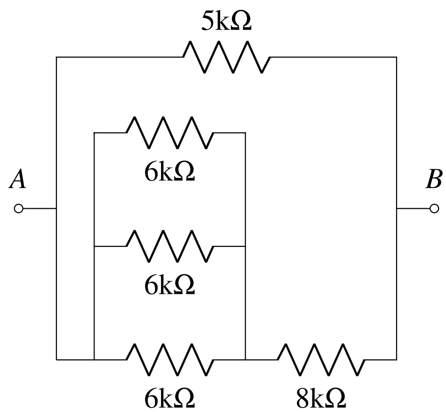
EECS 16A Designing Information Devices and Systems I
Spring 2022 Discussion 8A

1. Series and Parallel Combinations

For the resistor network shown below, find an equivalent resistance between the terminals A and B using the resistor combination rules for series and parallel resistors.

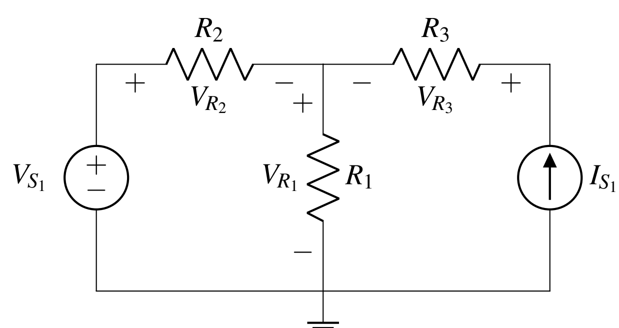


2. Superposition

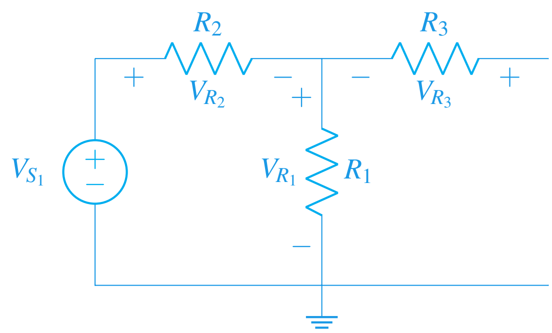
For the following circuits:

i. Use the superposition theorem to solve for the voltages across the resistors. First, redraw the circuits with just one source (while zero-ing the other source). Then, for each circuit solve for each element voltage. Finally, sum the voltages at each node.

(a)

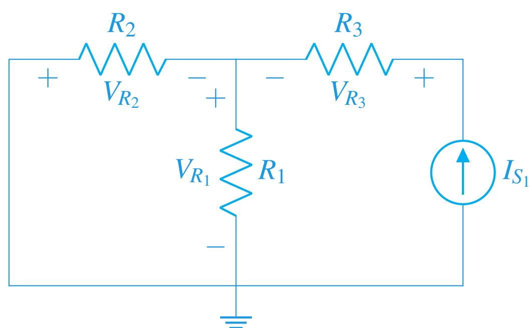


Answer: Turning on only V_{S_1} and zeroing the current source ($I_{S_1} = 0$A so an open circuit), we have the following circuit and voltages across the resistors. Notice that because we have an open circuit, R_3 no longer contributes to the circuit network, and we are left with a classic resistive voltage divider.

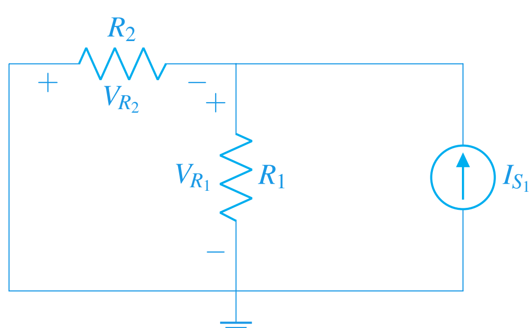


$$V_{R_1} = \frac{R_1}{R_1 + R_2} V_{S_1}$$
$$V_{R_2} = \frac{R_2}{R_1 + R_2} V_{S_1}$$
$$V_{R_3} = 0$$

Then turning only I_{S_1} and zeroing the voltage source ($V_{S_1} = 0\text{V}$ so a closed circuit), we have the following circuit and voltages:



The current going through R_3 is simply I_{S_1} and then splits between R_1 and R_2. Let us focus on the R_1 and R_2 components.



We know that $V_{R_1} = I_1 R_1$ and $V_{R_2} = I_2 R_2$. We also know I_{R_2}+I_{S_1}=I_{R_1} and $V_{R_1} = -V_{R_2}$. If we solve this system, we find

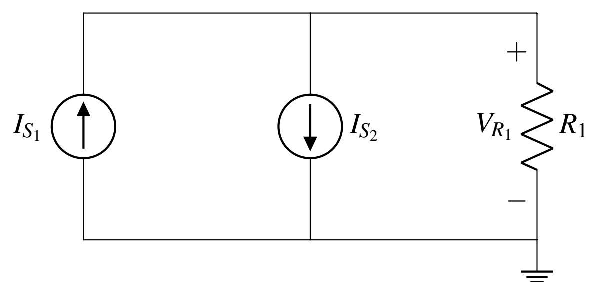
$$I_1 = \frac{R_2}{R_1 + R_2} I_{S_1}$$

$$I_2 = -\frac{R_1}{R_1+R_2} I_{S_1}$$
$$V_{R_1} = \frac{R_1 R_2}{R_1+R_2} I_{S_1}$$
$$V_{R_2} = -\frac{R_1 R_2}{R_1+R_2} I_{S_1}$$
$$V_{R_3} = I_{S_1} R_3$$

Using superposition we can sum up the contributions from both V_{S_1} and I_{S_1} to get:

$$V_{R_1} = \frac{R_1}{R_1+R_2} V_{S_1} + \frac{R_1 R_2}{R_1+R_2} I_{S_1}$$
$$V_{R_2} = V_{S_1} - V_{R_1} = \frac{R_2}{R_1+R_2} V_{S_1} - \frac{R_1 R_2}{R_1+R_2} I_{S_1}$$
$$V_{R_3} = I_{S_1} R_3$$

(b)



Answer:

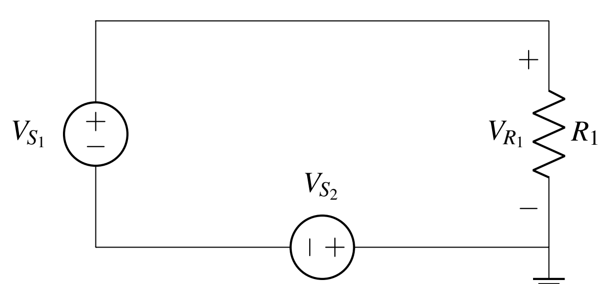
i. While we could apply the algorithm we have learned in class, let's see if there's a way to find the answer quicker than before. We're looking for the voltage across the resistor, which could be found quickly using Ohm's law if we knew the current. If we were to apply KCL at the node at the top of the circuit, one source is coming in, the other source is leaving, and the current through the resistor is leaving. From KCL, we then know $i_{R_1} = I_{S_1} - I_{S_2}$. Applying Ohm's Law we find:

$$V_{R_1} = (I_{S_1} - I_{S_2}) R_1$$

We could also solve this using superposition. Turning on I_{S_1} gives $V_{R_1} = I_{S_1} R_1$. Turning on I_{S_2} gives $V_{R_1} = -I_{S_2} R_1$. Finally, the total V_{R_1} is the sum of the individual V_{R_1}'s or

$$V_{R_1} = (I_{S_1} - I_{S_2}) R_1$$

(c) (**PRACTICE**)



Answer:

i. Once again, we could apply the circuit analysis algorithm or find the answer directly. Notice the circuit only has one loop, so we can use KVL to find the voltage across the resistor.

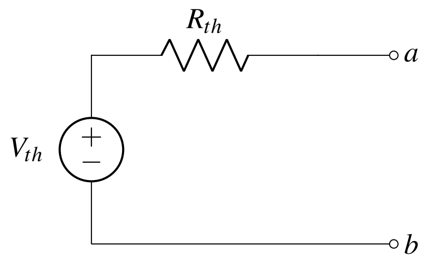
$$V_{R_1} = V_{S_1} - V_{S_2}$$

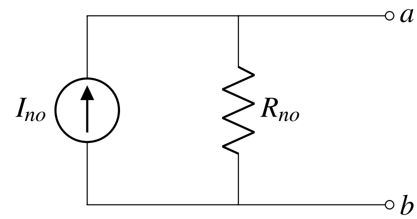
We could also solve with superposition. Turning on V_{S_1} gives $V_{R_1} = V_{S_1}$. Turning on V_{S_2} gives $V_{R_1} = -V_{S_2}$. The overall voltage is then the sum.

$$V_{R_1} = V_{S_1} - V_{S_2}$$

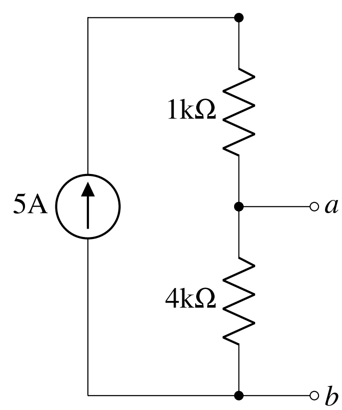
3. Thevenin and Norton Equivalence

The general Thévenin and Norton equivalents are shown below:





Find the Thévenin and Norton equivalents across terminals a and b for the circuit given below.



Answer:

The open circuit voltage across ab is given by Ohm's law:

$$V_{th} = 5\text{A} \times 4\,\text{k}\Omega = 20\,\text{kV}$$

To find R_{th}, zero out any independent sources (5A source becomes an open). Then, from the open circuit terminals ab, find the equivalent resistance of the rest of the circuit. This is just a single resistor.

$$R_{th} = 4\,\text{k}\Omega = R_{no}$$

Now to find I_{no},

$$I_{no} = \frac{V_{th}}{R_{th}} = \frac{20\,\text{kV}}{4\,\text{k}\Omega} = 5\,\text{A}$$