EECS16A DIS 3B

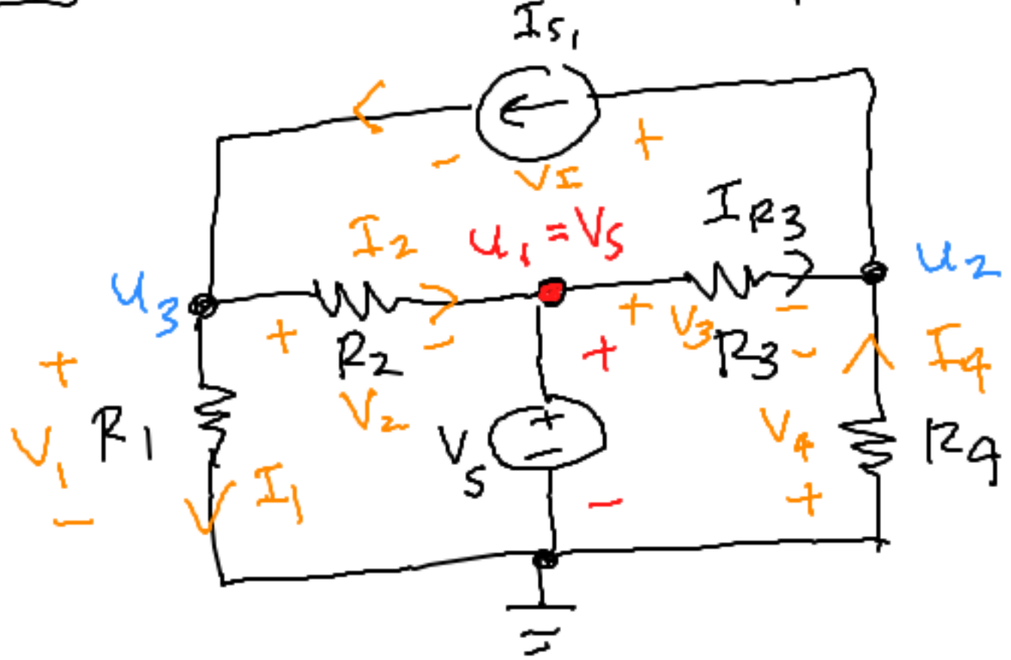
Today's Topics

1. Move NVA/Nodal analysis
2. Current Divider derivation
3. Current Divider demo
4. If time leftover: DIS3BQ3

Some notes from DIS3A

"Element voltages" ↔ "Branch voltages" ← same thing

1) Use nodal analysis to find all node voltages



KCL @ u_2 : $I_{R_3} + I_4 = I_{S_1}$

KCL @ u_3 : $I_{S_1} = I_1 + I_2$

$V_1 = R_1 I_1$, $V_2 = R_2 I_2$, $V_3 = R_3 I_{R_3}$, $V_4 = I_4 R_4$

$$I_1 = \frac{u_3 - 0}{R_1}, \quad I_2 = \frac{u_3 - V_S}{R_2}, \quad I_{R_3} = \frac{V_S - u_2}{R_3}$$

$$I_4 = \frac{0 - u_2}{R_4}$$

✓ (1) Pick and label ground

✓ (2) Label all nodes w/ voltages set by voltage sources

$V_S = u_1 - 0$ (branch V)

✓ (3) Label remaining nodes u_3, u_2

✓ (4) Label branch/element I's V's according to passive sign convention

I_1, V_I, I_2, I_3, I_4, V_2, V_1, V_3, V_4

✓ (5) KCL eqns. (write only for unk. nodes)

✓ (6) Element I's → Element V's (ohm's)

✓ (7) Write Element V's → node V's

(8) Sub. into KCL, solve.

⑧ Sub. into KCL (and solve for node V's)

$$I_{R_3} + I_4 = I_{S_1} \quad I_{S_1} = I_1 + I_2$$

$$\Leftarrow \begin{cases} I_1 = \dfrac{u_3}{R_1} & I_2 = \dfrac{u_3 - V_S}{R_2} \\ I_{R_3} = \dfrac{V_S - u_2}{R_3} & I_4 = \dfrac{-u_2}{R_4} \end{cases}$$

$$\left\{ \frac{V_S - u_2}{R_3} + \frac{-u_2}{R_4} = I_{S_1}, \quad I_{S_1} = \frac{u_3}{R_1} + \frac{u_3 - V_S}{R_2} \right\}$$ system of eqns. in u_2, u_3

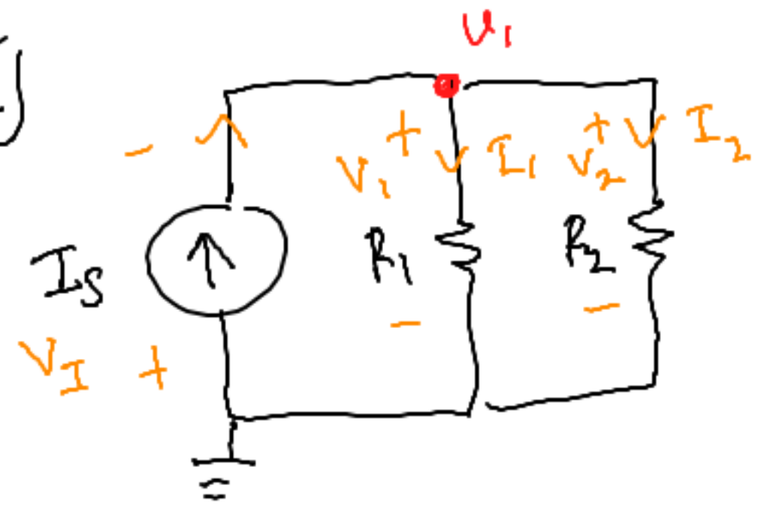
$$-I_{S_1} + \frac{u_2}{R_3} + \frac{u_2}{R_4} \qquad -I_{S_1} + \frac{u_2}{R_3} + \frac{u_2}{R_4}$$

$$\frac{V_S}{R_3} - I_{S_1} = u_2\left(\frac{1}{R_3} + \frac{1}{R_4}\right) \qquad I_{S_1} + \frac{V_S}{R_2} = u_3\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \rightarrow u_2, u_3 \text{ can be found}$$

⑥ Find: $I_{R_3} = \dfrac{V_S - u_2}{R_3}$ ✓

$$u_2 = \frac{R_4}{R_3 + R_4} V_S - \frac{R_3 R_4}{R_3 + R_4} I_{S_1}$$

2)



KCL @ u_1 : $I_S = I_1 + I_2$

Branch I / Branch V relations: $V_1 = R_1 I_1$

$V_2 = R_2 I_2$

Node voltages ~ I's : $I_1 = \frac{u_1 - 0}{R_1}$

$I_2 = \frac{u_1 - 0}{R_2}$

Sub, solve:

$$I_S = \frac{u_1}{R_1} + \frac{u_1}{R_2}$$

$$u_1 = \frac{R_1 R_2 \cdot I_S}{R_1 R_2 \frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_2 + R_1} I_S$$

Current divider "splits up current into two paths"

What is I_1 and I_2?

$$I_1 = \frac{u_1}{R_1} = \frac{1}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2} I_S\right)$$

$$\boxed{I_1 = \frac{R_2}{R_1 + R_2} I_S}$$

$$\boxed{I_2 = \frac{R_1}{R_1 + R_2} I_S}$$

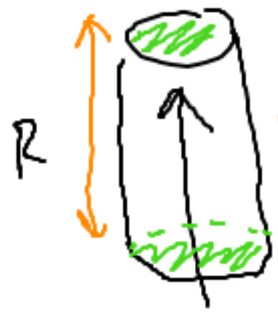
Case 1: $R_1 \gg R_2$

$\frac{R_1}{R_1 + R_2} \sim 1$ $\quad \frac{R_2}{R_1 + R_2} \sim 0$

$I_2 \approx I_S$ $\quad I_1 \approx 0$

$\Rightarrow$ "current takes path of least resistance"

3 $R = \rho \frac{L}{A}$



ρ : material property

L : length of resistor

A : cross sectional