EECS 16A Designing Information Devices and Systems I
Summer 2020 Discussion 6B

Reference: Inner products

Let $\vec{x}$, $\vec{y}$, and $\vec{z}$ be vectors in real vector space $\mathbb{V}$. A mapping $\langle \cdot, \cdot \rangle$ is said to be an inner product on $\mathbb{V}$ if it satisfies the following three properties:

(a) Symmetry: $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle$

(b) Linearity: $\langle \vec{x}, \vec{y} + \vec{z} \rangle = \langle \vec{x}, \vec{y} \rangle + \langle \vec{x}, \vec{z} \rangle$ and $\langle c\vec{x}, \vec{y} \rangle = c\langle \vec{x}, \vec{y} \rangle$

(c) Positive-definiteness: $\langle \vec{x}, \vec{x} \rangle \geq 0$, with equality if and only if $\vec{x} = \vec{0}$.

We define the norm of $\vec{x}$ as $\|\vec{x}\| = \sqrt{\langle \vec{x}, \vec{x} \rangle}$.

Cross-correlation:

The cross-correlation between two signals $r[n]$ and $s[n]$ is defined as follows:

$$\text{corr}_r(s)[k] = \sum_{i=-\infty}^{\infty} r[i]s[i-k].$$

1. Geometric Interpretation of the Inner Product In this problem, we will explore the geometric interpretation of the Euclidean inner product, restricting ourselves to vectors in $\mathbb{R}^2$.

(a) For each of the following cases, pick two vectors that satisfy the condition and find the inner product.

i. Parallel Vectors

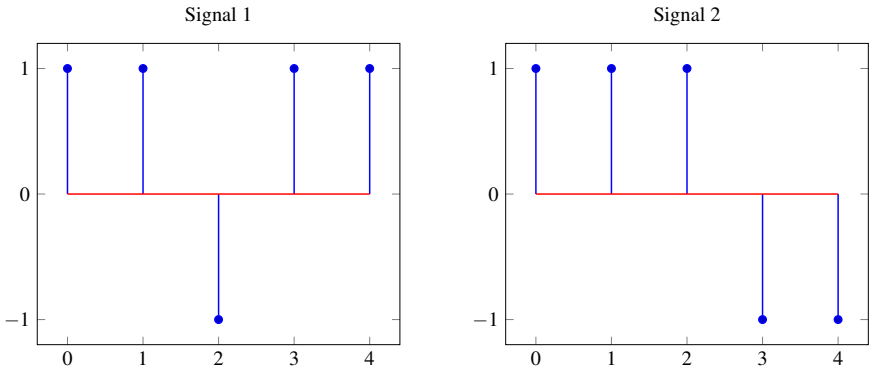
ii. Anti-parallel

iii. Perpendicular

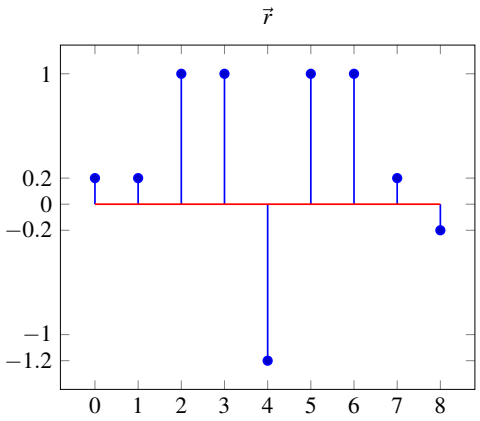
(b) Now, derive a formula for the inner product of two vectors in terms of their magnitudes and the angle between them.

2. Identifying satelites and their delays

We are given the following two signals, $\vec{s_1}$ and $\vec{s_2}$ respectively, that are signatures for two satellites.



(a) Your cellphone antenna receives the following signal $r[n]$. You know that there may be some noise present in $r[n]$ in addition to the transmission from the satellite.



Which satellites are transmitting? What is the delay between the satellite and your cellphone? Use cross-correlation to justify your answer. You can use iPython to compute the cross-correlation.

(b) Now your cellphone receives a new signal $r[n]$ as below. What the satellites that are transmitting and what is the delay between each satellite and your cellphone?