# EECS 16A Designing Information Devices and Systems I
# Summer 2020 Discussion 1D

**1. Inverses**

In general, the *inverse* of a matrix "undoes" the operation that the matrix performs. Mathematically, we write this as

$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{I},$$

where $\mathbf{A}^{-1}$ is the inverse of $\mathbf{A}$. Intuitively, this means that applying a matrix to a vector and then subsequently applying its inverse is the same as leaving the vector untouched.

**Properties of Inverses**

For a matrix $\mathbf{A}$, if its inverse exists, then:

$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$$
$$\left(\mathbf{A}^{-1}\right)^{-1} = \mathbf{A}$$
$$(k\mathbf{A})^{-1} = k^{-1}\mathbf{A}^{-1} \qquad \text{for a nonzero scalar } k$$
$$\left(\mathbf{A}^{T}\right)^{-1} = \left(\mathbf{A}^{-1}\right)^{T}$$
$$(\mathbf{A}\mathbf{B})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1} \qquad \text{assuming } \mathbf{A}, \mathbf{B} \text{ are both invertible}$$

(a) Prove that $(\mathbf{A}\mathbf{B}\mathbf{C})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1}$.

(b) Now consider the following four matrices.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix}$$

i. What do each of these matrices do when you multiply them by a vector $\vec{x}$? Draw a diagram.

ii. Intuitively, can these operations be undone? Why or why not? Make an intuitive argument.

iii. Are the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ invertible?

iv. Can you find anything in common about the rows (and columns) of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$? (*Bonus:* How does this relate to the invertibility of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$?)

v. Are all square matrices invertible?

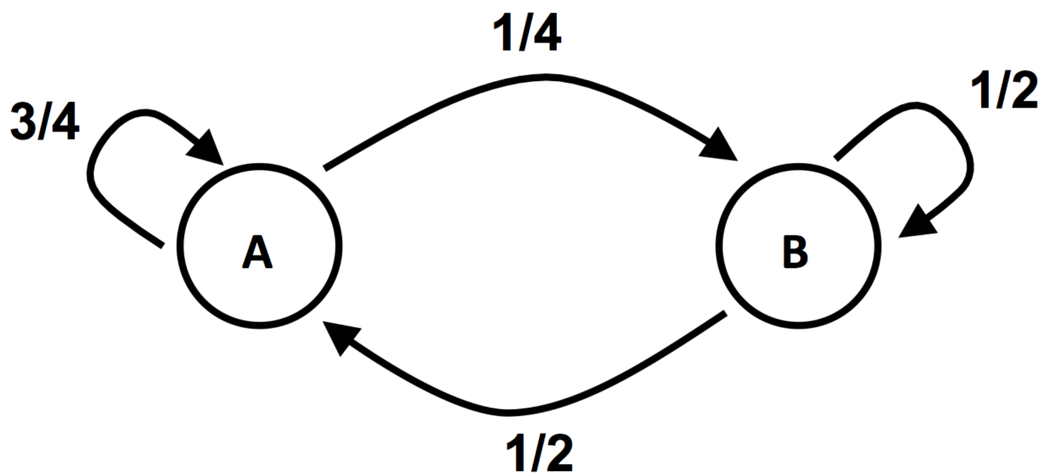
vi. How can you find the inverse of a general $n \times n$ matrix?

**2. Transition Matrix**

(a) Suppose there exists some network of pumps as shown in the diagram below. Let $\vec{x}(n) = \begin{bmatrix} x_A(n) \\ x_B(n) \end{bmatrix}$ where $x_A(n)$ and $x_B(n)$ are the states at timestep $n$.

Find the state transition matrix $S$, such that $\vec{x}(n+1) = S\vec{x}(n)$.




(b) Let us now find the matrix $S^{-1}$ such that we can recover $\vec{x}(n-1)$ from $\vec{x}(n)$. Specifically, solve for $S^{-1}$ such that $\vec{x}(n-1) = S^{-1}\vec{x}(n)$.

(c) Now draw the state transition diagram that corresponds to the $S^{-1}$ that you just found.

(d) Redraw the diagram from the first part of the problem, but now with the directions of the arrows reversed. Let us call the state transmission matrix of this "reversed" state transition diagram $T$. Does $T = S^{-1}$?

## 3. Identifying a Basis

Does each of these sets of vectors describe a basis for $\mathbb{R}^3$? If the vectors do not form a basis for $\mathbb{R}^3$, can they be thought of as a basis for some other vector space? If so, write an expression describing this vector space.

$$V_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\} \qquad V_2 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \qquad V_3 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$