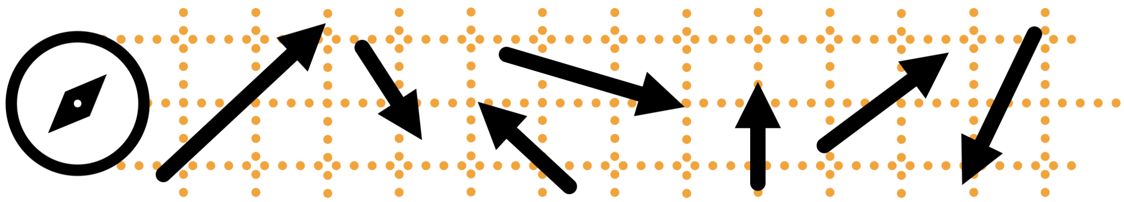
EECS 16A Designing Information Devices and Systems I
Spring 2022 Discussion 2A

1. Vectors



A vector is an ordered list of numbers. For instance, a point on a plane (x, y) is a vector! We label vectors using an arrow overhead $\vec{v}$, and since vectors can live in ANY dimension of space we'll need to leave our notation general $\vec{v} = (v_1, v_2, \ldots)$. Below are few more examples (the left-most form is the general definition):

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n \qquad \vec{a} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^3 \qquad \vec{b} = \begin{bmatrix} 2.4 \\ 5.3 \end{bmatrix} \in \mathbb{R}^2$$

Just to unpack this a bit more, $\vec{b} \in \mathbb{R}^3$ in english means "vector $\vec{b}$ lives in 3-Dimensional space".

- The $\in$ symbol literally means "in"
- The $\mathbb{R}$ stands for "real numbers" (FUN FACT: $\mathbb{Z}$ means "integers" like $-2, 4, 0, \ldots$)
- The exponent $\mathbb{R}^{n\leftarrow}$ indicates the dimension of space, or the number of elements in the vector.

One last thing: it is standard to write vectors in column-form, like seen with $\vec{a}, \vec{b}, \vec{x}$ above. We call these *column vectors*, in contrast to horizontally written vectors which we call *row vectors*.

Okay, let's dig into a few examples:

(a) Which of the following vectors live in $\mathbb{R}^2$ space?

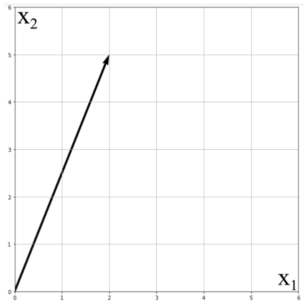
$$i. \begin{bmatrix} 3 \\ 6 \end{bmatrix} \qquad ii. \begin{bmatrix} 5 \\ 0 \\ 3 \\ 5 \end{bmatrix} \qquad iii. \begin{bmatrix} -4.76 \\ 1.32 \\ 0.01 \end{bmatrix} \qquad iv. \begin{bmatrix} -20 \\ 100 \end{bmatrix}$$

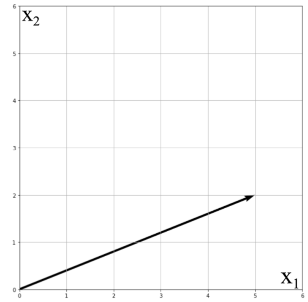
Answer:
i. Yes ii. No iii. No iv. Yes
Remember $\mathbb{R}^2$ means 2D space, which hosts vectors with 2 terms.
We count and see only i. and iv. have 2 terms.

(b) Graphically show the vectors (either in a sketch with axes, or a plot on a computer):

$$i. \begin{bmatrix} 2 \\ 5 \end{bmatrix} \qquad ii. \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Answer: Although these vectors look similar, remember that the ordering matters!

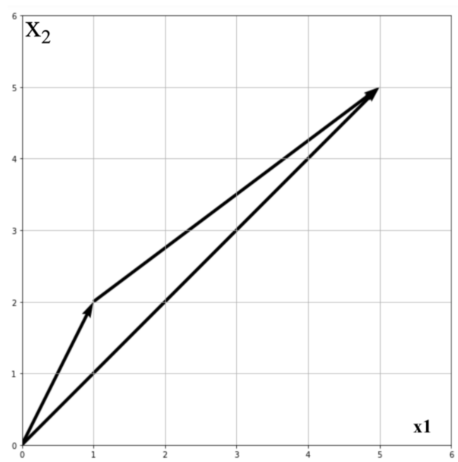




(c) Compute the sum $\vec{a}+\vec{b}=\vec{c}$ from the vectors below, and then graphically sketch or plot these vectors. (show them in a way that forms a triangle; also is there only one possible triangle?)

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

Answer:



Computation is done element-wise:

$$\vec{c} = \vec{a}+\vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

2. Computations: matrix-vector multiplication

For each matrix vector multiplication problem, find the product by hand

(a)

$$A = \begin{bmatrix} 1 & 6 \\ 2 & -7 \end{bmatrix} \vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 1 & 9 & 2 \\ 7 & 10 & -7 \\ -1 & 2 & -8 \end{bmatrix} \vec{b} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

Answer:

(a) To Let $a_i \in \mathbb{R}^3$ represent the transpose of the i_{th} row of the A matrix. To find the i_{th} entry of the $A\vec{b}$ vector we find the inner product of a_i and b. In this case, we get

$$A\vec{b} = \begin{bmatrix} 13 \\ -12 \end{bmatrix}$$

(b) Let $A_i \in \mathbb{R}^3$ represent the i_{th} column of A and let $b_i \in \mathbb{R}$ represent the i_{th} component of b.

$$A\vec{b} = A_1 b_1 + A_2 b_2 + A_3 b_3 = 1 \times \begin{bmatrix} 1 \\ 7 \\ -1 \end{bmatrix} + 3 \times \begin{bmatrix} 2 \\ -7 \\ -8 \end{bmatrix} = \begin{bmatrix} 7 \\ -14 \\ -25 \end{bmatrix}$$

3. Matrix Multiplication

Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

$$\mathbf{E} = \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} 5 & 5 & 8 \\ 6 & 1 & 2 \\ 4 & 1 & 7 \\ 3 & 2 & 2 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix}$$

For each matrix multiplication problem, *if the product exists*, find the product by hand.
Otherwise, explain why the product does not exist.

(a) **A B** **Answer:** A is a 1×2 matrix and B_1 is a 2×1 matrix, so the product exists!
$\mathbf{AB} = 1 \cdot 3 + 4 \cdot 2 = 11$

(b) **C D** **Answer:** Since both **C** and **D** are 2×2 matrices, the product exists and is a 2×2 matrix.
$\mathbf{CD} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 3 + 4 \cdot 2 & 1 \cdot 2 + 4 \cdot 1 \\ 2 \cdot 3 + 3 \cdot 2 & 2 \cdot 2 + 3 \cdot 1 \end{bmatrix} = \begin{bmatrix} 11 & 6 \\ 12 & 7 \end{bmatrix}$.

(c) **D C** **Answer:** Since both **C** and **D** are 2×2 matrices, the product exists and is a 2×2 matrix.
$\mathbf{DC} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 \cdot 1 + 2 \cdot 2 & 3 \cdot 4 + 2 \cdot 3 \\ 2 \cdot 1 + 1 \cdot 2 & 2 \cdot 4 + 1 \cdot 3 \end{bmatrix} = \begin{bmatrix} 7 & 18 \\ 4 & 11 \end{bmatrix}$.

(d) **C E** **Answer:** Since **C** is a 2×2 matrix and **E** is a 2×4 matrix, the product exists and is a 2×4 matrix.
$\mathbf{CE} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 4 \cdot 4 & 1 \cdot 9 + 4 \cdot 3 & 1 \cdot 5 + 4 \cdot 2 & 1 \cdot 7 + 4 \cdot 2 \\ 2 \cdot 1 + 3 \cdot 4 & 2 \cdot 9 + 3 \cdot 3 & 2 \cdot 5 + 3 \cdot 2 & 2 \cdot 7 + 3 \cdot 2 \end{bmatrix} = \begin{bmatrix} 17 & 21 & 13 & 15 \\ 14 & 27 & 16 & 20 \end{bmatrix}$.

(e) **F E** (only note whether or not the product exists) **Answer:**
Since **E** is a 2×4 matrix and **F** is a 4×3 matrix, the product does not exist.
This is because the number of columns in the first matrix (**F**) should match the number of rows in the second matrix (**E**) for this product to be defined.

(f) **E F** (only note whether or not the product exists) **Answer:**
Since **E** is a 2×4 matrix and **F** is a 4×3 matrix, the product exists and is a 2×3 matrix.

$$\mathbf{EF} = \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} 5 & 5 & 8 \\ 6 & 1 & 2 \\ 4 & 1 & 7 \\ 3 & 2 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 1\cdot5+9\cdot6+5\cdot4+7\cdot3 & 1\cdot5+9\cdot1+5\cdot1+7\cdot2 & 1\cdot8+9\cdot2+5\cdot7+7\cdot2 \\ 4\cdot5+3\cdot6+2\cdot4+2\cdot3 & 4\cdot5+3\cdot1+2\cdot1+2\cdot2 & 4\cdot8+3\cdot2+2\cdot7+2\cdot2 \end{bmatrix}$$
$$= \begin{bmatrix} 100 & 33 & 75 \\ 52 & 29 & 56 \end{bmatrix}$$

(g) **G H** (Practice on your own) **Answer:** Since **G** and **H** are both 3×3 matrices, the product exists and is another 3×3 matrix.

$$\mathbf{GH} = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 8\cdot5+1\cdot1+6\cdot2 & 8\cdot3+1\cdot8+6\cdot3 & 8\cdot4+1\cdot2+6\cdot5 \\ 3\cdot5+5\cdot1+7\cdot2 & 3\cdot3+5\cdot8+7\cdot3 & 3\cdot4+5\cdot2+7\cdot5 \\ 4\cdot5+9\cdot1+2\cdot2 & 4\cdot3+9\cdot8+2\cdot3 & 4\cdot4+9\cdot2+2\cdot5 \end{bmatrix} = \begin{bmatrix} 53 & 50 & 64 \\ 34 & 70 & 57 \\ 33 & 90 & 44 \end{bmatrix}.$$

(h) **H G** (Practice on your own) **Answer:** Since **H** and **G** are both 3×3 matrices, the product exists and is another 3×3 matrix.

$$\mathbf{GH} = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix} \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 5\cdot8+3\cdot3+4\cdot4 & 5\cdot1+3\cdot5+4\cdot9 & 5\cdot6+3\cdot7+4\cdot2 \\ 1\cdot8+8\cdot3+2\cdot4 & 1\cdot1+8\cdot5+2\cdot9 & 1\cdot6+8\cdot7+2\cdot2 \\ 2\cdot8+3\cdot3+5\cdot4 & 2\cdot1+3\cdot5+5\cdot9 & 2\cdot6+3\cdot7+5\cdot2 \end{bmatrix} = \begin{bmatrix} 65 & 56 & 59 \\ 40 & 59 & 66 \\ 45 & 62 & 43 \end{bmatrix}.$$

