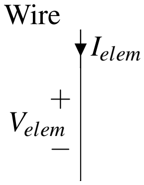
EECS 16A Spring 2022 — Designing Information Devices and Systems I

Discussion 3A

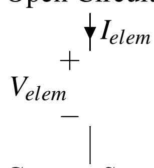
1. Circuit Components and Ohm's Law

(a) We will look at the $I-V$ characteristics of different circuit components. For each of the components listed below plot the $I_{elem} - V_{elem}$ characteristic curves.

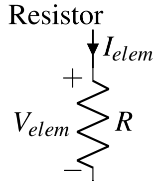
i. Wire



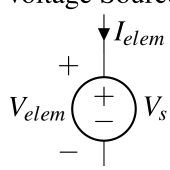
ii. Open Circuit



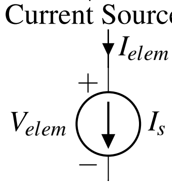
iii. Resistor



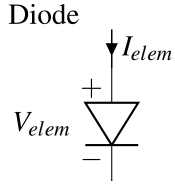
iv. Voltage Source



v. Current Source

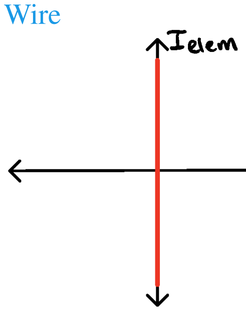


vi. Diode

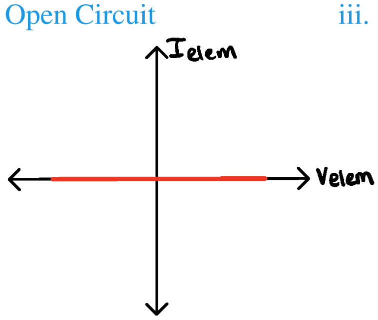


Answer:

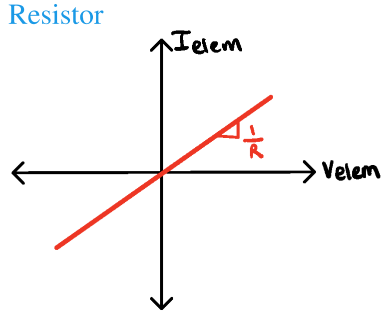
i. Wire



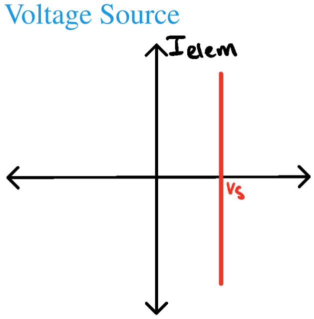
ii. Open Circuit



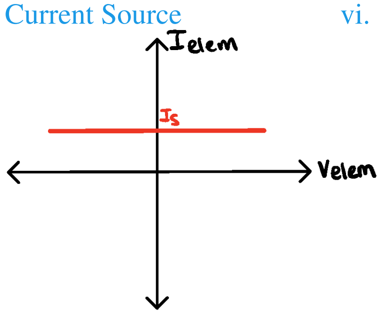
iii. Resistor



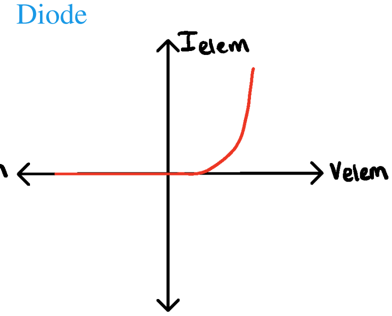
iv. Voltage Source



v. Current Source

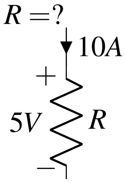


vi. Diode

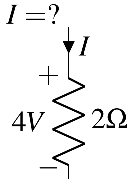


(b) Use Ohm's Law to find the missing component values in the circuits below.

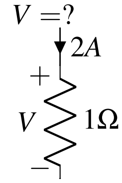
i. $R = ?$



ii. $I = ?$



iii. $V = ?$



Answer:

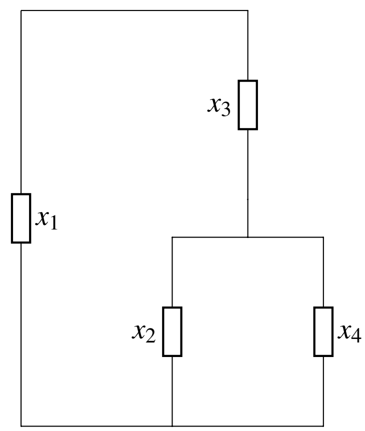
i. $R = \frac{5V}{10A} = 0.5\Omega$

ii. $I = \frac{4V}{2\Omega} = 2A$

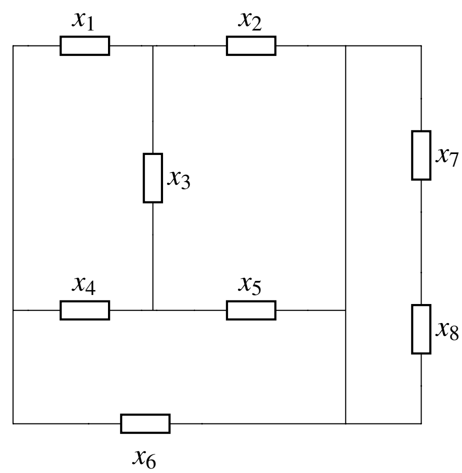
iii. $V = 2A \times 1\Omega = 2V$

2. Label the nodes

In the circuits shown below, label all the nodes.



(a)

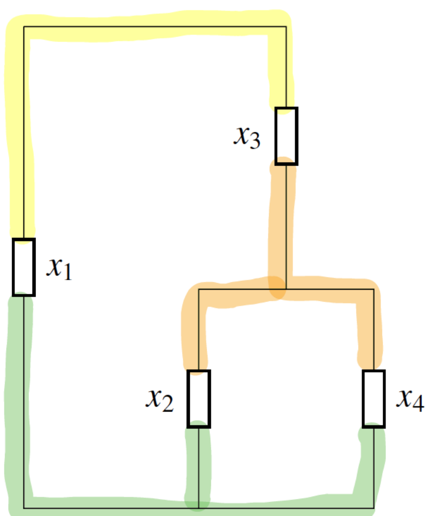


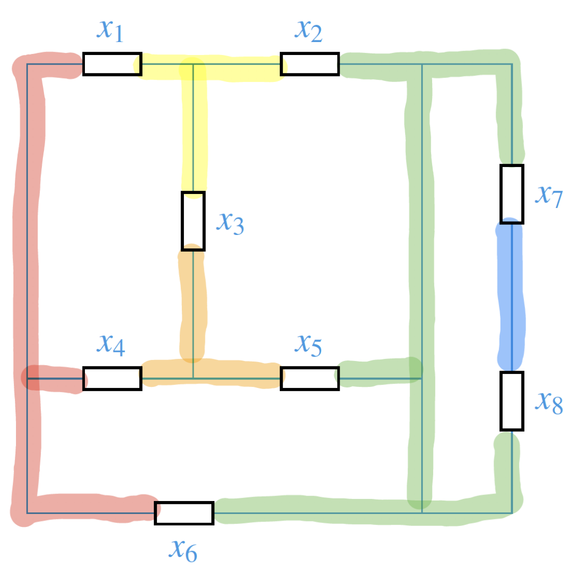
(b)

Answer:

Here are the nodes marked with different colors on each circuit diagram.

There are three nodes in (a) and five in (b).





3. Quest Review

This problem will review concepts of linearity, systems of equations, Gaussian elimination and matrix multiplication.

(a) Determine whether the following functions are linear or nonlinear.

i.
$$f(x_1,x_2) = 3x_1 + 4x_2$$

Answer: To check for linearity, check for superposition (additivity) and homogeneity (multiplicative scaling).
In other words we must check that:

$$f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) = \alpha f(x_1,x_2) + \beta f(y_1,y_2) \quad \forall \alpha,\beta,x_1,x_2,y_1,y_2 \in \mathbb{R}$$

Linear

$$\begin{aligned} f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) &= 3(\alpha x_1 + \beta y_1) + 4(\alpha x_2 + \beta y_2) \\ &= \alpha(3x_1 + 4x_2) + \beta(3y_1 + 4y_2) \\ &= \alpha f(x_1,x_2) + \beta f(y_1,y_2) \end{aligned}$$

Alternatively you can state that this function is linear because it is of the form:

$$f(x_1,x_2) = a_1x_1 + a_2x_2$$

where a_1 and a_2 are constants.

ii.
$$f(x_1,x_2) = e^{x_2} + x_1^2$$

Answer: To check for linearity, check for additivity and homogeneity (multiplicative scaling).
In other words we must check that:

$$f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) = \alpha f(x_1,x_2) + \beta f(y_1,y_2) \quad \forall \alpha,\beta,x_1,x_2,y_1,y_2 \in \mathbb{R}$$

Nonlinear

$$\begin{aligned} f(\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2) &= e^{\alpha x_2 + \beta y_2} + (\alpha x_1 + \beta y_1)^2 \\ &\neq e^{\alpha x_2} + (\alpha x_1)^2 + e^{\beta y_2} + (\beta y_1)^2 \\ &= \alpha f(x_1,x_2) + \beta f(y_1,y_2) \end{aligned}$$

Alternatively you can state that this function is nonlinear because it is NOT of the form:

$$f(x_1,x_2) = a_1x_1 + a_2x_2$$

where a_1 and a_2 are constants.

(b) For each system of equations given as an augmented matrix, use Gaussian elimination to determine whether the system has a unique solution, infinite solutions or no solution.

i.

$$\left[\begin{array}{ccc|c} 2 & 6 & 4 & 10 \\ 1 & -3 & 3 & 13 \\ 0 & 0 & 3 & 12 \end{array}\right] \tag{1}$$

Answer:
Unique solution!!! Start by using Gaussian elimination to reduce the rows:

$$\left[\begin{array}{ccc|c} 2 & 6 & 4 & 10 \\ 1 & -3 & 3 & 13 \\ 0 & 0 & 3 & 12 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 1 & -3 & 3 & 13 \\ 0 & 0 & 3 & 12 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & -6 & 1 & 8 \\ 0 & 0 & 3 & 12 \end{array}\right] \tag{2}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & 1 & \frac{-1}{6} & \frac{-8}{6} \\ 0 & 0 & 3 & 12 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 2 & 5 \\ 0 & 1 & \frac{-1}{6} & \frac{-8}{6} \\ 0 & 0 & 1 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 0 & -3 \\ 0 & 1 & 0 & \frac{-4}{6} \\ 0 & 0 & 1 & 4 \end{array}\right] \tag{3}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{-4}{6} \\ 0 & 0 & 1 & 4 \end{array}\right] \tag{4}$$

In this form, we can read off the single unique solution: $x = -1, y = \frac{-4}{6}, z = 4$.

ii.

$$\left[\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ 0 & 0 & 2 & 1 \end{array}\right] \tag{5}$$

Answer:
Infinite! Start by using Gaussian elimination to reduce the rows:

$$\left[\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ 0 & 0 & 2 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & \frac{-1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & \frac{-1}{3} & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right] \tag{6}$$

The second equation determines that the third unknown, z is $\frac{1}{2}$ and then there are an infinite combination of the first and second unknowns that can satisfy the first equation. To describe this set of solutions, let the second unknown, y, be a free variable, $t \in \mathbb{R}$, and solve for the first unknown, x, in terms of y. Exactly,

$$x = \frac{t}{3}, \quad y = t, \quad z = \frac{1}{2}. \tag{7}$$

(c) Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 5 & 5 & 8 \\ 6 & 1 & 2 \\ 4 & 1 & 7 \\ 3 & 2 & 2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix}$$

For each matrix multiplication problem note whether the product exists, and *if the product exists*, find the dimensions of the resulting matrix.

i. **A B** **Answer:** A is a 2×4 matrix and B is a 4×3 matrix, so the product exists! The resulting matrix has dimensions of 2×3.

ii. **A C** **Answer:** A is a 2×4 matrix and C is a 3×3 matrix, so the product does not exist.

iii. **B D** **Answer:** B is a 4×3 matrix and D is a 3×3 matrix, so the product exists! The resulting matrix has dimensions of 4×3.