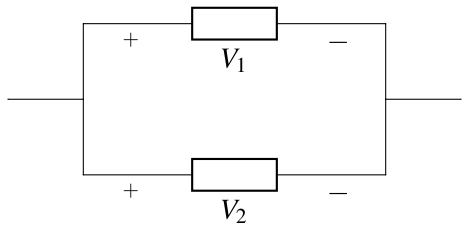
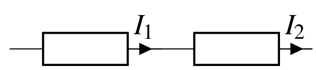
EECS 16A Designing Information Devices and Systems I
Summer 2020 Module 2 Practice Handout

1. Circuits Intuition Practice

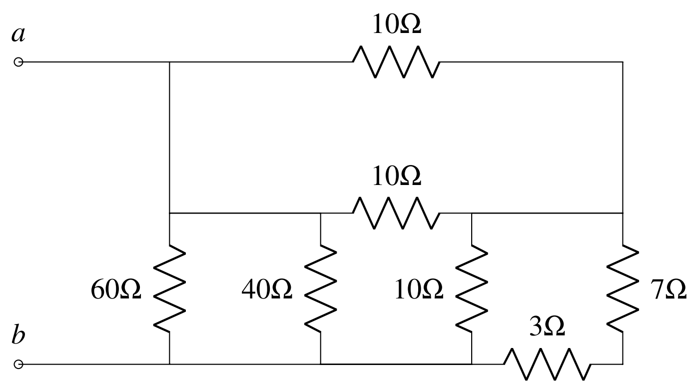
(a) What does KVL tell you about V_1 and V_2 for any elements connected to the same pair of nodes?



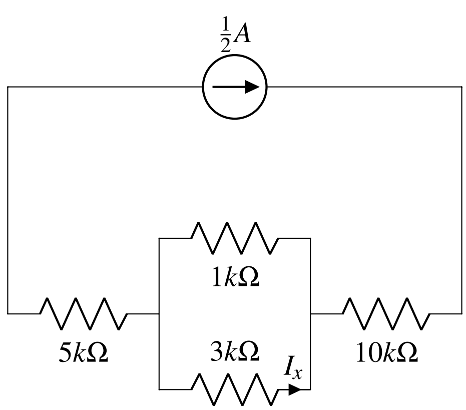
(b) What does KCL tell you about I_1 and I_2 for any two elements connected to a node with nothing else connected to that node?



(c) Find R_{ab}, the equivalent resistance between terminals a and b. Give your answer as a number, or an expression involving no more than one use of $||$.



(d) Find I_x. (Hint: Can you see the current divider?)



2. Digital to Analog Converter (DAC)

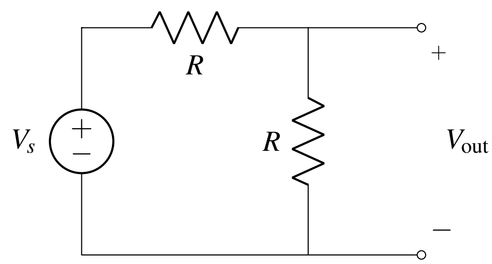
For some outputs, such as audio applications, we need to produce an analog output, or a continuous voltage from 0 to V_s. These analog voltages must be produced from digital voltages, that is sources, that can only be V_s or 0. A circuit that does this is known as a Digital to Analog Converter. It takes a binary representation of a number and turns it into an analog voltage.

The output of a DAC can be represented with the equation shown below:

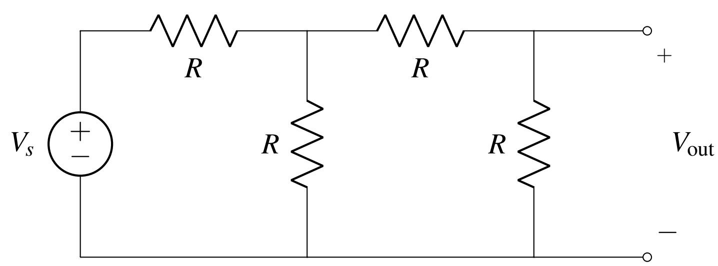
$$V_{\text{out}} = V_s \sum_{n=0}^{N} \frac{1}{2^n} \cdot b_n$$

where each binary digit b_n is multiplied by $\frac{1}{2^n}$.

(a) We know how to take an input voltage and divide it by 2:



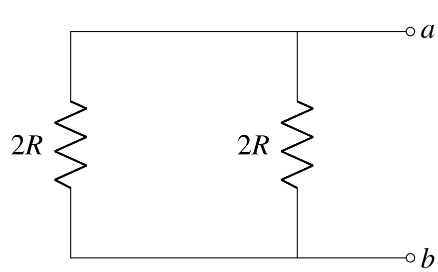
To divide by larger powers of two, we might hope to just "cascade" the above voltage divider. For example, consider:



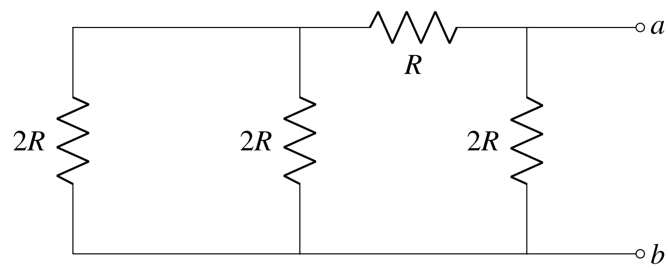
Calculate V_{out} in the above circut. Is $V_{\text{out}} = \frac{1}{4}V_s$?

(b) The R-$2R$ ladder, shown below, has a very nice property. For each of the circuits shown below, find the equivalent resistance looking in from points a and b. Do you see a pattern?

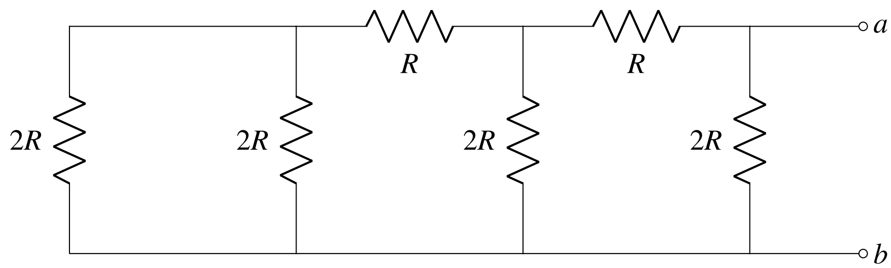
i.



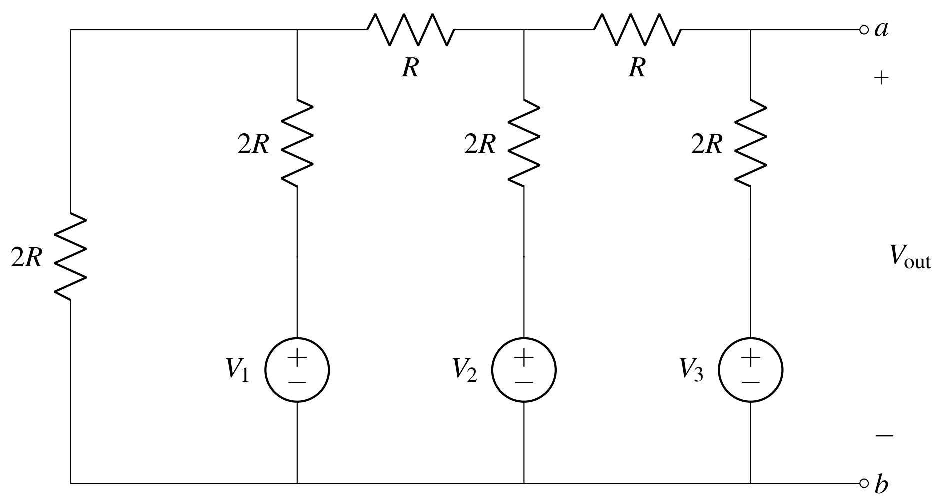
ii.



iii.



(c) The following circuit is an R-$2R$ DAC. To understand its functionality, use superposition to find V_{out} in terms of each V_k in the circuit.



(d) We've now designed a 3-bit R-$2R$ DAC. What is the output voltage V_{out} if $V_2 = 1\,\text{V}$ and $V_1 = V_3 = 0\,\text{V}$?

3. Measuring Voltage and Current

In order to measure quantities such as voltage and current, engineers use voltmeters and ammeters. A simple model of a voltmeter is a resistor with a very high resistance, R_{VM}. **The voltmeter measures the voltage across the resistance** R_{VM}. The measured voltage is then relayed to a microprocessor (such as the MSP430s used in Lab).

This model of an voltmeter is shown in Figure **??**. Let us explore what happens when we connect this voltmeter to various circuits to measure voltages.
Throughout this problem assume $R_{\text{VM}} = 1M\Omega$. Recall that the SI prefix M or Mega is 10^6.

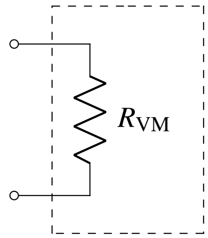


Figure 1: Our model of a voltmeter, $R_{VM} = 1M\Omega$

(a) Suppose we wanted to measure the voltage across R_2 (v_{out}) produced by the voltage divider circuit shown in Figure **??** on the left. The circuit on the right in Figure **??** shows how we would connect the voltmeter across R_2. Assume $R_1 = 100\Omega$ and $R_2 = 200\Omega$.
First calculate the value of v_{out}. Then calculate the voltage the voltmeter would measure, i.e. v_{meas}.

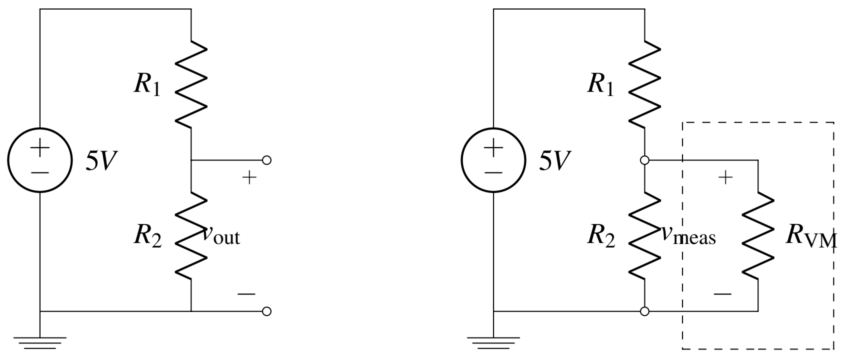


Figure 2: Left: Circuit without the voltmeter connected, Right: Voltmeter measuring voltage across R_2

(b) Repeat part (a), but now $R_1 = 10M\Omega$ and $R_2 = 10M\Omega$. Is this voltmeter still a good tool to measure the output voltage?

(c) Now suppose we are working with the same circuit as in Part (a), but we know that $R_2 = R_1$. What is the maximum value of R_1 that ensures that the difference between voltage measurement of the voltmeter (v_{meas}) and the actual value (v_{out}) remains within $\pm 10\%$ of v_{out}?

(d) Using the combination of our voltmeter and an additional resistor R_x, we can make an ammeter and measure the current through an element. Using the circuit shown in Figure **??**, where $R_x = 1\,\Omega$, then the measured current through R_x is $I_{meas} = \frac{V_{VM}}{R_x}$ where V_{VM} is the voltage across the voltmeter.
In Figure **??**, the voltmeter is connected to measure the current through resistor $R_1 = 1k\Omega$. For the circuit on the left, find the current through R_1 without the voltmeter connected (i.e. I_1). Then, for the circuit on the right, find the current measured by the voltmeter when it is connected as an ammeter (i.e. I_{meas}).

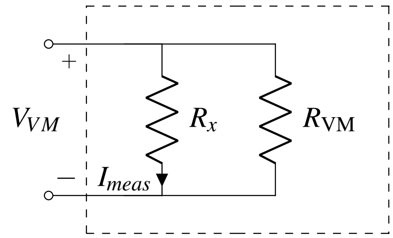


Figure 3: The voltmeter combined with resistor R_x to function as an ammeter (i.e. to measure current), $R_{\text{VM}} = 1M\Omega$.

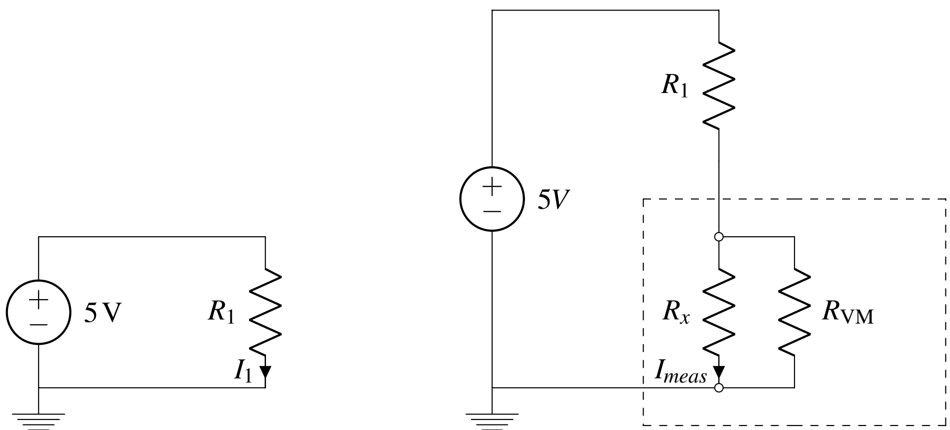


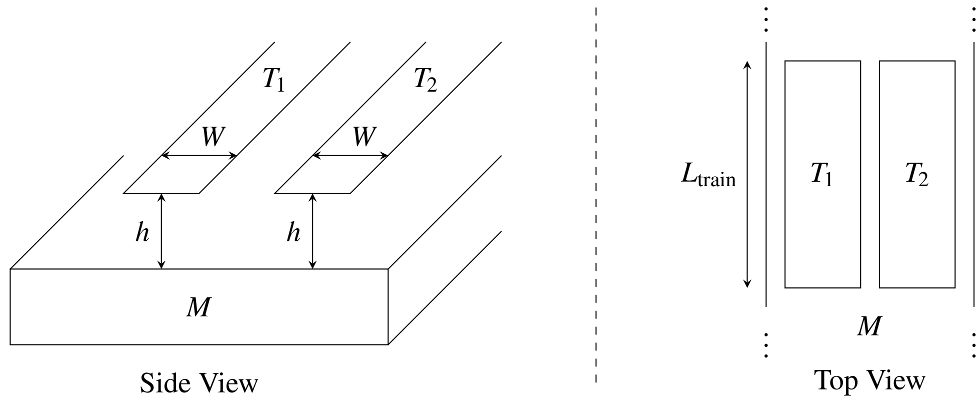
Figure 4: Circuits for Part (d) Left: Original circuit; Right: Circuit with the voltmeter connected as an ammeter.

(e) What is the minimum value of R_1 that ensures the difference between current measurement (I_{meas}) and the the actual value (I_1) stays within $\pm 10\%$ of I_1?

4. Maglev Train Height Control System

One of the fastest forms of land transportation are trains that actually travel slightly elevated from the ground using magnetic levitation (or "maglev" for short). Ensuring that the train stays at a relatively constant height above its "tracks" (the tracks in this case are what provide the force to levitate the train and propel it forward) is critical to both the safety and fuel efficiency of the train. In this problem, we'll explore how maglev trains use capacitors to stay elevated. (Note that real maglev trains may use completely different and much more sophisticated techniques to perform this function, so if you get a contract to build such a train, you'll probably want to do more research on the subject.)

(a) As shown below, we put two parallel strips of metal (T_1, T_2) along the bottom of the train and we have one solid piece of metal (M) on the ground below the train (perhaps as part of the track).



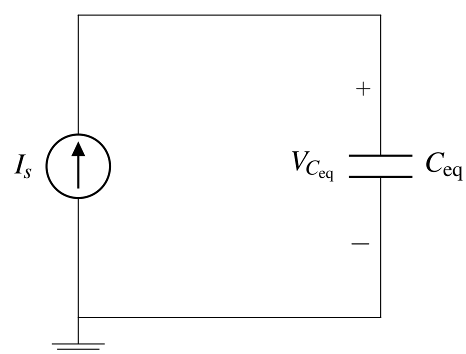
Assuming that the entire train is at a uniform height above the track and ignoring any fringing fields (i.e., we can use the simple equations developed in lecture to model the capacitance), as a function of L_{train} (the length of the train), W (the width of T_1 and T_2), and h (the height of the train off of the track), what is the capacitance between T_1 and M? What is the capacitance between T_2 and M?

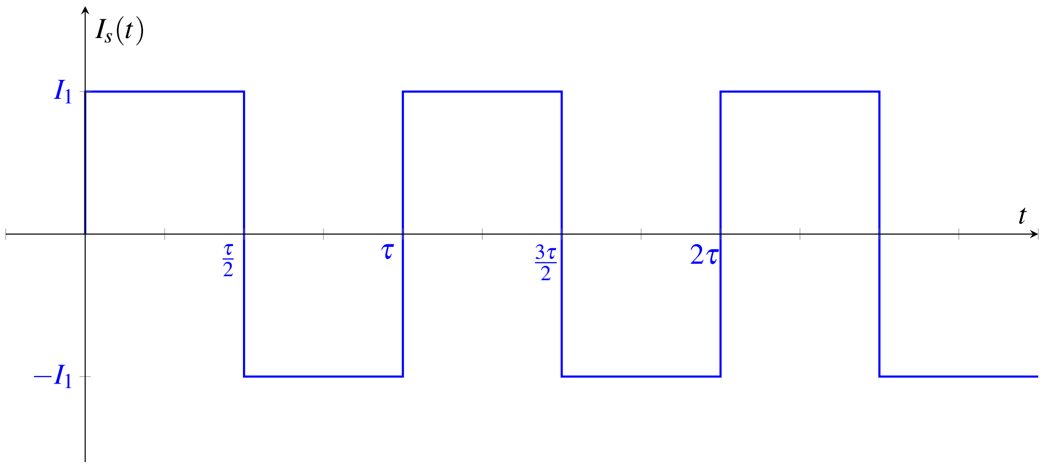
(b) Any circuit on the train can only make direct contact at T_1 and T_2. Thus, you can only measure the equivalent capacitance between T_1 and T_2. Draw a circuit model showing how the capacitors between T_1 and M and between T_2 and M are connected to each other.

(c) Using the same parameters as in part (a), provide an expression for the equivalent capacitance between T_1 and T_2.

(d) We want to build a circuit that creates a voltage waveform with an amplitude that changes based on the height of the train. Your colleague recommends you start with the circuit as shown below, where I_s is a periodic current source, and C_{eq} is the equivalent capacitance between T_1 and T_2. The graph below shows I_s, a square wave with period τ and amplitude I_1, as a function of time.

Find an equation for and draw the voltage $V_{C_{\text{eq}}}(t)$ as a function of time. Assume the capacitor C_{eq} is discharged at time $t = 0$, so $V_{C_{\text{eq}}}(0) = 0\,\text{V}$.





(e) We now want to develop an indicator that alerts us when the train is too high above the tracks. We want to output a series of 5 V pulses that can be used to drive a horn when the train is above 1 cm, and not output anything when the train is below 1 cm.

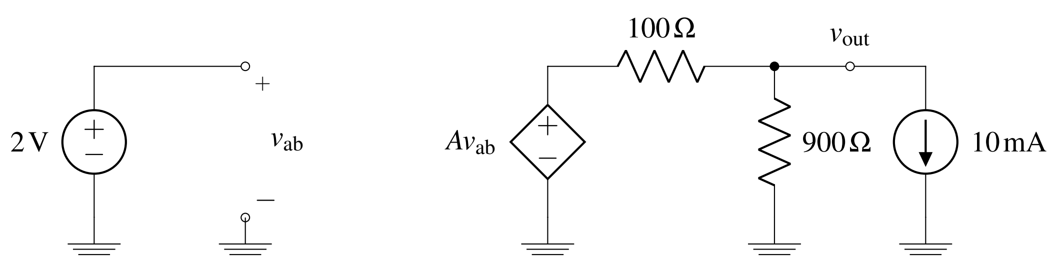
We will assume the train has length $L_{\text{train}} = 100\,\text{m}$ and that the metals, T_1 and T_2, have width $W = 1\,\text{cm}$ and permitivity $\varepsilon = 8.85 \times 10^{-12}\,\text{F}\,\text{m}^{-1}$.

Design a circuit using a square wave current source (i.e. I_s in part (d)) with period $\tau = 1\,\mu\text{s}$ and pulses of amplitude $I_1 = 1\,\text{mA}$, a comparator, and any number of voltage sources to implement this function. **Hint: you should use the circuit you analyzed in part (d).**

(f) So far we've assumed that the height of the train off of the track is uniform along its entire length, but in practice, this may not be the case. Suggest and sketch a modification to the basic sensor design (i.e., the two strips of metal T_1 and T_2 along the entire bottom of the train) that would allow you to measure the height at the train at 4 different locations.

5. Superposition with a Dependent Source

Given $A = 5$, find the voltage v_{out} indicated in the circuit diagram below using superposition.



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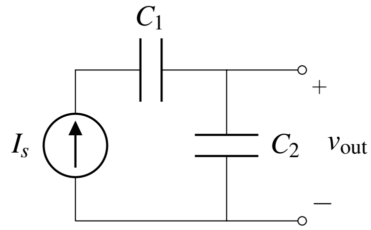
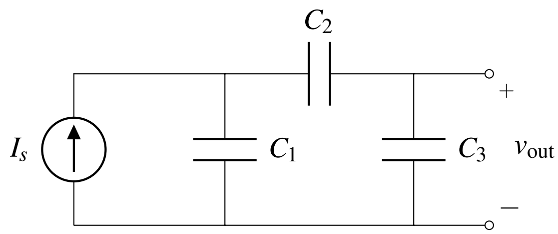
6. Current Sources And Capacitors

For the circuit given below, give an expression for $v_{\text{out}}(t)$ in terms of I_s, C_1, C_2, and t. Assume that all capacitors are initially uncharged, i.e. the initial voltage across each capacitor is 0 V.



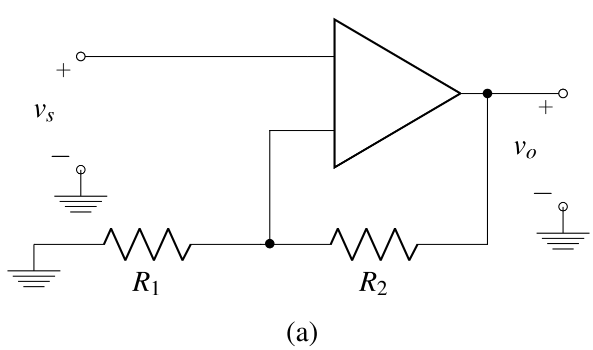
7. More Current Sources And Capacitors

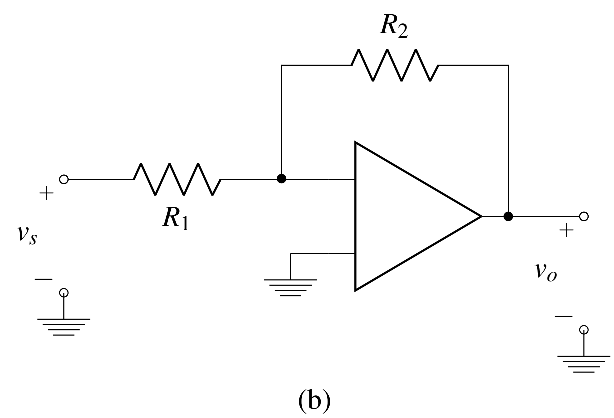
For the circuit given below, give an expression for $v_{\text{out}}(t)$ in terms of I_s, C_1, C_2, C_3, and t. Assume that all capacitors are initially uncharged, i.e. the initial voltage across each capacitor is 0 V.



8. Basic Amplifier Building Blocks

The following amplifier stages are used often in many circuits and are well known as (a) the non-inverting amplifier and (b) the inverting amplifier.



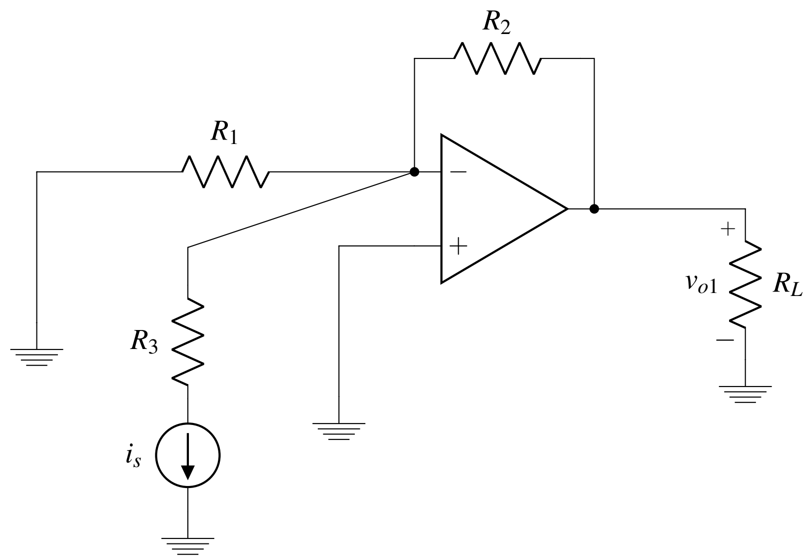


(a) Label the input terminals of the op-amp labeled (a), so that it is in negative feedback. Then derive the voltage gain ($A_v = \frac{v_o}{v_s}$) of the non-inverting amplifier using the Golden Rules. Explain the origin of the name of the amplifier.

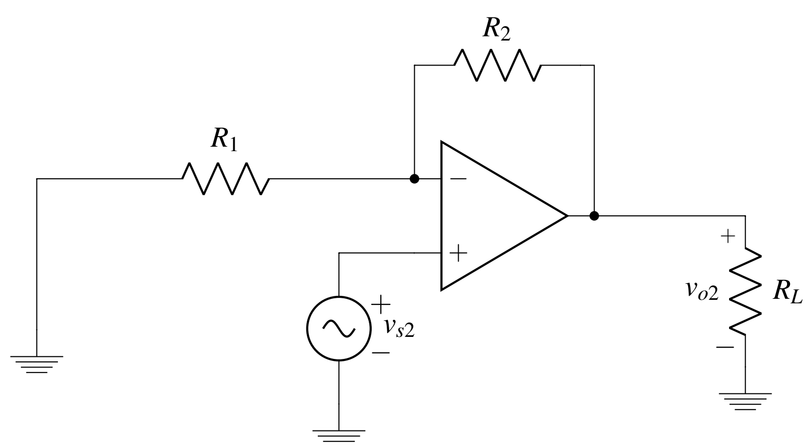
(b) Label the input terminals of the op-amp labeled (b), so that it is negative feedback. Then derive the voltage gain ($A_v = \frac{v_o}{v_s}$) of the inverting amplifier using the Golden Rules. Explain the origin of the name of the amplifier.

9. Amplifier with Multiple Inputs

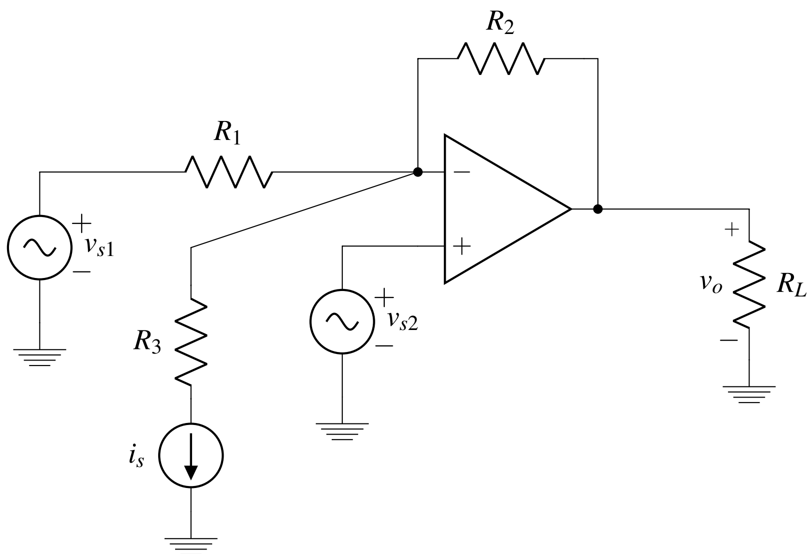
(a) Use the Golden Rules to find v_{o1} for the circuit below.



(b) Use the Golden Rules to find v_{o2} for the circuit below.

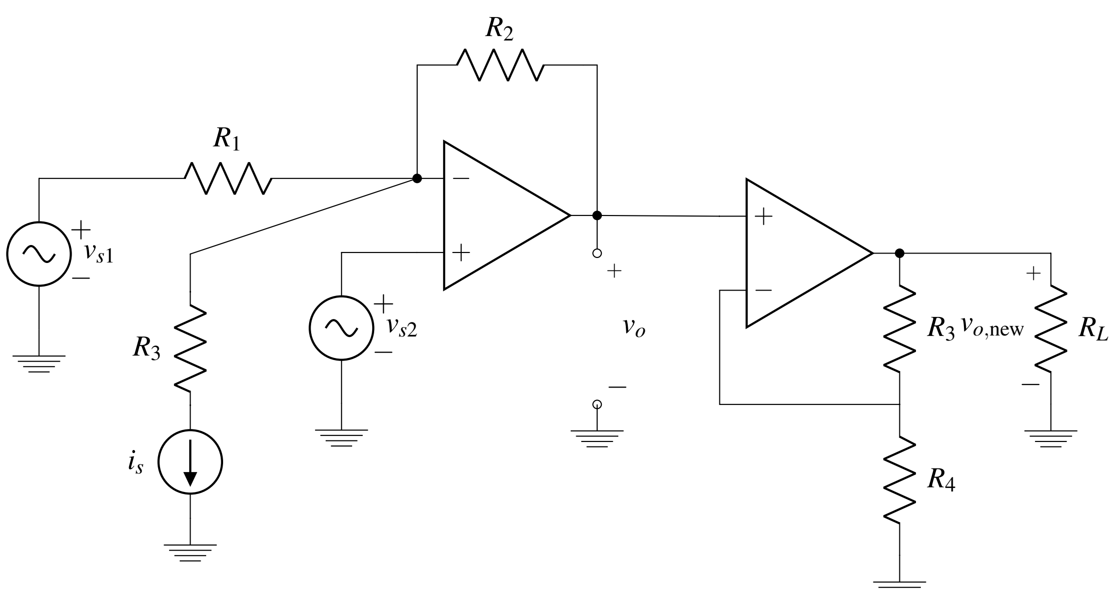


(c) Use the Golden Rules to find the output voltage v_o for the circuit shown below.



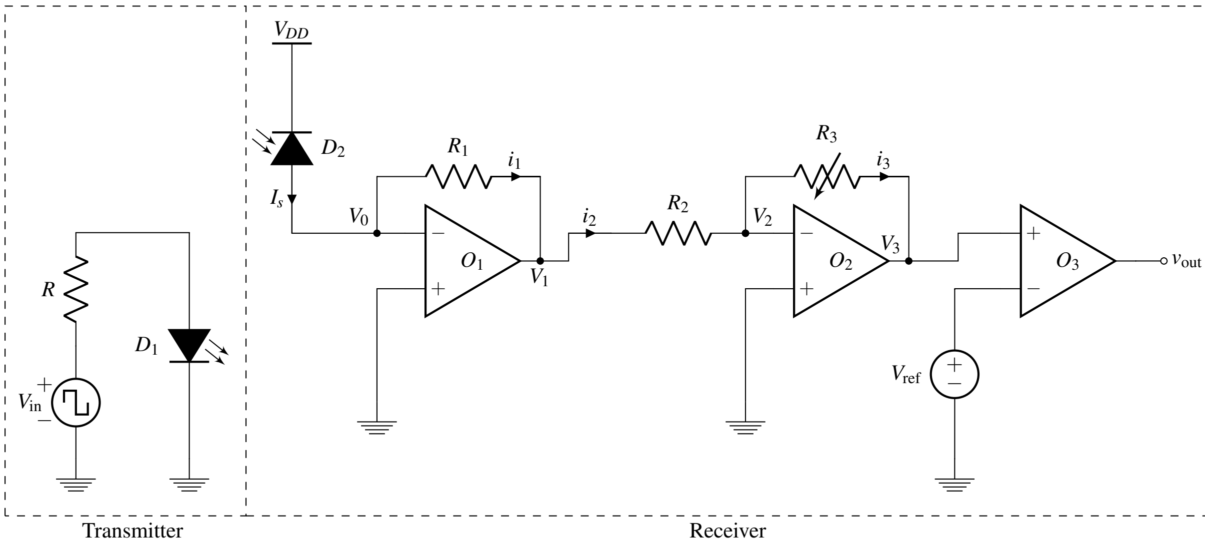
(d) Use superposition and the answers to the first few parts of this problem to check your work.

(e) Now add a second stage as shown below. What is $v_{o,\text{new}}$? Does v_o change between part (c) and this part? Does the voltage $v_{o,\text{new}}$ depend on R_L?



10. Wireless Communication With An LED

In this question, we are going to analyze the system shown in the figure below. It shows a circuit that can be used as a wireless communication system using visible light (or infrared, very similar to remote controls).



The element D_1 in the transmitter is a light-emitting diode (LED). An LED is an element that emits light and whose brightness is controlled by the current flowing through it. You can recall controlling the light emitted by an LED using your MSP430 in Touchscreen Lab 1. In our circuit, the current across the LED, hence its brightness, can be controlled by choosing the applied voltage V_{in} and the value of the resistor R. In the receiver, the element labeled as D_2 is a reverse biased solar cell. You can recall using an ambient light sensor in Imaging Labs 1 to 3 as a light-controlled current source. We will denote the current supplied by the solar cell by I_S. In this circuit, the LED D_1 is used as a means for transmitting information with light, and the reverse-biased solar cell D_2 is used as a light receiver to see if anything was transmitted.

Remark: In Imaging Lab 3, we talked about how non-idealities, such as background light, affect the performance of a system that does light measurements. In this question, we will assume ideal conditions, that is, there is no source of light around except for the LED.

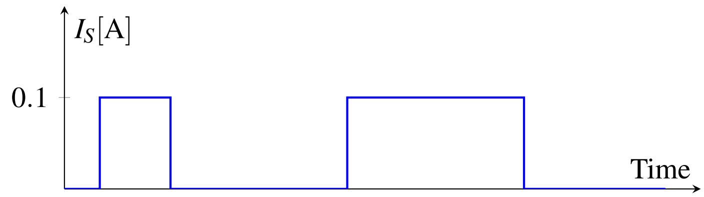
In our system, we define two states for the transmitter: the *transmitter is sending something* when they turn on the LED and the *transmitter is not sending anything* when they turn off the LED. On the receiver side, the goal is to convert the current I_S generated by the solar cell into a voltage and amplify it, so that we can read the output voltage V_{out} to see if the transmitter was sending something or not. The circuit implements this operation through a series of op-amps. It might look look complicated at first glance, but we can analyze it one section at a time.

(a) Currents i_1, i_2 and i_3 are labeled on the diagram. Assuming the Golden Rules hold, is $I_S = i_1$? $i_1 = i_2$? $i_2 = i_3$? Treat the solar cell as an ideal current source.

(b) Use the Golden Rules to find V_0, V_1, V_2 and V_3 in terms of I_S, R_1, R_2 and R_3.
Hint: Solve for them from left to right, and remember to use the Golden Rules.

(c) In the previous part, how could you check your work to gain confidence that you got the right answer?

(d) Now, assume that the transmitter has chosen the values of V_{in} and R to control the intensity of light emitted by LED, such that when the *transmitter is sending something*, I_S is equal to 0.1 A and when the *transmitter is not sending anything*, I_S is equal to 0 A. The following figure shows a visual example of how this current I_S might look like as time changes (note that this is just to help you visualize the shape of the current supplied by the solar cell).

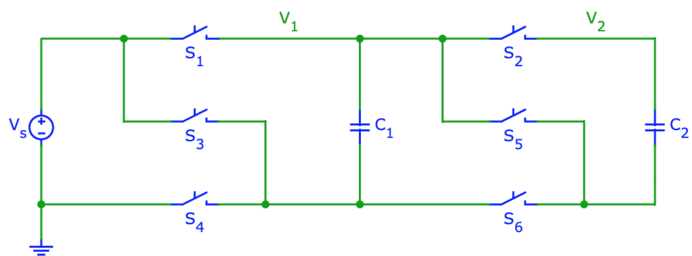


For the receiver, suppose $V_{\text{ref}} = 2\,\text{V}$, $R_1 = 10\,\Omega$, $R_2 = 1000\,\Omega$, and the supply voltages of the op-amps are $V_{\text{DD}} = 5\,\text{V}$ and $V_{\text{SS}} = -5\,\text{V}$. Pick a value of R_3 such that V_{out} is V_{DD} when the *transmitter is sending something* and V_{SS} when the *transmitter is not sending anything*?

(e) In the previous part, how could you check your work to gain confidence that you got the right answer?

11. Voltage Booster

We have made extensive use of resistive voltage dividers to reduce voltage. What about a circuit that boosts voltage to a value greater than the supply $V_S = 5V$? We can do this with capacitors!



(a) In the circuit above switches S1, S2, S4 and S6 are initially closed and switches S3 and S5 open. Calculate voltages V_1 and V_2.

(b) Now, after the capacitors are charged, switches S1, S2, S4 and S6 are opened and switches S3 and S5 closed. Calculate the new voltages V_1 and V_2.

12. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

