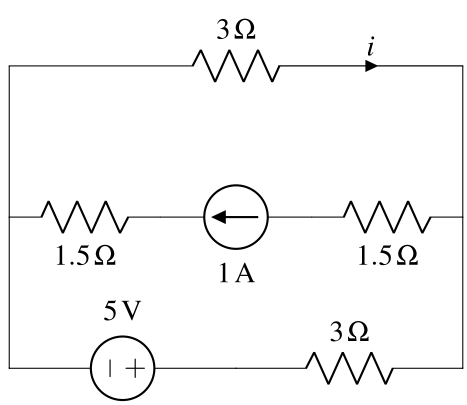


$$V_{\text{out}} = \frac{R_{AB}}{R_{\text{fixed}} + R_{AB}} V_{\text{in}}$$

$$V_{\text{out}} = \frac{\frac{2\rho(L - \frac{kF}{2})}{A_c}}{R_{\text{fixed}} + \frac{2\rho(L - \frac{kF}{2})}{A_c}} V_{\text{in}} = \frac{2\rho(L - \frac{kF}{2})}{R_{\text{fixed}} A_c + 2\rho(L - \frac{kF}{2})} V_{\text{in}}$$

6. Superposition

Find the current i indicated in the circuit diagram below using superposition.

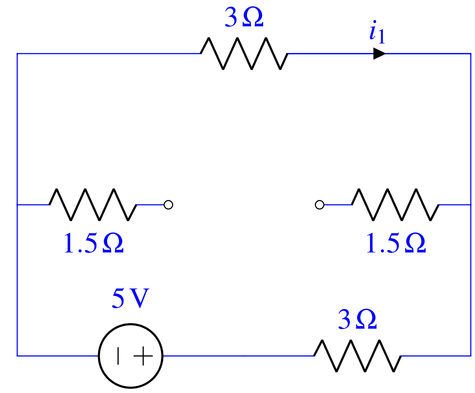


Solution:

$$i = -1/3\,\text{A}$$

Consider the circuits obtained by:

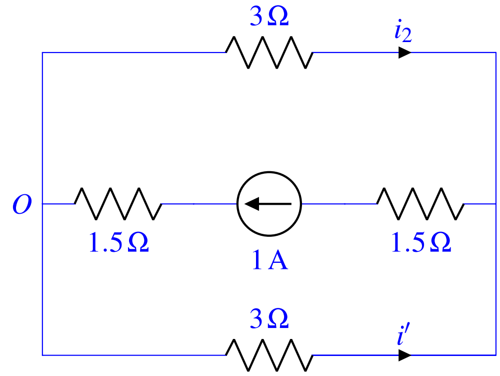
(a) Turning off the 1 A current source:



In the above circuit, no current is going to flow through the middle branch, as it is an open circuit. Thus this is just a 5 V voltage source connected to two 3 Ω resistors in series so

$$i_1 = -5/6\,\text{A}$$

(b) Turning off the 5 V voltage source:



In the above circuit, notice that the 3 Ω resistors are in parallel and therefore form a current divider. Since the values of the resistances are equal, the current flowing through them will also be equal, that is $i_2 = i'$. Applying KCL to node O, we get

$$1 - i_2 - i' = 0$$

which gives us

$$i_2 = 1/2\,\text{A}$$

Now, applying the principle of superposition, we have $i = i_1 + i_2 = -5/6\,\text{A} + 1/2\,\text{A} = -1/3\,\text{A}$.

7. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.

