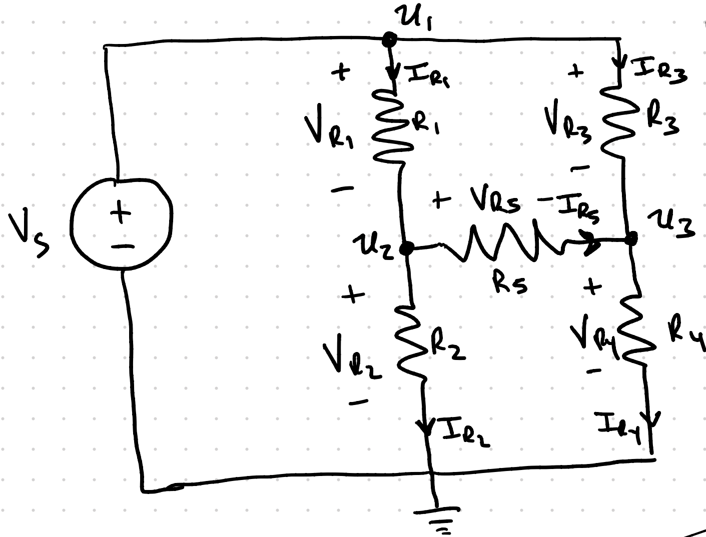
Steps of NVA

1) Define/label a reference node
2) Label all nodes whose voltage is set by a voltage source
3) Label all other nodes
4) Label all element voltages and currents following the "passive sign convention"
5) Write KCL expressions for all nodes with unknown voltages.
6) Substitute element voltages and resistances for the currents in step $\boxed{5}$ (using Ohm's Law)
7) Substitute node voltages for element voltages
8) solve system for unknown node voltages



Q: Find all unknown node voltages

$u_1 = V_s$

KCL for u_2: $I_{R_1} = I_{R_2} + I_{R_5}$

KCL for u_3: $I_{R_5} + I_{R_3} = I_{R_4}$

$$\frac{V_{R_1}}{R_1} = \frac{V_{R_2}}{R_2} + \frac{V_{R_5}}{R_5}$$

$$\frac{V_{R_5}}{R_5} + \frac{V_{R_3}}{R_3} = \frac{V_{R_4}}{R_4}$$

$$\frac{u_1 - u_2}{R_1} = \frac{u_2}{R_2} + \frac{u_2 - u_3}{R_5}$$

$$\frac{u_2 - u_3}{R_5} + \frac{u_1 - u_3}{R_3} = \frac{u_3}{R_4}$$

$V_s = 5\,V$

$R_1 = 1\,\Omega$

$R_2 = 2\,\Omega$

$R_3 = 3\,\Omega$

$R_4 = 4\,\Omega$

$R_5 = 5\,\Omega$

$$\frac{u_1 - u_2}{R_1} = \frac{u_2}{R_2} + \frac{u_2 - u_3}{R_5}$$

$$\frac{u_2 - u_3}{R_5} + \frac{u_1 - u_3}{R_3} = \frac{u_3}{R_4} \Longrightarrow \frac{u_2}{5} + \frac{5}{3} = \frac{u_3}{4} + \frac{u_3}{5} + \frac{u_3}{3}$$

solve for u_3

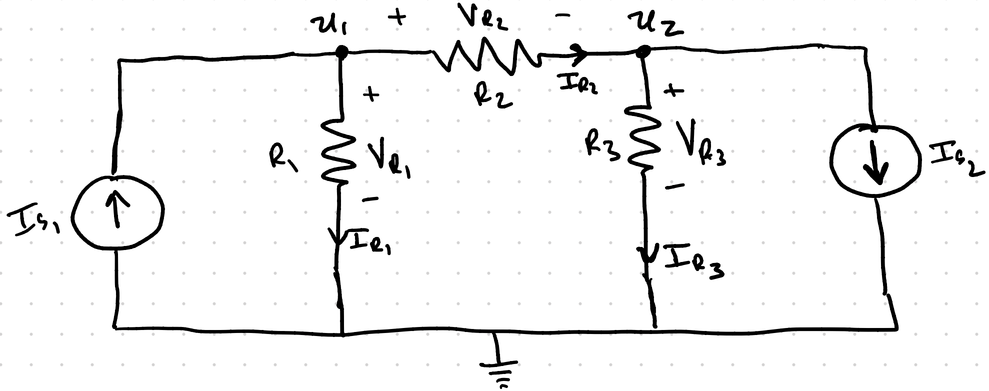
$$\Longrightarrow \left(\frac{u_2}{5} + \frac{5}{3}\right)\left(\frac{1}{4} + \frac{1}{5} + \frac{1}{3}\right)^{-1} = u_3$$

$$\underbrace{\left(\frac{u_2}{5} + \frac{5}{3}\right)(1.276596) = u_3}$$

then substitute for u_3

$u_2 \approx 3.31\,V$ (ish)

$u_3 \approx 2.98\,V$ (ish)



$R_1 = 10\,\Omega$ $\quad I_{S_1} = 1\,A$

$R_2 = 20\,\Omega$ $\quad I_{S_2} = 3\,A$

$R_3 = 50\,\Omega$

KCL for u_1: $I_{S_1} = I_{R_1} + I_{R_2} \Rightarrow 1 = I_{R_1} + I_{R_2} \Rightarrow 1 = \frac{V_{R_1}}{R_1} + \frac{V_{R_2}}{R_2}$

KCL for u_2: $I_{R_2} = I_{R_3} + I_{S_2}$ $\quad 3 = I_{R_2} - I_{R_3} \Rightarrow 3 = \frac{V_{R_2}}{R_2} - \frac{V_{R_3}}{R_3}$

$$\left.\begin{aligned} 1 &= \frac{V_{R_1}}{R_1} + \frac{V_{R_2}}{R_2} \\ 3 &= \frac{V_{R_2}}{R_2} - \frac{V_{R_3}}{R_3} \end{aligned}\right\} \quad \boxed{\begin{aligned} 1 &= \frac{u_1}{R_1} + \frac{u_1 - u_2}{R_2} \\ 3 &= \frac{u_1 - u_2}{R_2} - \frac{u_2}{R_3} \end{aligned}}$$

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ \frac{1}{R_2} & -\frac{1}{R_2} - \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{10} + \frac{1}{20} & -\frac{1}{20} \\ \frac{1}{20} & -\frac{1}{20} - \frac{1}{50} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Solve for u_1, u_2!

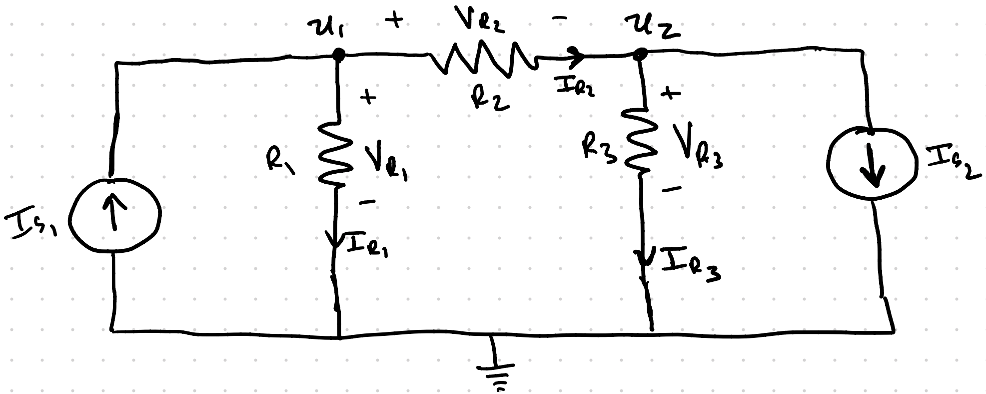
we found $u_1 = -10$, $u_2 = -50$

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{u_1}{R_1} = \frac{-10}{10} = -1\,A$$

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{-50}{50} = -1\,A$$

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{u_1 - u_2}{R_2} = \frac{(-10v) - (-50v)}{20\Omega} = \frac{40v}{20\Omega} = 2\,A$$

Using superposition to solve this... (on next page)

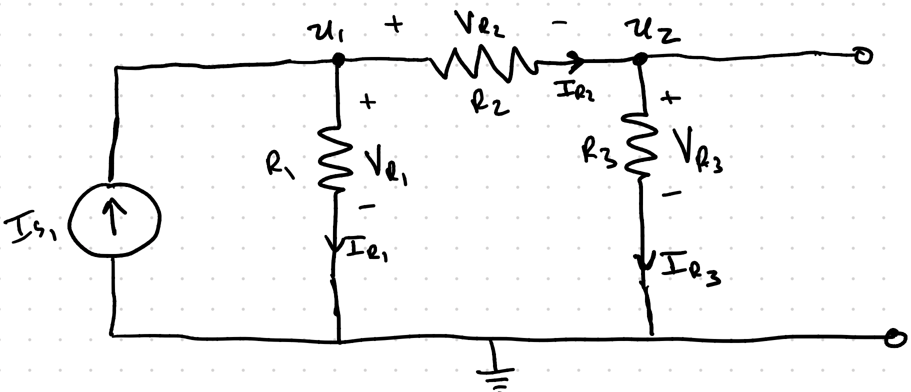


$R_1 = 10\,\Omega$ $\quad I_{s_1} = 1A$

$R_2 = 20\,\Omega$ $\quad I_{s_2} = 3A$

$R_3 = 50\,\Omega$

Zeroing-out I_{s_2} to solve for I_{s_1} alone:



$$I_{s_1} = I_{R_1} + I_{R_2}$$

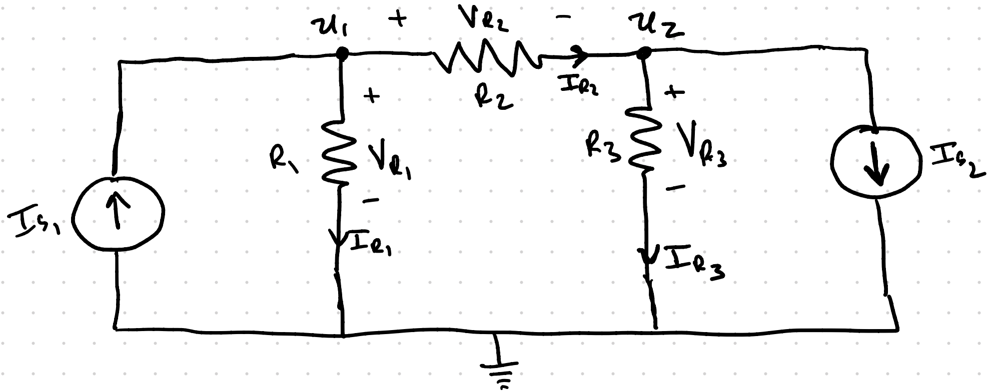
$$1 = \frac{u_1}{10} + \frac{u_1 - u_2}{20}$$

$$u_2 = \frac{R_3}{R_2 + R_3} u_1 = \frac{5}{7} u_1$$

$$20 = 2u_1 + u_1 - \frac{5}{7} u_1$$

$$\Rightarrow 20 = \frac{16}{7} u_1 = u_1 = \frac{70}{8}$$

$$u_2 = \frac{50}{8}$$



$R_1 = 10\,\Omega$

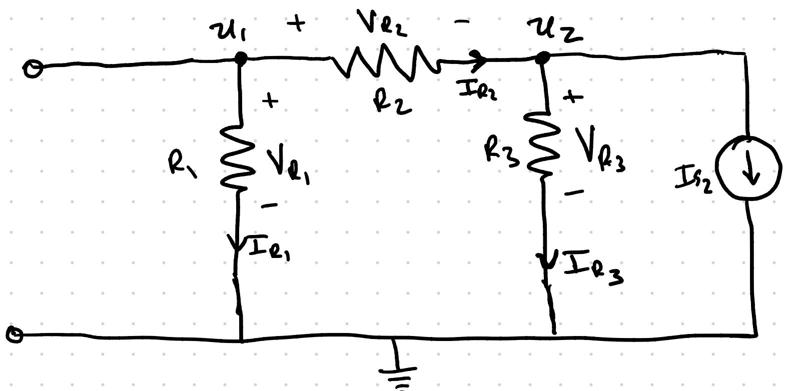
$R_2 = 20\,\Omega$

$R_3 = 50\,\Omega$

$I_{s_1} = 1\,A$

$I_{s_2} = 3\,A$

Zeroing-out I_{s_1} to solve for I_{s_2} alone:



$I_{s_2} = I_{R_2} - I_{R_3}$

$3 = \frac{u_1 - u_2}{R_2} - \frac{u_2}{R_3} = \frac{u_1}{20} - \frac{u_2}{20} - \frac{u_2}{50} = \frac{u_1}{20} - \frac{7}{100} u_2$

Also

$u_1 = \frac{R_1}{R_1 + R_2} u_2 = \frac{1}{3} u_2 \Rightarrow u_2 = 3u_1$

$3 = \frac{u_1}{20} - \frac{7}{100}(3u_1)$

$= \left(\frac{5}{100} - \frac{21}{100}\right) u_1 \Rightarrow u_1 = \frac{-300}{16} = \frac{-150}{8}$

$\Rightarrow u_2 = \frac{-450}{8}$

Putting the two cases together...

Just I_{s1}

$$u_1 = \frac{70}{8}$$

$$u_2 = \frac{50}{80}$$

Just I_{s2}

$$u_1 = -\frac{150}{8}$$

$$u_2 = -\frac{450}{8}$$

$$u_1 = \frac{70}{8} - \frac{150}{8} = \frac{-80}{8} = -10$$

$$u_2 = \frac{50}{8} - \frac{450}{8} = \frac{-400}{8} = -50$$