Q&A until Berkeley Time

EECS16A Summer 2020 Review Session: Time Integration for Cap Ckts, Charge Sharing

August 13, 2020

Moses Won

Concept Dependencies

Capacitor I-V Relationships

charge, capacitance, voltage

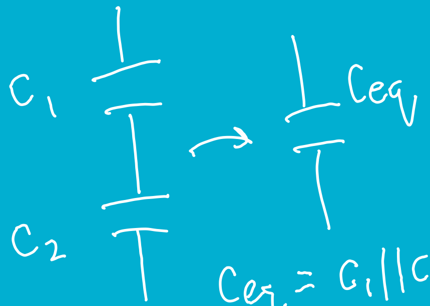
$$\underbrace{Q}_{\text{charge}} = \underbrace{C}_{\text{capacitance}}\underbrace{V}_{\text{voltage}}$$

$[C] \quad [F][V]$

Capacitor Equivalences

$A \| B = \frac{AB}{A+B}$

Series

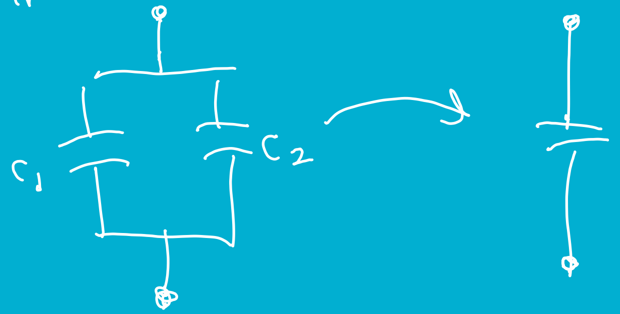


$C_{eq} = C_1 \| C_2$

d's increase

$C = \varepsilon \frac{A}{d}$

Parallel

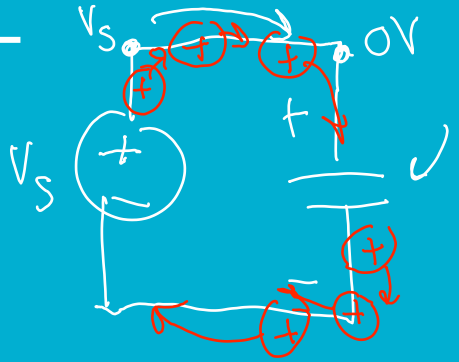


$C_{eq} = C_1 + C_2$

areas add
so C's add

“Physics” of Charge on Capacitors

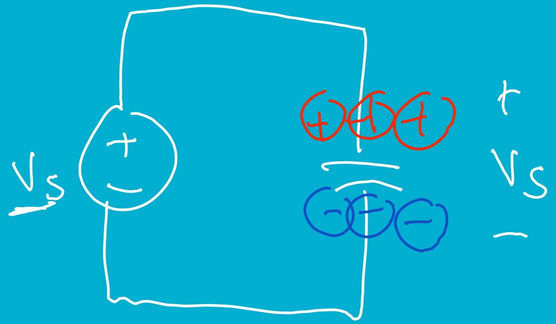
Capacitors Charging by Voltage Source



at time 0, uncharged $(Q = 0)$

$$Q = CV$$
$$= C\,\cancelto{0}{V(t=0)}$$

$$\underline{Q = 0} \Rightarrow V(t=0) = 0V$$

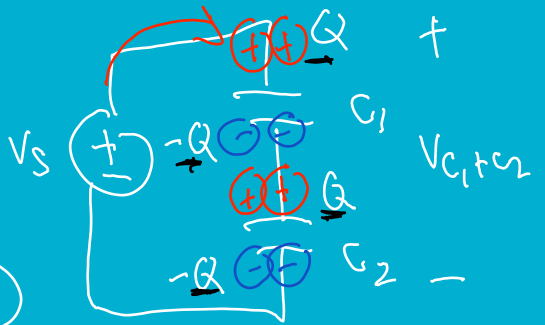


a full cup

Capacitors Charging by Voltage Source

Examples

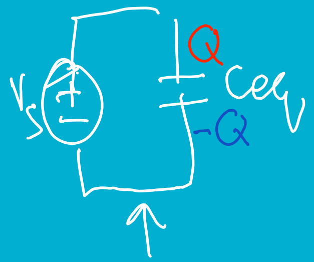
- In series
⇒ same I
(both uncharged)

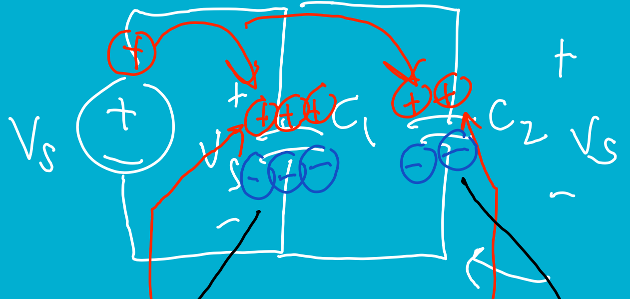


$$V_{c_1+c_2} = V_1 + V_2$$

$$= \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$V_s = \frac{Q}{C_1} + \frac{Q}{C_2} \Leftarrow$$





$Q_1 = C_1 V_s$

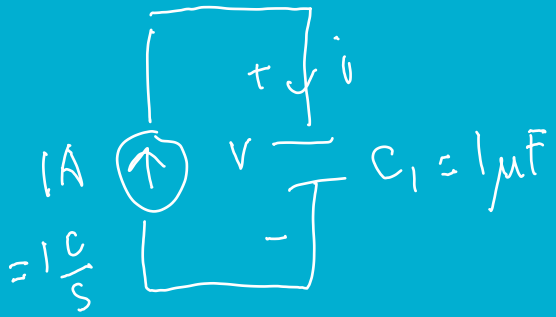
$Q_2 = C_2 V_s$

$-Q_1$

$-Q_2$

Capacitors Charging by Current Source

Current is a flow of charge → builds up charge on cap
→ builds up voltage on cap over time

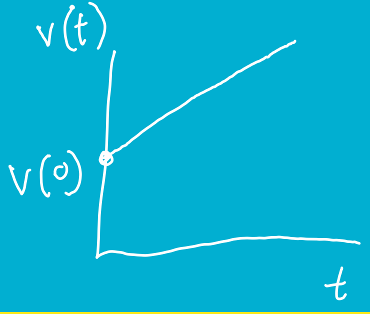


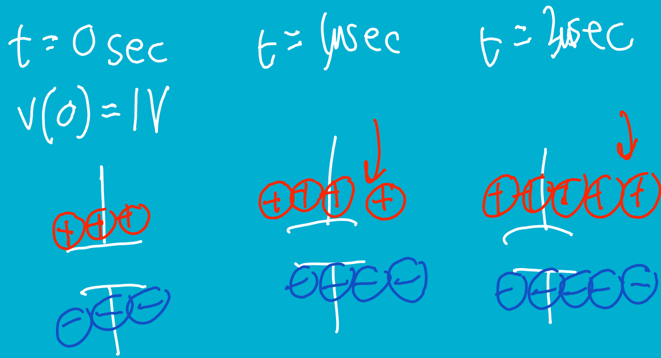
$$i = C\frac{dV}{dt}$$

$$\int_{t=0}^{t=t_f} \frac{i}{C}\,dt = \int_{v(0)}^{v(t_f)} dV$$

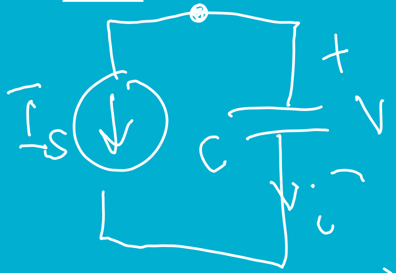
$$\frac{i}{C}(t_f - 0) = V(t_f) - V(0)$$

$$\Rightarrow V(t) = \frac{i}{C}t + V(0)$$



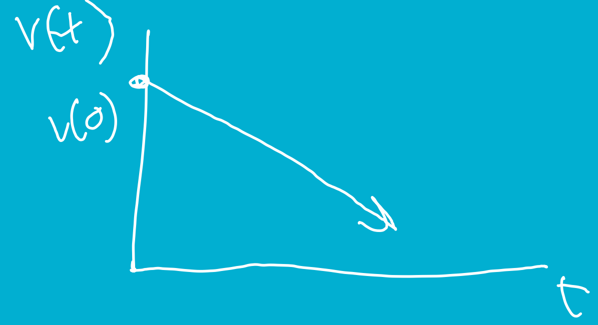


Capacitors Charging by Current Source



$i = -I_s$

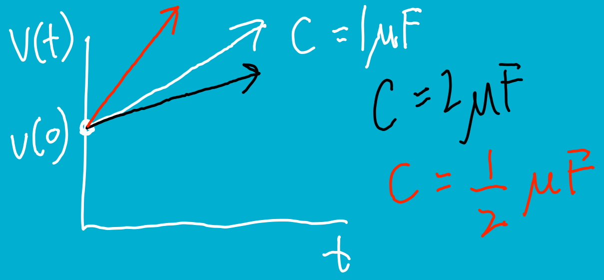
$$v(t) = -\frac{I_s}{C}t + v(0)$$



(Discharging)

What happens if C changes?

I_s goes in (charging)

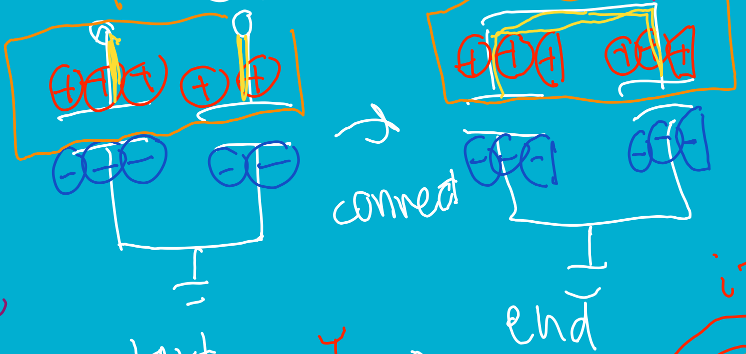


$$v(t) = v(0) + \frac{I_s}{C}t$$

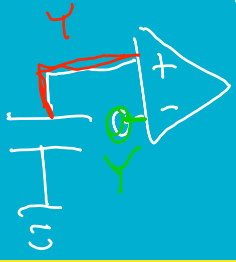
Identifying Floating Nodes

Def: A floating node is a node from which charge cannot escape → charge conservation applies

$Q_{start} = Q_{end}$



top node is the floating node @ which charge was conserved



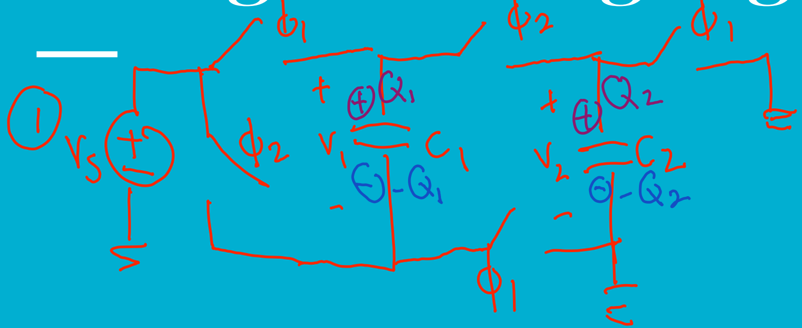
Charge Sharing Algorithm

1. Label Q's, V's in ckt
2. Draw each eq. ckt for each phase (label Q's V's)
3. Identify floating nodes (in phases for which we want to find V)
4. Apply Q-conservation @ each floating node (one eqn 4 each floating node)

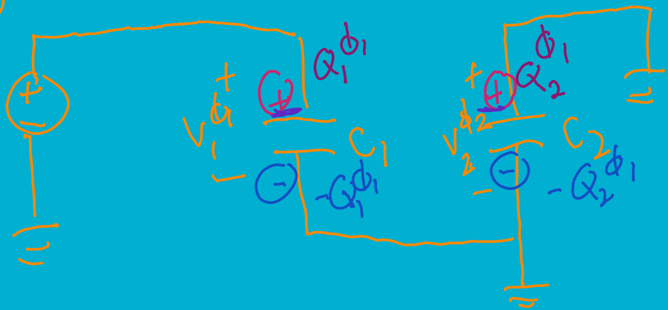
Charge Sharing Algorithm

Goal: find all node voltages after ϕ_1, ϕ_2

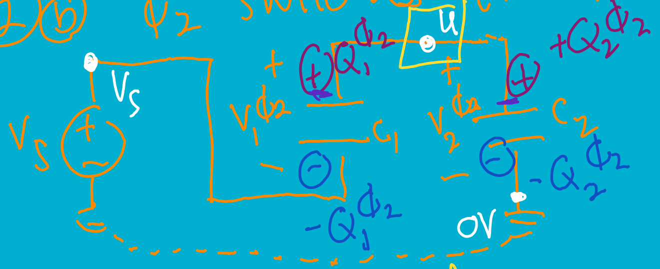
1



2 a ϕ_1 switches closed



2 b ϕ_2 switches closed



find V's here

3 Find floating nodes

u node is floating

4 Write Q cons. eqns

Q start = Q end

$$Q_1^{\phi_1} + Q_2^{\phi_1} = Q_1^{\phi_2} + Q_2^{\phi_2}$$

$$C_1(V_s - 0) + C_2(0-0) = C_1(u - V_s) + C_2(u - 0)$$

$$\boxed{u = \frac{2C_1V_s}{C_1+C_2}}$$

Charge Sharing Algorithm