#	Question	Answer(s)
1	How do we determine whether the reservoir and pump system is conservative?	If the summation of the numbers in each column equals 1 then it's conservative
2	what do we call these matrices again?	These are transition matrices
3	to determine if its conservative, do we check if each row adds to 1 of i if each column adds up to 1?	Each column
4	in this matrice is this for when t = 0	The matrix P doesn't represent just one point in time, but the transition from t=0 to t=1
5	why does the sum of all outflows per reservoir = 1	We sum to 1 so that we describe what happens to 100% of the water. If its greater than 1, then we would be adding more water into the system. Similarly, less than 1 would mean we are losing water in the system
6	What would it,Äôs transpose represent?	live answered
7	should the sum of the elements of the column always equal to 1 or is it okay to have leaks which will make the sum elements of the column less than 1?	Yes there may be leaks depending on what information the problem gives you. So not always add up to 1
8	what's the difference between transpose vs inverse?	Tranpose just rearranges the values in the matrix by mirroring across the diagonal. Inverse is a matrix that, when multiplied with the original matrix, gives the identity. In general, they are not the same
9	the trasnpose is the inverse? i didnt hear	No they are not the same. You will see more details in the discussion sections.
10	but can't they go over 1?	Yes they can, if there is gain
11	can you re-explain what is conservative/nonconservative system? I missed that part	If the numbers in each column add up to 1, the system is conservative (not losing or gaining water)
12	Shouldn,Äôt sum of the rows equal to 1?	Not necessarily. Since the column describes outflow, we can't (generally) output more than 100% of what we have. There is generally no restriction on the inflow.
13	What if we run the pump twice in a non conservative system, and the second run on of the pumps doesnt have enough water to pump out what it did last time?	live answered
14	How do you get the vector 2, 6, 0?	This was some chosen initial state.
15	How is this system conservative, westarted with 2,6,0 and ended with 2.5, 2.5, 2.5	live answered
16	Is there a valid shorthand for (t=#)?	We will generally pick some variable x(t=N) and use that N in the equation.
17	what's the diff between conservative and non-conversative system again?	The total amount of water in a conservative system will always remain the same
18	will it always be P^2*x(t=0) for after two runs? or just in this case?	It's always P^2 after two runs
19	how is it conservative if 2+6+0 = 8 (state at t = 0) and 2.5+2.5+2.5 = 7.5 (state at t = 2). The amoount of water decreased	live answered
20	how is it still conserved if the system loses 0.5 of water?	live answered
21	If the system is conservative, why is the sum of the last vector 7.5 compared to the last 2 states where the sum of water is 8?	live answered
22	the total is 8 in t = 1, and 7.5 and in t = 2? It,Äôs not conserved right?	live answered
23	Shouldn't all the elements add up to 8 in this case?	live answered
24	if the system is conservative why does the vector have elements 2, 3, 3 after the first time step which add up to 8 and after the second time step it,Äôs 2.5, 2.5, 2.5 which adds to 7.5? how is that still conservative?	live answered
25	How come the sum of the amount of water in the system in the first vector (2, 6, 0) wasn,Äôt equal to the second (2.5, 2.5, 2.5)?	live answered
26	Shouldnt third vector sum to eight?	live answered
27	why does the vector only add up to 7.5 when its run twice? Before it was 8	live answered
28	will pumps always pump out a fraction of what they have or can they also pump out certain values instead?	For our examples, they will pump out a fraction.
29	if the columns don,Äôt add up to 1 like this case, how can we tell if the system is conservative	For conservative system the numbers in each column of the transition matrix add up to 1.
30	a conservative system means that the sum in the output vector is constant?	Yes
31	Why was running the pumps twice conservative again? i missed that part	live answered
32	Why did we lose 0.5 in the system? (2+3+3) - (2.5)*3 = 0.5. Is	live answered
33	Why is it conservative?	live answered
34	Should the components of x(t + 2) sum to 8?	live answered
35	if we,Äôre checking columns for conservation, and each reservoir pumps our 100% of the amount of water it had, each reservoir could then receive differing amounts of that water from the other reservoirs-how is that conservative?	The water each reservoir pumps out always goes into some reservoirs. They don't disappear and there is also no gain. So the total amount of water in all the reservoirs remain the same
36	How is this system conservative, if we started with 2,6,0 and ended with 2.5, 2.5, 2.5	live answered
37	i,Äôm still confused on conservative/nonconservative in this current practice problem, the sum of the columns of x(t+1) doens,Äôt add up to 1,Ä¶?	For conservative system the numbers in each column of the [transition matrix] add up to 1. Not the state vectors x(t).
38	How do we have {2*5, 2*5, 2*5}?	live answered
39	take limits	live answered
40	What is I here?	I is the identity matrix

41	how is it conservative if 2+6+0 = 8 (state at t = 0) and 2.5+2.5+2.5 = 7.5 (state at t = 2). The amoount of water decreased	live answered
42	Where does the I come from?	I is the identity matrix. We bringing it in so we can isolate (P-I). Multiplying by I does not affect the equation.
43	whats i again?	identity matrix (diagonal elements are 1, others are all 0)
44	can you please explain why we are multiplying the identity matrix to the x vector or subtracting the I matrix?	we're trying to reach (P-I)x=0, which is a problem we know how to solve using GE. This is mainly foreshadowing right now, we'll cover this again in coming lectures and discussion.
45	What does the identitiy matrix represent in this problem?	The identity matrix is a square matrix with the same size as P with just 1s along the diagonal. I * x = x, so it doesn't affect the equation, but it helps us get to (P-I).
46	why do we need to isolate (P-I)?	We want to set up a system of equations of x_star and solve that by Gaussian elimination
47	how is these infinite steady states?	The exact steady state will depend on the initial amount of water in the system, but the final steady state will be that vector times some scalar.
48	how did we get 8 6 9 if there are infinite solutions	any scalar alpha times [8 6 9] are also a solutions
49	wait where are the 8 6 9 from?	By Gaussain elimination and writing the parametric solution. It's [8 6 9] times any scalar alpha
50	Are we guaranteed that any given initial state will converge to a steady state?	Not necessarily
51	How do we know which steady state solution it actually goes to?	live answered
52	How did the Professor get to [8 6 9] if the system had infinite solutions?	[8 6 9] time any scalar alpha
53	I see that the water level does not change at the steady state. But can we be sure that we can always reach the steady state from any initial state?	We will get into this more when we look at eigenspaces next week
54	What does it mean if there are multiple steady state solutions? If we let alpha=2, is there double the amount of water compared with alpha=1?	Yes, but once you fixed the alpha (fixed the amount of water in the initial state), the amount of water will not change by applying the transition matrix
55	the reason why it‚Äôs x = P * x is because the system is conservative?	No, the idea comes from steady states. We call it steady because at infinite time, the transition doesn't change the state.
56	Mathematically, I understand how there are infinite solutions. But conceptually, how is it that we can apply any constant alpha to our resovoir volume? i.e. if 8, 6, 9 were valid volumes, how could 80, 60, 90 also simultaneously be valid?	It depends on the total initial amount of water.
57	What was the parametric answer again?	[8 6 9] times any scalar alpha
58	what happened after we solved the Gaussian Equilibrium for the steady state problem and got infinite solutions? I didn‚Äôt fully understand that	That means each of the infinite solutions will be a possible steady state
59	no	live answered
60	is there an inverse ‚Äúfunction‚Äù to get the previous states?	Yes, it will be the inverse of the P matrix, which we will show in a moment
61	so we use the inverse to find out information about the past?	In the context of these transition matrices yes. In general, the point of an inverse is to undo some operation.
62	how did we get 8, 6, 9 again?	By gaussian elimination to the (P-I)x = 0 equation
63	So if alpha was 1, what does that mean for the input?	We don't know about the initital state, but when you run the system for a lot of times you will finally get the [8 6 9] state (steady state)
64	Once you reach steady state is it impossible to ‚Äúsee into the past‚Äù	That's a good tricky question. Technically, once we reach infinite time, we can't go back in time (inf -1 = inf). as long as we are before inf time, then we can go into the past.
65	how did we get exactly 8 6 9 though. after doing gaussian elimination regardless of what we get on the left, doesnt the 3x1 vector on the right of the equation have zeroes as every element. so how do we equate and solve it	We're not always guaranteed a solution to (P-I), but if we do, we should get a parametric solution from GE
66	Is our inverse matrix to the left or to the right	Inverse matrix works both to the left and to the right
67	arcsine has a range of -90 to 90. Does that have an impact here?	Yes you are right. We need to limit the variable range first.
68	sorry for the redundant question, but how the transition doesn‚Äôt change the state at infinite times? does that come from that infinity plus one is just infinity?	That's more or less the idea, and it gives us the setup Px=x
69	waut how x = p x1	x(t) = P^(-1) * x(t+1)
70	is the x(t + N) formula only true if x is a vector or could it be a matrix of any size?	It's the state vector in this case
71	Does it not also work for other matrices?	It depends on the matrix. There are plenty of matrices that are not invertible, and we will look at how to determine that.
72	how about 1	f(x) = 1 is also not invertible
73	yes but i dont get the initial equation in black and after she added the red p-1	
74	Does the steady state only happen when we run the transition matrix infinite times?	We may reach the steady state early if we start with some special initial state. But generally if we run the system for infinite times it will reach the steady state regardless the initial state
75	what‚Äôs a parametric solution? (in the case of GE)	When we have infinite solutions, we can generally describe those solutions with some restriction of variables and vectors. Those are the parametric solutions.
76	why did we multiply system Q into system R?	It's an example showing if we first do R and then do Q the system will be back to the initial state.

77	so x(t+2) = x(t)	For this example where the flows changes for t=2N and t=2N+1, yes
78	what does this say about the system if they will constantly undo each other	One thing we can say is that its oscillating between 2 states.
79	is A(A^-1(x)) always equal to A^-1(A(x))?	Yes
80	Is it always true that if A * A-1 = I, then A-1 * A = I, or was that more specific to this case?	It's always true
81	how do you find the inverse of a matrix again?	We haven't formally shown that just yet, but we will show a technique using GE.
82	does this mean that any two matrices multiplied with each other which give rise to an identity matrix are inverses of each other?	Yes if they are square matrices.
83	will the inverse always be a reciprical?	Generally no
84	How come x(t+2) = Q*x(t+1)? Isn‚Äôt it supposed to be R?	We apply R as the first step, then apply Q as the second step
85	If P Q = I, is it then also always true that Q P = I?	Yes if P and Q are both square matrices
86	Why do we apply Q in second step and not R?	It's an example showing if we do R first and Q second the system will be back to the initial state, so Q is an inverse of R
87	if the nodes were not clearly labeled with letters how would we know which one to start with.	We will nicely label all our graphs. Generally, you get to pick your own labels.
88	why is x_c half of x_a?	live answered
89	Is it necessary to write out the equations or can you go straight to the matrix using the AA, BA, CA, etc etc method?	You can go straight to the matrix if you are comfortable
90	If PQ = I de we assume that QP also = I?	Yes in this case, since they are both square matrices
91	can we only find inverse matrices for systems that are conservative?	not necessarily, in the previous example, [2 0; 0 2] and [0.5 0; 0 0.5] are not conservative
92	i‚Äôm a little confused, why did she highlight P? why is it important? I missed that, and what do the colors represent?	The matrix P times each of the columns of Q will give the corresponding column in I. For example, P times the green q1 will give the green b1, etc.
93	Will the right hand side be Q after solving this?	Yes, that's the goal
94	How do we go from three matrix-vector multiplications to this augmented matrix?	Since we already know how to solve one M-V multiplication with GE, we just stack extra columns on the right hand side, and the operations are the same as the GE we've seen so far.
95	Can‚Äôt we do determinant * A transpose to get the inverse?	Nope, the equation is the inverse of A is the transpose of the cofactor matrix times the reciprocal of the determinant of A. We will generally not use this equation, but for 2D matrix we will derive it in discussions.
96	P*Q = I, [P | I] = Q	That's more or less the idea with Gauss Jordon
97	wait so not all matrices are invertable?	Correct. we will develop tools to explain and show which matrices are and are not invertible
98	does every square matrix have an inverse?	Nope. we will develop tools to explain and show which matrices are and are not invertible.
99	does not every matrix have an inverse? why?	Not every square matrix has an inverse. we will develop tools to explain and show which matrices are and are not invertible
100	is the if statement in the proof an assunption?	Generally yes
101	what does qed mean?	live answered
102	are most proofs done using proof by contradiction?	There are a variety of proof techniques, no one better than the others
103	is the inverse of the identity matrix the identity matrix itself?	Yes
104	Will we get a lot of proof practice in homeworks?	We're doing a decent amount in discussion, and we'll have a few more coming in homeworks. If you want more practice, there are some resources on the course website; or drop by OH.
105	what does QED stand for again?	live answered
106	would we have to write out the definition of linear independence?	If your proof uses the defn of L.I., you need to use it enough to setup the start of your proof. ex. setting up the linear combination.
107	how‚Äôd we go from the original matrix to the identity one? I might have missed something	For proof by contradiction, we assumed that the inverse exists. Then we left multiplied by the inverse, so A^(-1)*A = I
108	"if a invertible ‚Äî> then cols of A are lin dependent" is the asumption that we proove false yes?	We proved if A is invertible, then the cols of A cannot be linearly dependent
109	is there a quicker way to find out if a matrix has an inverse or not?	Checking linear indepedence of the columns ia good fast check. Another method is using the determinant, but depending on the size of the matrix, that may be slower. There are also some specific matrix forms that have / don't have inverse.
110	does d have to not equal zero as well or else the determinant could be zero?	Even if both a and d are zero, the determinant can still be nonzero if b and c are nonzero. We assume a to be nonzero here just for a easier Gaussian elimination
111	Why did she highlight those two columns again? what does that represent	She's recognizing them as 2 matrix-vector multplications.
112	How did we know that a was already 1	live answered
113	why didn‚Äôt she start by normalizing row 1, and making a 1	Technically, she didn't have to do that first. You can normalize the rows at the end, as long as you get the pivots through row reduction.
114	how is r1 - br2 = 0?	we already normalized r2 before r1 - b r2

115	wait isnt that 1 / the determinant	