EECS 16A Designing Information Devices and Systems I
Summer 2020 Homework 4B

This homework is due Sunday, July 26, 2020, at 23:59.
Self-grades are due Wednesday, July 29, 2020, at 23:59.

Submission Format
Your homework submission should consist of a single PDF file that contains all of your answers (any handwritten answers should be scanned).

Homework Learning Goals: *The objective of this homework is to familiarize you with capacitive modeling and measurement, as well as the use of capacitors as batteries. It also introduces circuit design using Op-Amps in negative feedback.*

1. **Capacitive Touchscreen**

The model for a capacitive touchscreen can be seen in Figure 1. See Table 1 for values of the dimensions. The green area represents the contact area of the finger with the top insulator. It has dimensions w_2 x d_1.

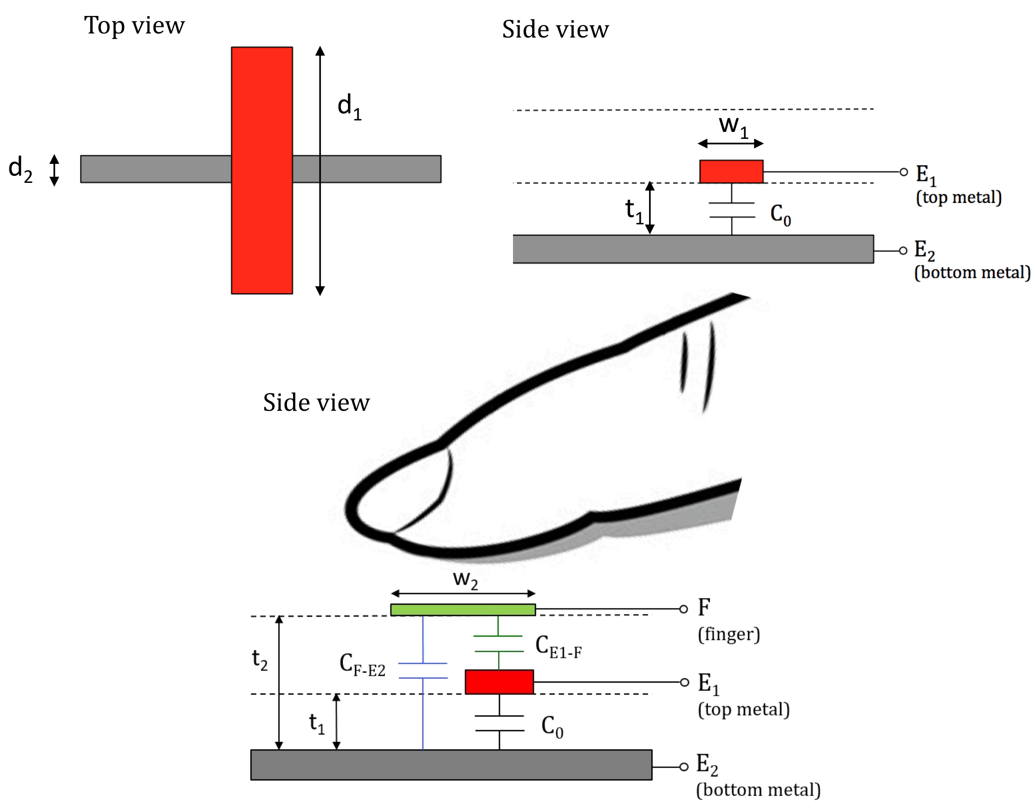


Figure 1: Model of capacitive touchscreen.

Table 1: Touchscreen Dimension Values

d_1	10 mm
d_2	1 mm
t_1	2 mm
t_2	4 mm
w_1	1 mm
w_2	2 mm

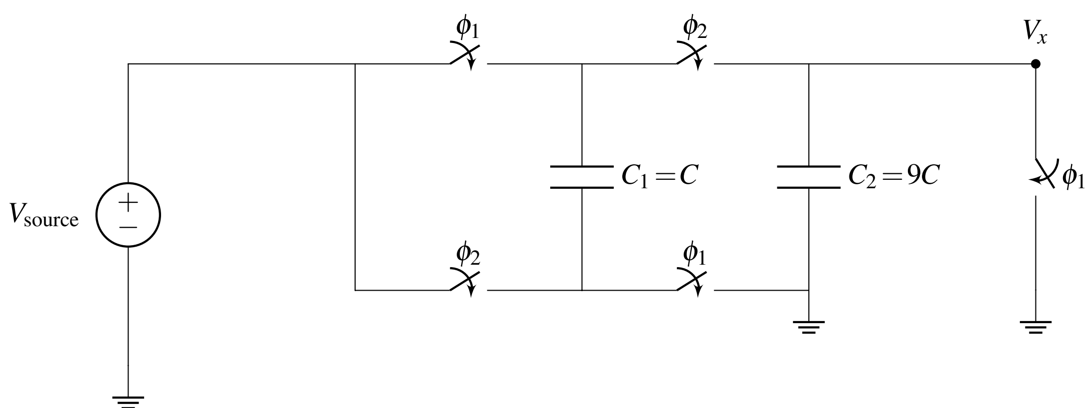
(a) Draw the equivalent circuit of the touchscreen that contains the nodes F, E_1, and E_2 when there no finger present and when there is a finger present. Express the capacitance values in terms of C_0, C_{F-E1}, and C_{F-E2}.

(b) What are the values of C_0, C_{F-E1}, and C_{F-E2}? Assume that the insulating material has a permittivity of $\varepsilon = 4.43 * 10^{-11} F/m$ and that the thickness of the metal layers is small compared to t_1.

(c) What is the difference in effective capacitance between the two metal plates (nodes E_1 and E_2) when a finger is present?

2. Charge Sharing

Consider the following circuit:



In the first phase, all of the switches labeled ϕ_1 will be closed and all switches labeled ϕ_2 will be open. In the second phase, all switches labeled ϕ_1 are opened and all switches labeled ϕ_2 are closed.

(a) Draw the polarity of the voltage (using $+$ and $-$ signs) across the two capacitors C_1 and C_2. (It doesn't matter which terminal you label $+$ or $-$; just remember to keep these consistent through phase 1 and 2!)

(b) Draw the circuit in the first phase and in the second phase. Keep your polarity from part (a) in mind.

(c) Find the voltage across and the charge on C_1 and C_2 in phase 1. Be sure to keep the polarities of the voltages the same!

(d) Now, in the second phase, find the voltage V_x.

(e) If the capacitor C_2 did not exist (i.e. had a capacitance of 0F), what would the voltage V_x be?

3. Super-Capacitors

In order to enable small devices for the "Internet of Things" (IoT), many companies and researchers are currently exploring alternative means of storing and delivering electrical power to the electronics within these devices. One example of these are "super-capacitors" - the devices generally behave just like a "normal" capacitor but have been engineered to have extremely high values of capacitance relative to other devices that fit in to the same physical volume. They can function as a power supply for low power applications such as IoT devices and have the advantage that they can be charged and discharged many times without losing maximum charge capacity. That property makes super-capacitors suitable to store energy from intermittent power sources such as those from energy harvesting. Suppose you are tasked with designing a power supply with a super-capacitor in an IoT device.

(a) Assuming that your electronic device (load) can be modeled as a constant current source with a value of i_{load}, draw circuit models for your device using super-capacitors as the power supply with the following configurations:

- Config 1: a single super-capacitor as the power supply
- Config 2: two super-capacitors stacked in series as the power supply
- Config 3: two super-capacitors connected in parallel as the power supply

(b) If each super-capacitor is charged to an initial voltage v_{init} and has a capacitance of C_{sc}, for each of the three configurations above, write an expression for the voltage supplied to your electronic device as a function of time after the device has been activated (i.e. connected to the super-capacitor(s)).

(c) Now let's assume that your electronic device requires some minimum voltage v_{min} in order to function properly. For each of the three super-capacitor configurations, write an expression for the lifetime of the device.

(d) Assume that a single super-capacitor doesn't provide you sufficient lifetime and so you have to spend the extra money (and device volume) for another super-capacitor. You consider the two following configurations:

- Config 2: two super-capacitors stacked in series
- Config 3: two super-capacitors connected in parallel

When is Config 3 (parallel) better than Config 2 (series)? Your answer should involve conditions on v_{init} and v_{min}.

(e) Calculate the amount of energy delivered by the super-capacitors in Config 3 (parallel) over the device's lifetime.

4. Op-Amp in Negative Feedback

In this question, we analyze op amp circuits that have finite gain. We replace the op amp with its circuit model with parameterized gain and observe the gain's effect on terminal and output voltages as the gain approaches infinity. Figure 2 shows the equivalent model of the op-amp.

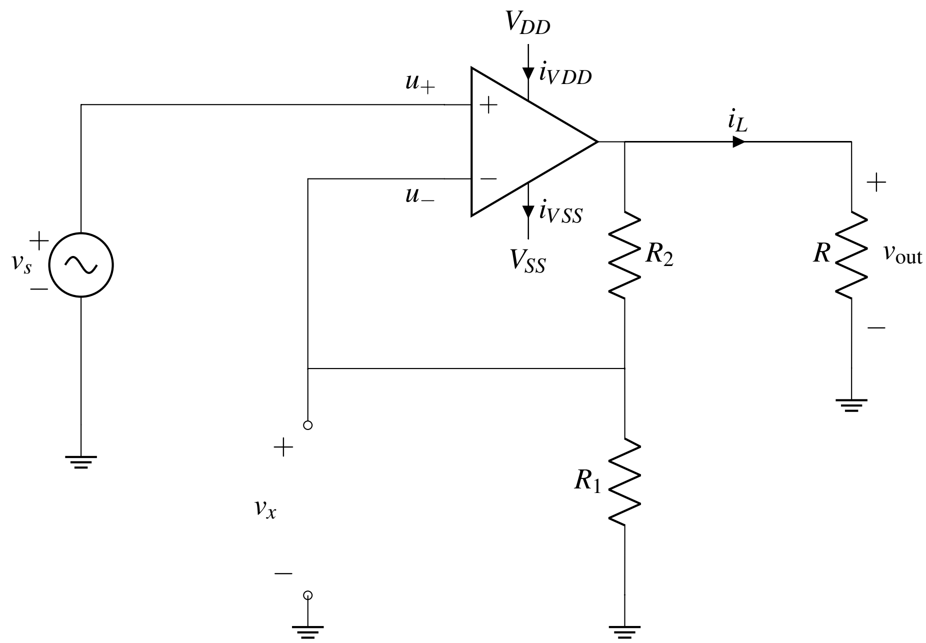


Figure 3: Non-inverting amplifier circuit

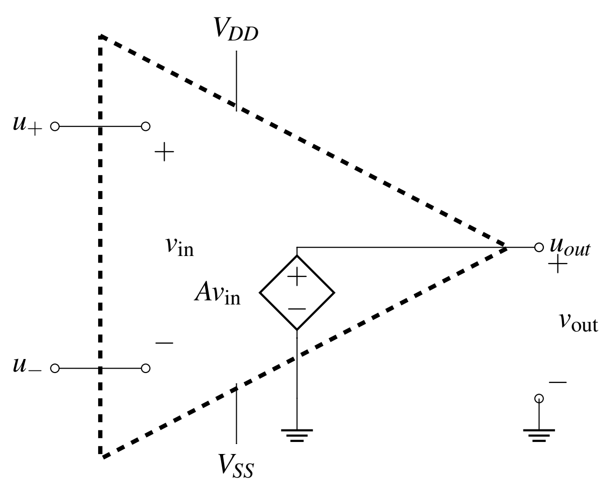


Figure 2: Op-amp model

(a) Consider the circuit shown in Figure 3. Assume that the op amp is ideal ($A \to \infty$) for parts (a) through (e). What is $u_+ - u_-$?

(b) Find v_x as a function of v_{out}.

(c) What is the current flowing through R_2 as a function of v_s?

(d) Find v_{out} as a function of v_s.

(e) What is the current i_L through the load resistor R? Give your answer in terms of v_{out}.

(f) We will now examine what happens when A is not ∞. Draw an equivalent circuit by replacing the op-amp with the op-amp model shown in Figure 2 and calculate v_{out} and v_x in terms of A, v_s, R_1, R_2

and R. Is the magnitude of v_x larger or smaller than the magnitude of v_s? Do these values depend on R?

(g) Using your solution to the previous part, calculate the limits of v_{out} and v_x as $A \to \infty$. Do you get the same answer as in part (d)?

(h) Now you want to make a circuit whose gain is nominally $G_{nom} = \frac{v_{out}}{v_s} = 4$ with a minimum error of 1% (a minimum gain of $G_{min} = 3.96$). What is the minimum required gain of the amplifier A_{min} to achieve that specification?

5. Cool For The Summer

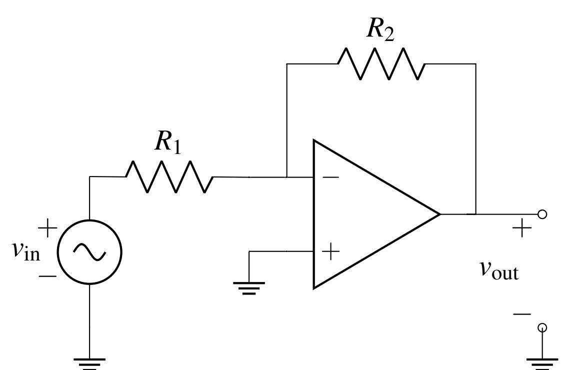
You and a friend want to make a box that helps control an air conditioning unit using both your inputs. You both have individual dials where you can set a control voltage: input of 0 means that you want to leave the temperature as it is. Negative voltage input would mean that you want to reduce the temperature. (It's hot, so we will assume that you never want to increase the temperature – so, we're not talking about a Berkeley summer...)

Your air conditioning unit, however, responds to positive voltages. The higher the magnitude of the voltage, the stronger it runs. At zero, it is off. You also need a system that sums up both you and your friend's control inputs.

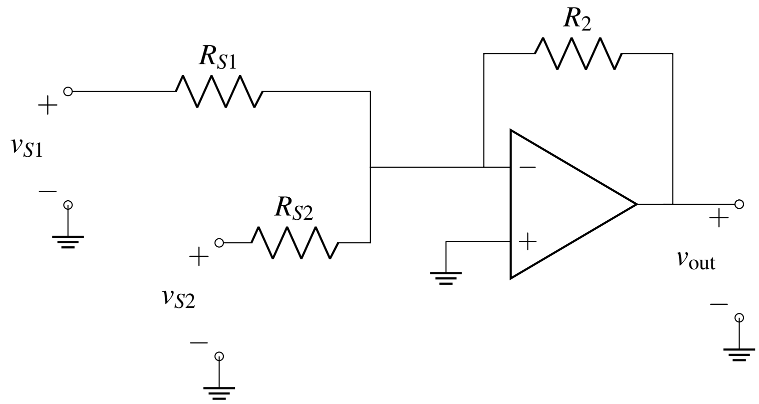
Therefore, you need a box that is **an inverting summer** – *it outputs a weighted sum of two voltages where the weights are both negative*. The sum is weighted because each of you has your own subjective sense of how much to turn the dial down, so you need to compensate for this.

This problem walks you through designing this inverting summer using an op-amp.

(a) As a first step, derive v_{out} in terms of R_2, R_1, v_{in}.



(b) Now we will add a second input to this circuit as shown below. Find v_{out} in terms of v_{S1}, v_{S2}, R_{S1}, R_{S2} and R_2.

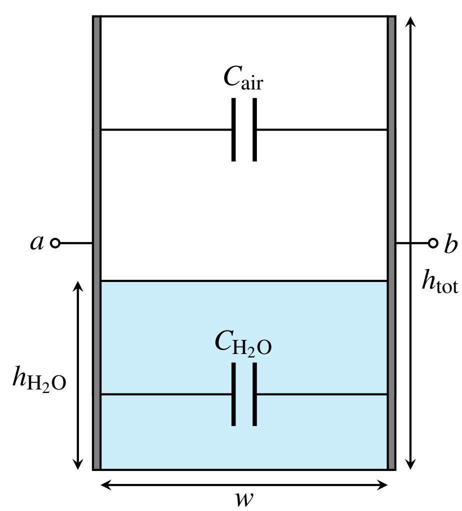


(c) Let's suppose that you want $v_{out} = -\left(\frac{1}{4}v_{S1} + 2v_{S2}\right)$ where v_{S1} and v_{S2} represent the input voltages from you and your friend. Select resistor values such that the circuit implements this desired relationship.

(d) Now suppose that you have a new AC unit that you want to use with your control inputs v_{S1} and v_{S2}. This unit, however, functions opposite to the previous unit; it responds to negative voltages. The higher the magnitude of the negative voltage, the stronger the AC runs.
Now design a circuit that *outputs a weighted sum of two control input voltages where both weights are positive*. Specifically, add another op-amp based circuit to your circuit in part (b), so that you invert the output of the circuit from part (b).

6. It's finally raining!

A lettuce farmer in Salinas Valley has grown tired of weather.com's imprecise rain measurements. Therefore, they decided to take matters into their own hands by building a rain sensor. They placed a rectangular tank outside and attached two metal plates to two opposite sides in an effort to make a capacitor whose capacitance varies with the amount of water inside.

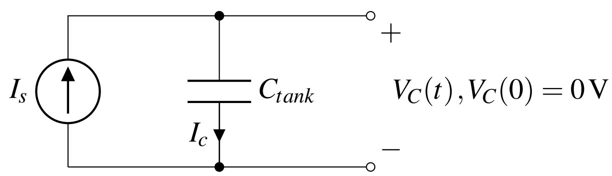


The width and length of the tank are both w (i.e., the base is square) and the height of the tank is h_{tot}.

(a) What is the capacitance between terminals a and b when the tank is full? What about when it is empty?
Note: the permittivity of air is ε, and the permittivity of rainwater is 81ε.

(b) Suppose the height of the water in the tank is h_{H_2O}. Modeling the tank as a pair of capacitors in parallel, find the total capacitance between the two plates. Call this capacitance C_{tank}.

(c) After building this capacitor, the farmer consults the internet to assist them with a capacitance-measuring circuit. A fellow internet user recommends the following:



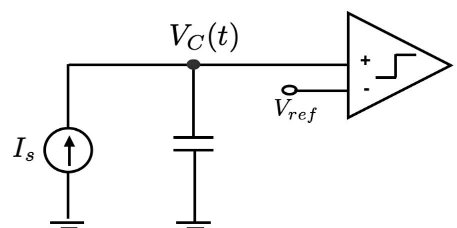
In this circuit, C_{tank} is the total tank capacitance that you calculated earlier. I_s is a known current supplied by a current source.

The suggestion is to measure V_C for a brief interval of time, and then use the difference to determine C_{tank}.

Determine $V_C(t)$, where t is the number of seconds elapsed since the start of the measurement. You should assume that before any measurements are taken, the voltage across C_{tank}, i.e. V_C, is initialized to $0\,\mathrm{V}$, i.e. $V_C(0) = 0$.

(d) Using the equation you derived for $V_C(t)$, describe how you can use this circuit to determine C_{tank} and h_{H_2O}.

(e) However, after spending some time thinking about different ways of measuring this capacitance you came up with a better idea. You decided to use the circuit proposed in part (c) along with a comparator, as show in the figure below. What you are basically interested in, is the time T_1 needed for V_c to reach V_{ref}. In order to measure time you use a timer. When voltage V_c becomes larger than V_{ref}, the comparator flips its value and you stop the timer. How would you measure in that case the value of the capacitance?



7. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?