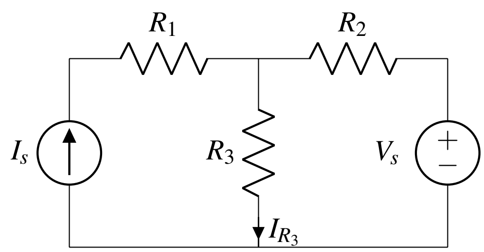
EECS 16A Designing Information Devices and Systems I
Spring 2022 Discussion 11A

1. **Superposition**

Consider the following circuit:



(a) With the current source turned on and the voltage source turned off, find the current I_{R_3}.

Answer:

We note that that I_s is split between R_2 and R_3 and therefore we can use the current divider relation (here the notation I_{R_3,I_s} represents the current across R_3 with only I_s turned on.):

$$I_{R_3,I_s} = \frac{I_s R_2}{R_2 + R_3}$$

(b) With the voltage source turned on and the current source turned off, find the voltage drop across R_3, V_{R_3}. **Answer:**

We note that when the current source is turn off it becomes an open circuit. Thus, we are left with a voltage divider.

$$V_{R_3,V_s} = \frac{V_s R_3}{R_2 + R_3}$$

(c) Find the power dissipated by R_3.

Answer:

We first find the missing quantities. The voltage drop across R_3 when the current source is on is given by:

$$V_{R_3,I_s} = I_{R_3,I_s} R_3 = \frac{I_s R_2 R_3}{R_2 + R_3}$$

$$I_{R_3,V_s} = \frac{V_{R_3,V_s}}{R_3} = \frac{V_s}{R_2 + R_3}$$

Thus, we have:

$$V_{R_3} = V_{R_3,V_s} + V_{R_3,I_s} = \frac{V_s R_3 + I_s R_2 R_3}{R_2 + R_3}$$

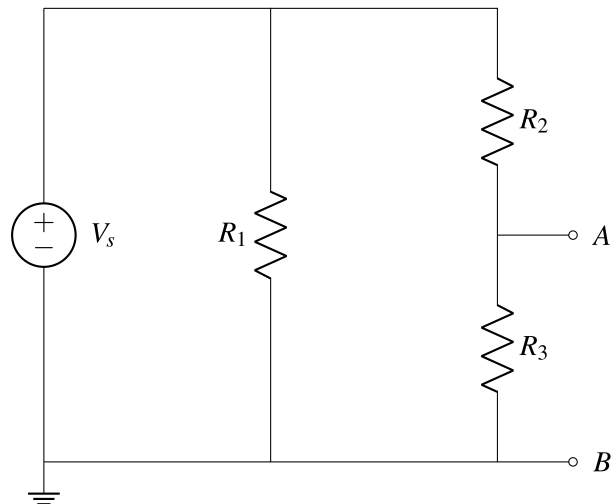
$$I_{R_3} = I_{R_3,V_s} + I_{R_3,I_s} = \frac{I_s R_2 + V_s}{R_2 + R_3}$$

Thus, the power dissipated is:

$$P_{R_3} = I_{R_3} V_{R_3} = \frac{R_3 (I_s R_2 + V_s)^2}{(R_2 + R_3)^2}$$

2. Thévenin/Norton Equivalence

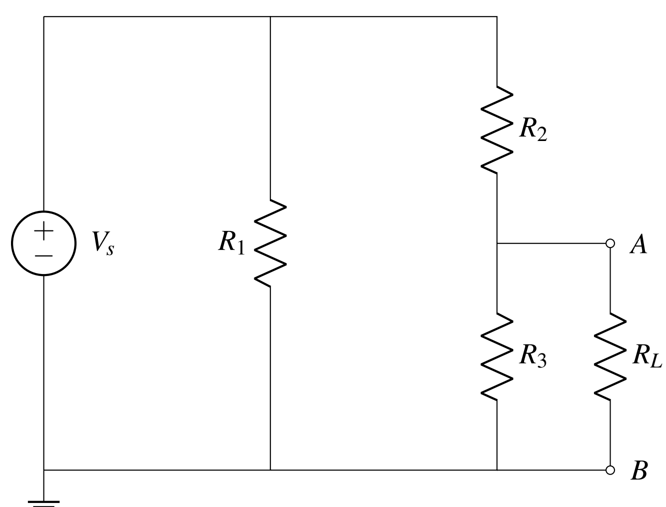
(a) Find the Thévenin resistance R_{th} of the circuit shown below, with respect to its terminals A and B.



Answer: To find the Thévenin resistance, we null out the voltage source (which shorts out R_1) and find the equivalent resistance which is simply:

$$R_{th} = R_2 \parallel R_3$$

(b) Now, a load resistor, $R_L = R$, is connected across terminals A and B as shown in the circuit below. Find the power dissipated in the load resistor in terms of given variables.

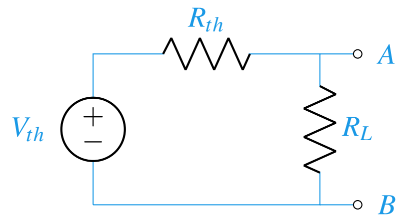


Answer:

To help simplify the analysis, we replace the circuit by its Thévenin equivalent circuit. In order to do so, we first need to finds the Thévenin voltage. That is the open circuit voltage, V_{AB}, in the original circuit, which is simply a voltage divider:

$$V_{th} = V_s \frac{R_3}{R_2 + R_3}$$

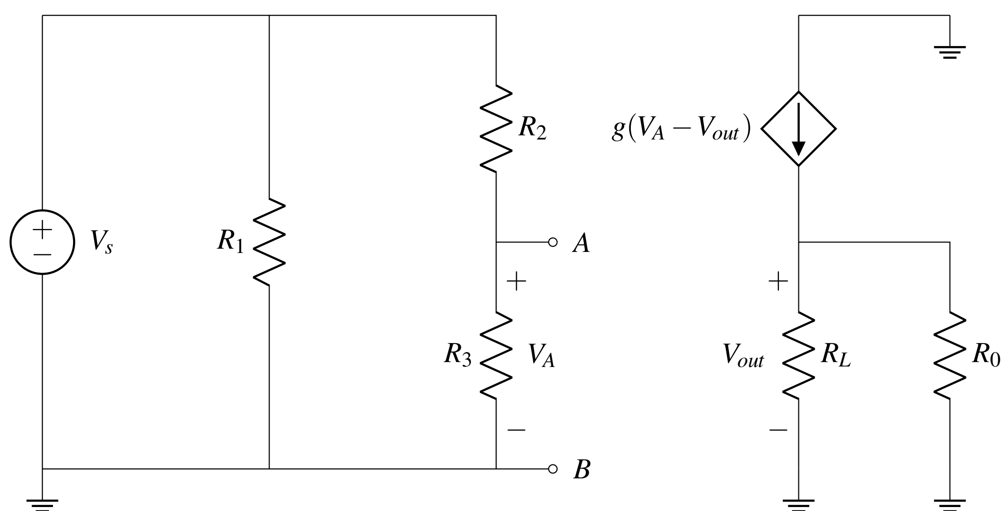
Thus, the circuit can be simplified to:



The power through the load resistor is given by:

$$P_{R_L} = (\frac{V_{th}}{R_L + R_{th}})^2 R_L = (V_s \frac{R_3}{R_2 + R_3} \cdot \frac{1}{R_L + R_{th}})^2 R_L$$

(c) We modify the circuit as shown below, where g is a known constant:



Find a symbolic expression for V_{out} as a function of V_s.
Hint: Redraw the left part of the circuit using its Thévenin equivalent.

Answer:

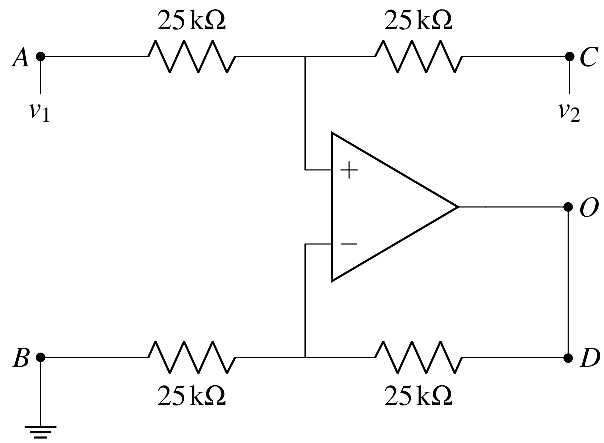
We note that V_{AB} simply equals $V_{th} = \frac{R_3}{R_2+R_3} V_s$. Then, noting that R_0 and R_L are in parallel, we have that $V_{out} = g(V_{th} - V_{out})(R_0 \parallel R_L)$. Solving for V_{out}, we get:

$$V_{out} = \frac{R_2}{R_2 + R_3} V_s \frac{gR_L \parallel R_0}{1 + gR_L \parallel R_0}$$

3. A Versatile Opamp Circuit

For each subpart, determine the voltage at O.

(a) Configuration 1:



Answer:

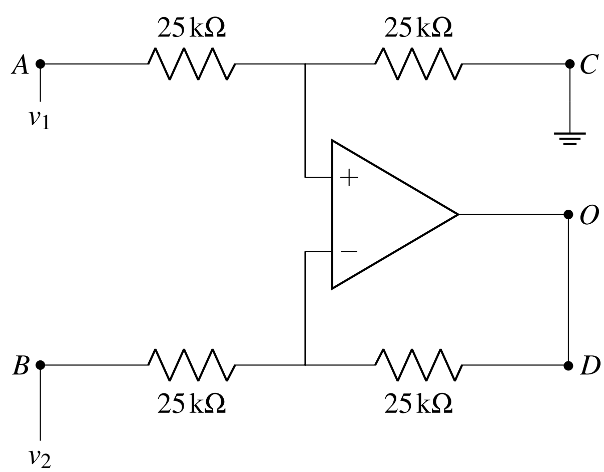
By superposition, we note that the voltage at v^+ is given by:

$$v^+ = \frac{v_1 + v_2}{2}$$

The rest of the circuit looks like a non-inverting amplifier with a gain of $1 + \frac{25\,\mathrm{k}\Omega}{25\,\mathrm{k}\Omega} = 2$. Therefore, the output voltage is:

$$v_O = 2v^+ = v_1 + v_2$$

(b) Configuration 2:



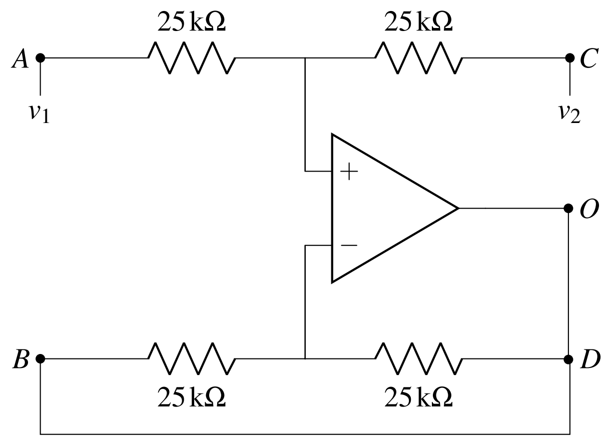
Answer:

Through the voltage divider equation, we note that the voltages at the input terminals of the op-amp are given by:

$$v^+ = \frac{v_1}{2}$$
$$v^- = \frac{v_2 + v_O}{2}$$

By the Golden Rules, these must be equal to each other since the op-amp is in negative feedback. Therefore, $v_O = v_1 - v_2$.

(c) Configuration 3:



Answer:

Like in part (i), by superposition, we note that the voltage at v^+ is given by:

$$v^+ = \frac{v_1 + v_2}{2}$$

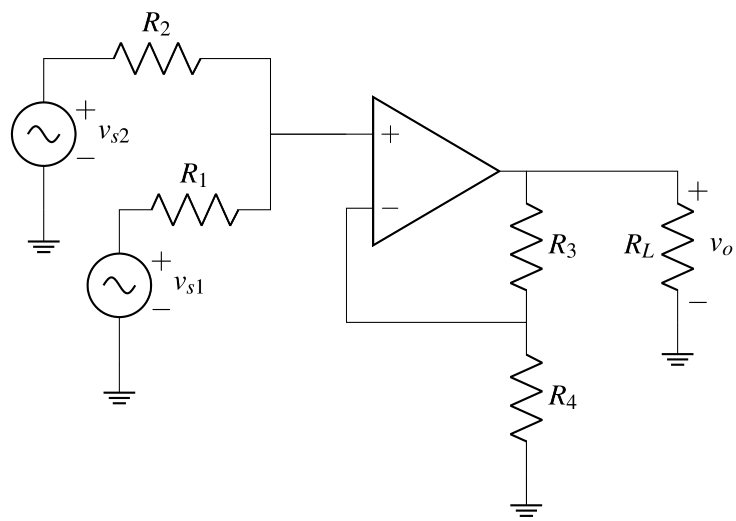
We note that the resistors connected to B and D do not affect the circuit as no current is flowing through them. Therefore, $v_O = v^- = v^+ = \frac{v_1+v_2}{2}$.

4. Multiple Inputs To One Op-Amp

Solution/Answer: Estimated Time: 20 min

Note to TAs: Reminder that this is the composition of different blocks we have seen before: a summer, non-inverting amplifier, and load resistor. So you can simply apply the equations for each individual block, rather than doing analysis from scratch. Be sure to clarify why we can hook these blocks together and still get the expected output.

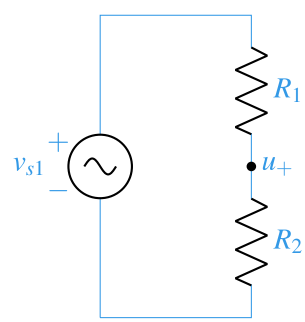
TAs should also mention the difference between voltage signal and voltage source. Voltage signals are usually mV level and don't dissipate much power. In contrast voltage sources are usually V level and are often used as power supplies (e.g. rails for op-amp).



(a) For the circuit above, find an expression for v_o. *(Hint: Use superposition.)*

Solution/Answer:

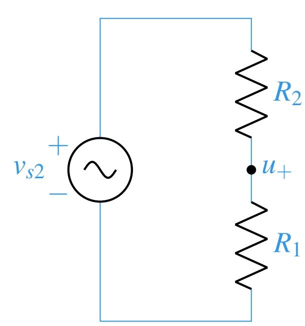
Let's call the potential at the positive input of the op-amp u_+. Using superposition, we first turn off v_{s2} and find u_+. The circuit then looks like:



We recognize the above circuit as a voltage divider. Thus,

$$u_{+,vs1} = \frac{R_2}{R_1 + R_2} v_{s1}$$

By symmetry, we expect v_{s2} to have a similar circuit and expression. The circuit for v_{s2} looks like:



The expression for u_+ with v_{s2} is then:

$$u_{+,vs2} = \frac{R_1}{R_1 + R_2} v_{s2}$$

From superposition, we know the output must be the sum of these.

$$u_+ = \frac{R_2}{R_1+R_2}v_{s1} + \frac{R_1}{R_1+R_2}v_{s2}$$

With u_+ determined, we can find the output voltage directly from the formula for a non-inverting amplifier. We can also derive it using the process below.

From the negative feedback rule, $u_+ = u_-$. Using voltage dividers, we can express u_- in terms of v_o:

$$u_- = \frac{R_4}{R_3+R_4}v_o$$

$$v_o = \left(1+\frac{R_3}{R_4}\right)u_- = \left(1+\frac{R_3}{R_4}\right)u_+$$

Now, to find the final output, we can set u_+ to our earlier expression.

$$v_o = \left(1+\frac{R_3}{R_4}\right)\left(\frac{R_2}{R_1+R_2}v_{s1} + \frac{R_1}{R_1+R_2}v_{s2}\right)$$

(b) How could you use this circuit to find the sum of different signals, i.e. $V_{s1}+V_{s2}$? What about taking the sum and multiplying by 2, i.e. $2(V_{s1}+V_{s2})$?

Answer:

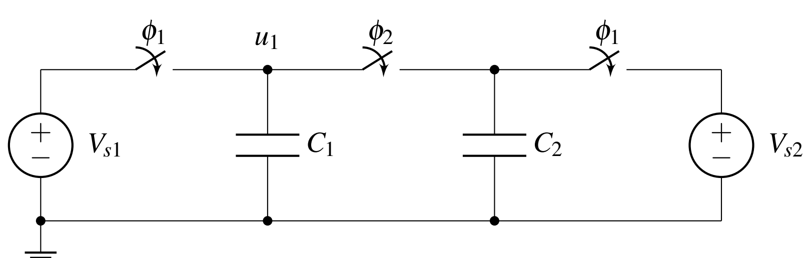
The circuit already finds the weighted sum of two inputs. By setting $R_1 = R_2$ and $R_3 = R_4$, we can take the exact sum of two inputs.

$$v_o = \left(1+\frac{R_3}{R_4}\right)\left(\frac{R_2}{R_1+R_2}v_{s1} + \frac{R_1}{R_1+R_2}v_{s2}\right) = (1+1)\left(\frac{1}{2}v_{s1}+\frac{1}{2}v_{s2}\right) = v_{s1}+v_{s2}$$

Notice that the first half of this circuit (R_1 and R_2) form a voltage summer with coefficients less than one; the second half is just a non-inverting amplifier. Thus we can always use R_1 and R_2 to take an equally weighted sum of the inputs and then multiply greater than 1 using the non-inverting amplifier. If we set $R_1 = R_2$, we get $\left(\frac{1}{2}v_{s1}+\frac{1}{2}v_{s2}\right)$ into the op-amp. To get a total gain of 2, then the non-inverting op-amp needs a gain of 4, so we can pick $R_3 = 3R_4$.

5. Capacitive Charge Sharing (from Spring 2020 Midterm 2)

Consider the circuit below with $C_1 = C_2 = 1\,\mu\text{F}$ and three switches ϕ_1, ϕ_2. Suppose that initially the switches ϕ_1 are closed and ϕ_2 is open such that C_1 and C_2 are charged through the corresponding voltage sources $V_{s1} = 1\,\text{V}$ and $V_{s2} = 2\,\text{V}$.



(a) How much charge is on C_1 and C_2?

Solution/Answer:

$$q_1 = C_1 V_1 = 1\,\mu\text{C}$$
$$q_2 = C_2 V_2 = 2\,\mu\text{C}$$

(b) Now suppose that some time later, switch ϕ_1 opens and switch ϕ_2 closes. What is the value of voltage u_1 at steady state?

Answer: The total charge on node u_1 will be conserved after switch S_1 is opened. That charge is $Q_{tot} = q_1 + q_2$. Also note that during phase 2 the capacitors are connected in parallel so they will both have $V_{C1} = V_{C2} = u_1$.

$$Q_{tot} = C_1 u_1 + C_2 u_1$$
$$u_1 = \frac{q_1 + q_2}{C_1 + C_2} = 1.5\,\text{V}$$