EECS 16A Designing Information Devices and Systems I
Spring 2022 Homework 1

1. **Reading Assignment**

   For this homework, please read Note 0, Note 1A, and Note 1B. These will provide an overview of linear equations and augmented matrices. You are always welcome and encouraged to read ahead as well. How does the content you read in these notes relate to what you've learned before? What content is unfamiliar or new?

2. **Counting Solutions**

   Learning Goal: *(This problem is designed to illustrate the different types of systems of equations. Some sub-parts will have a unique solution and others have no solutions or infinitely many solutions. In this class, we will build up the mathematical machinery to systematically determine which case applies.)*

   Directions: For each of the following systems of linear equations, determine if there is a unique solution, no solution, or an infinite number of solutions. If there is a unique solution, find it. If there is an infinite number of solutions, explicitly state this, describe the set of all solutions, and then write one of such solutions. You can represent the set of all solutions by writing two variables as a function of the last variable (i.e. $x = 2z+1, y = 4z-4$). If there is no solution, explain why. **Show your work**.

   Example: The below example shows how to methodically solve systems of linear equations using the substitution method.

$$\begin{array}{ccccc} 2x & + & 3y & = 5 \\ x & + & y & = 2 \end{array}$$

   Example Solution

$$2x+3y = 5 \tag{1}$$
$$x+ \ y = 2 \tag{2}$$

   Subtract: Eq (1) - 2*Eq (2)

$$y = 1 \tag{3}$$

   Now we plug in Eq (3) into Eq (2) and solve for x

$$x+ \ 1 = 2$$
$$\rightarrow \quad x = 1 \tag{4}$$

   From Eq (3) and Eq (4), we get the unique solution:

$$x = 1$$
$$y = 1$$

   (a)

$$\begin{array}{ccccccc} x & + & y & + & z & = 3 \\ 2x & + & 2y & + & 2z & = 5 \end{array}$$

(b)

$$\begin{aligned} - y + 2z &= 1 \\ 2x + z &= 2 \end{aligned}$$

(c)

$$\begin{aligned} x + 2y &= 5 \\ 2x - y &= 0 \\ 3x + y &= 5 \end{aligned}$$

(d)

$$\begin{aligned} x + 2y &= 3 \\ 2x - y &= 1 \\ x - 3y &= -5 \end{aligned}$$

3. Magic Square

In an $n \times n$ "magic square," all of the sums across each of the n rows, n columns, and 2 diagonals equal magic constant k. For example, in the below magic square, each row, column, and diagonal sums to 34.

4	14	15	1
9	7	6	12
5	11	10	8
16	2	3	13

The magic square is a classic math puzzle, and some of you may have solved these as children by guessing. However, it turns out they can be solved systematically by setting up a system of linear equations!

(a) How many linear equations can you write for an $n \times n$ magic square?

(b) For the generalized magic square below, write out a system of linear equations.
Hint: Set the sum of entries in each row, column, and diagonal equal to k.

x_{11}	x_{12}	x_{13}
x_{21}	x_{22}	x_{23}
x_{31}	x_{32}	x_{33}

(c) Now consider the following square, with some entries filled in. Substitute the known entries into the linear equations you wrote in part (b) to solve for the missing entries x_{11}, x_{12}, x_{32}. Please show the equations you use to solve; credit will not be given for solving by inspection.

x_{11}	x_{12}	8
9	5	1
2	x_{32}	6

4. Word Problems

Learning Objective: *Understand how to setup a system of linear equations from word problems.*

For these word problems, represent the system of linear equations as an augmented matrix. Then, solve the system using substutition.

(a) Gustav is collecting soil samples. Each soil sample contains some sand, some clay, and some organic material. He wants to know the density of each material. His first sample has 0.5 liters of sand, 0.25 liters of clay, and 0.25 liters of organic material, and weighs 1.625 kg. His second sample contains 1 liter of sand, 0 liters of clay, and 1 liter of organic material, and weighs 3 kg. His third sample contains 0.25 liters of sand, 0.5 liters of clay, and 0 liters of organic material, and weighs 1.375 kg. That is,

$$0.5s + 0.25c + 0.25m = 1.625 \tag{5}$$
$$1s + 0c + 1m = 3 \tag{6}$$
$$0.25s + 0.5c + 0m = 1.375 \tag{7}$$

where s is the density of sand, c is the density of clay, and m is the density of organic material, all measured in kg/L. Solve for the density of each material.

(b) Alice buys 3 apples and 4 oranges for 17 dollars. Bob buys 1 apple and 10 oranges for 23 dollars (Bob really likes oranges). How much do apples and oranges cost individually?

(c) Jack, Jill, and James are driving from Berkeley to Las Vegas. Each of them takes a different route. Jack takes a short route and ends up going through Toll Road A and Toll Road B, costing him $10. Jill takes a slightly longer route and goes through Toll Road B and Toll Road C, costing her $15. Finally, James takes a wrong turn and takes Toll Road A twice, then takes Toll Road B and finally Toll Road C, costing him $25. What is the toll cost on each road?

5. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

Please remember to submit both your homework as well as the self-grade assignment following the release of the solutions. A full description of the submission process is listed on the class website (eecs16a.org).