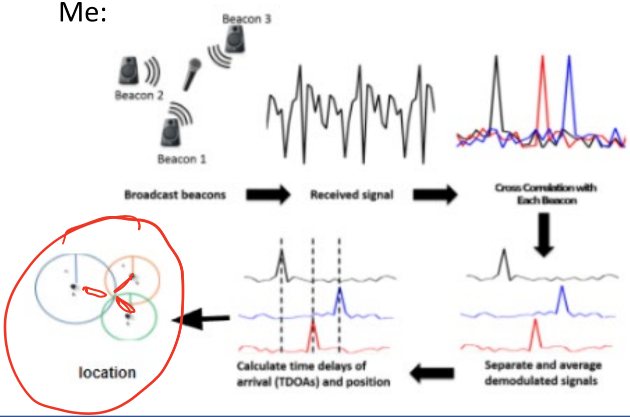
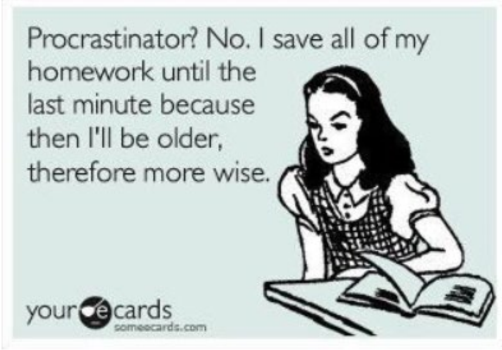
Friend: Come over!
Me: I have no idea where i am and all I have is this recording that sounds like trash
Friend: I have chocolate ☺
Me:



EECS 16A
Trilateration and projections

Admin

- Midterm 2 redo due TOMORROW – required for clobber policy



Previously on 16A... Inner products aka “dot product”

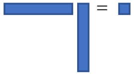
- a measure of how aligned two vectors are
- the sum of the element-wise product of two vectors:

$$\langle \vec{a}, \vec{b} \rangle = \vec{a}^T \vec{b}$$

$$= a_1 b_1 + a_2 b_2 + a_3 b_3 + \ldots a_n b_n$$

$$= \sum_{i=1}^{n} a_i b_i$$

$$= \|\vec{a}\| \|\vec{b}\| \cos(\Delta\theta)$$

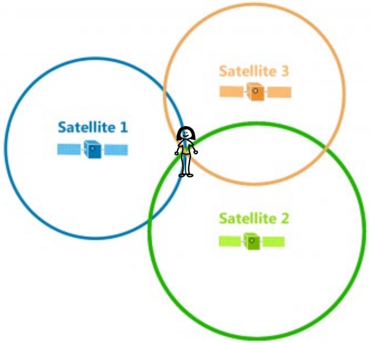


If 0, vectors are ***orthogonal (perpendicular)***

Cauchy-Schwartz Inequality $\langle \vec{a}, \vec{b} \rangle \leq \|\vec{a}\| \|\vec{b}\|$

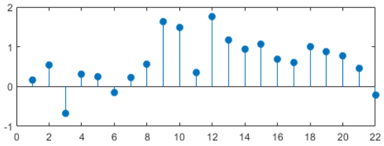
JORGE CHAM ©THE STANFORD DAILY

Last lecture: Classification

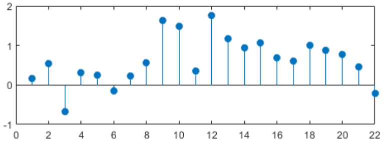


Which satellite am I talking to? **Satellite A**

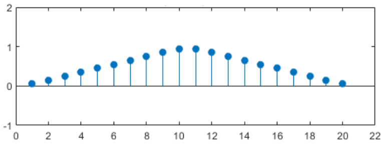
$\vec{\mathbf{s}}_A =$



$\vec{\mathbf{r}} =$



$\vec{\mathbf{s}}_B =$



$\langle \vec{\mathbf{r}}, \vec{\mathbf{s}}_A \rangle =$ large ✓

$\langle \vec{\mathbf{r}}, \vec{\mathbf{s}}_B \rangle =$ smaller

$\vec{s}_A$

want $\langle \vec{s}_A, \vec{s}_B \rangle$ = small

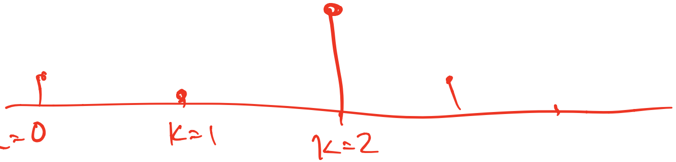
Last time: Cross-correlation

Each element of cross-correlation is the inner product of the first vector with a shifted version of the second vector

$$corr_{\vec{x}}(\vec{y})[k] = \sum_{i=0}^{\infty} x(i)\,y(i-k)$$

correlate $\vec{x}$ & $\vec{y}$

shift



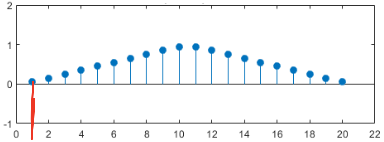
$$corr_{\vec{x}}(\vec{y}) = \left[\langle x, y_0\rangle \langle x, y_1\rangle \langle x, y_2\rangle \ldots \langle x, y_n\rangle\right]$$

0 shift

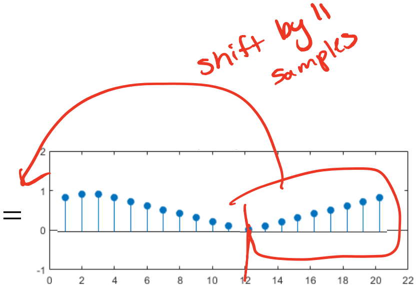
shifted by 1

Last time: Cross-correlation

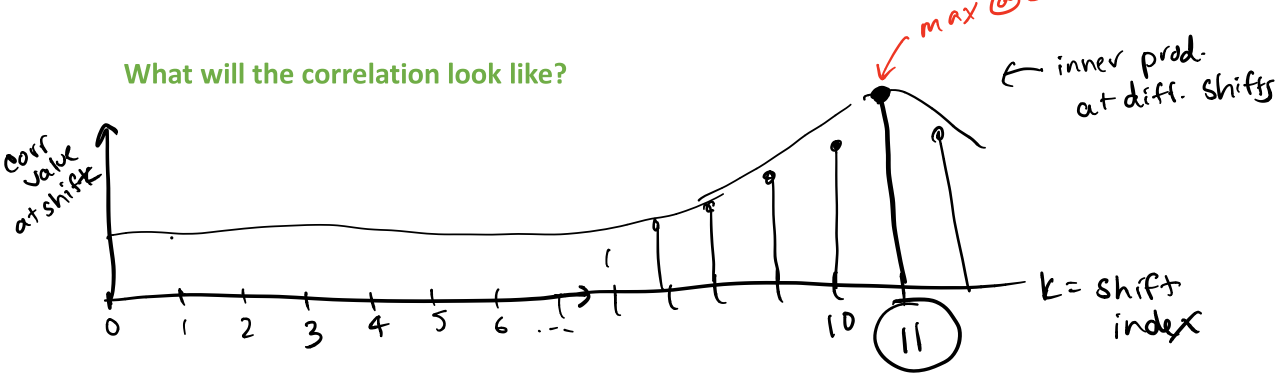
$\vec{s} =$



$\vec{r} =$

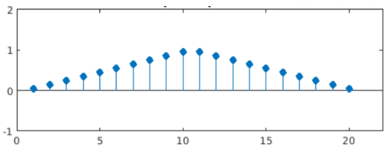


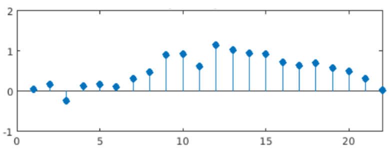
What will the correlation look like?



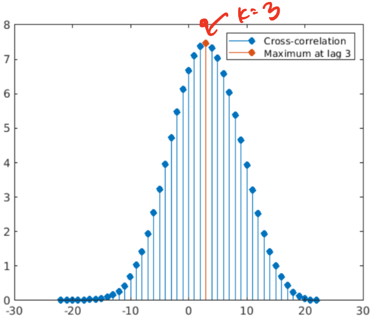
https://www.youtube.com/watch?v=Ma0YONjMZLI

Last time: Cross-correlation

$\vec{\mathbf{s}} =$ 

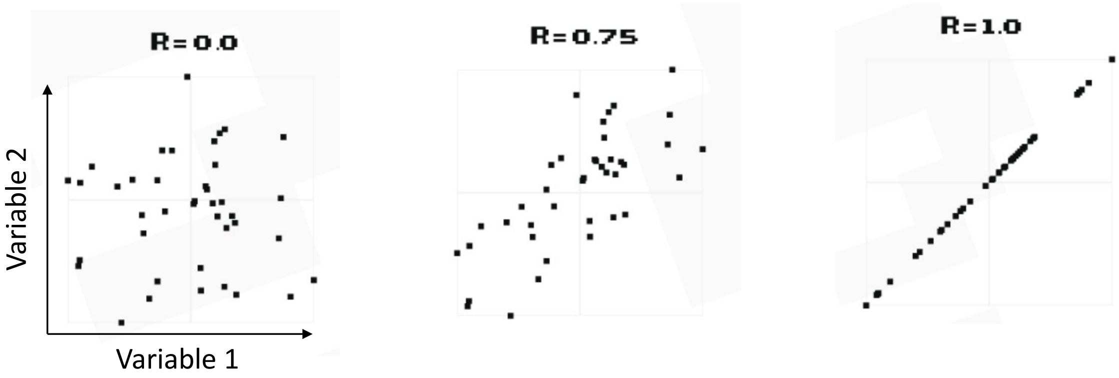


What will the correlation look like?



https://www.youtube.com/watch?v=Ma0YONjMZLI

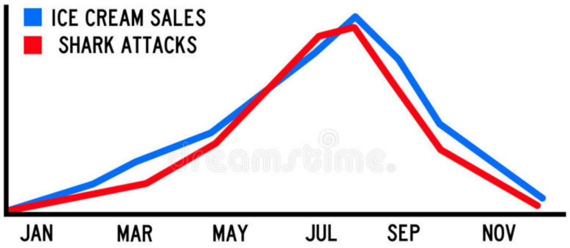
Correlation: scatter plot view



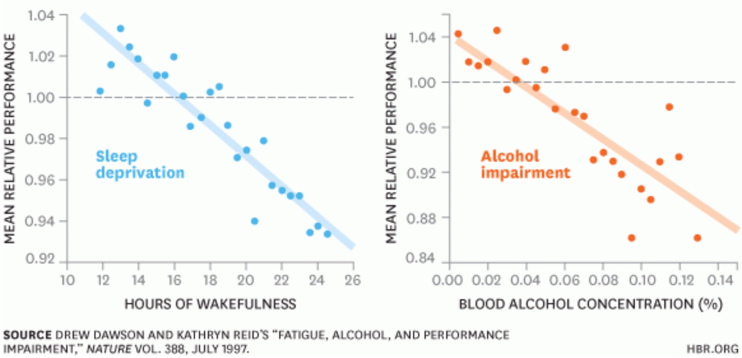
more correlated

www.guessthecorrelation.com

“Correlation is not causation”



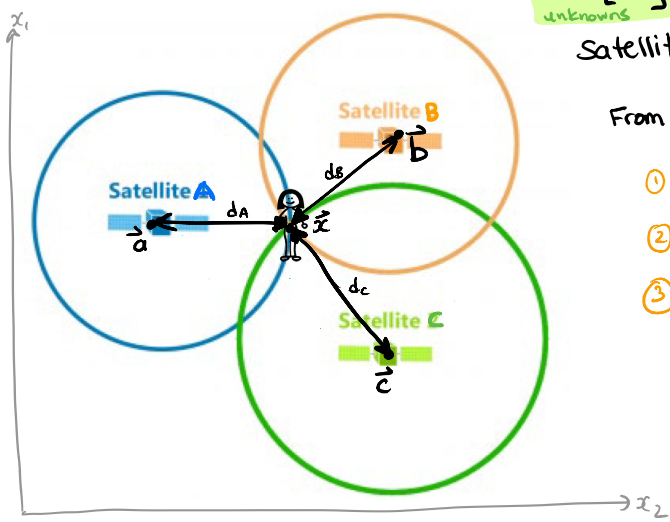
Both ice cream sales and shark attacks increase when the weather is hot and sunny, but they are not caused by each other (they are caused by good weather, with lots of people at the beach, both eating ice cream and having a swim in the sea)



Trilateration

Let's figure out how to solve for my coordinates in 2D world, $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, from distances d_A, d_B, d_C to 3 satellites with known positions: $\vec{a}, \vec{b}, \vec{c}$

unknowns — known



From picture, we can write:

square both sides → equivalent

① $\|\vec{x}-\vec{a}\|^2 = d_A^2$

② $\|\vec{x}-\vec{b}\|^2 = d_B^2$

③ $\|\vec{x}-\vec{c}\|^2 = d_C^2$

3 equations, 2 unknowns ☺

Problem: they're not linear ☹

① $(\vec{x}-\vec{a})^T(\vec{x}-\vec{a}) = d_A^2$

$\vec{x}^T\vec{x} + \vec{a}^T\vec{a} - \vec{x}^T\vec{a} - \vec{a}^T\vec{x} = d_A^2$

④ $\underbrace{\|\vec{x}\|^2}_{\text{This is the problem term → nonlinear + unknown!}} + \underbrace{\|\vec{a}\|^2}_{\text{known}} - 2\underbrace{\langle \vec{x}, \vec{a}\rangle}_{\text{unknown}} = \underbrace{d_A^2}_{\text{known}}$

⑤ $\|\vec{x}\|^2 + \|\vec{b}\|^2 - 2\langle \vec{x}, \vec{b}\rangle = d_B^2$

Trick! take ④-⑤ to get rid of evil term!

⑥ $\|\vec{a}\|^2 - \|\vec{b}\|^2 - 2\langle \vec{x}, \vec{a}\rangle + 2\langle \vec{x}, \vec{b}\rangle = d_A^2 - d_B^2$

All squared terms are known, so this is LINEAR

Now, do same with

④ $\|\vec{x}\|^2 + \|\vec{a}\|^2 - 2\langle \vec{x}, \vec{a}\rangle = d_A^2$

③ $\|\vec{x}\|^2 + \|\vec{c}\|^2 - 2\langle \vec{x}, \vec{c}\rangle = d_C^2$

④-③ ⑦ $\|\vec{a}\|^2 - \|\vec{c}\|^2 - 2\langle \vec{x}, \vec{a}\rangle + 2\langle \vec{x}, \vec{c}\rangle = d_A^2 - d_C^2$

⑥, ⑦ are 2 linear eqns, 2 unknowns! ☺

For clarity, let's write in $A\vec{x} = \vec{b}$ form:

⑥ $\|\vec{a}\|^2 - \|\vec{b}\|^2 - 2(x_1 a_1 + x_2 a_2) + 2(x_1 b_1 + x_2 b_2) = d_A^2 - d_B^2$

⑦ $\|\vec{a}\|^2 - \|\vec{c}\|^2 - 2(x_1 a_1 + x_2 a_2) + 2(x_1 c_1 + x_2 c_2) = d_A^2 - d_C^2$

$\begin{bmatrix} 2(\vec{b}-\vec{a})^T \\ 2(\vec{c}-\vec{a})^T \end{bmatrix}$ need lin. indep. → suggests satellites shouldn't be collinear or not invertible!

these are all known scalars!

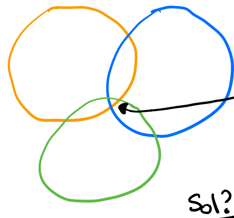
$$\begin{bmatrix} -2a_1+2b_1 & -2a_2+2b_2 \\ -2a_1+2c_1 & -2a_2+2c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} d_A^2 - d_B^2 - \|\vec{a}\|^2 + \|\vec{b}\|^2 \\ d_A^2 - d_B^2 - \|\vec{a}\|^2 + \|\vec{b}\|^2 \end{bmatrix}$$

knowns — solve for us!

→ Gaussian Elim to solve! EASY PEASY!

Problem: noise may make system of eqns inconsistent ☹

Example: if satellite positions or distances have some error, 3 circles won't intersect in single point!



Where am i?
Need a 'best estimate' when no perfect sol'n! } Least Squares Algorithm!

Sol?: → take more meas. & average?
→ add more satellites? hope one intersects
→ weight meas. by accuracy?

Least Squares finds best estimate for linear system, even with noise

$A\vec{x} = \vec{b} + \vec{e}$ ← error

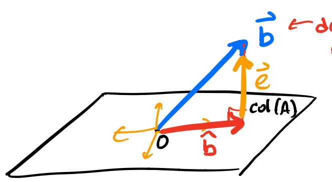
$$\underbrace{\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 2 \end{bmatrix}}_{\text{A isn't square, so can't invert}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

} more eqns than unknowns, seems good for averaging...

Want to find $\vec{x}$ such that $A\vec{x}$ is as close as possible to $\vec{b}$:

$$A = \begin{bmatrix} | & | & & | \\ \vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_n \\ | & | & & | \end{bmatrix} \qquad A\vec{x} = \begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \vec{a}_1 x_1 + \vec{a}_2 x_2 + \dots \vec{a}_n x_n$$

"column view"



← doesn't have to be in same plane if error/noise

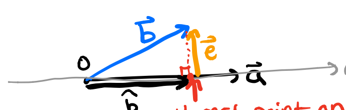
Find the vector in plane of col(A) ← the set of allowed solutions!
that is closest to $\vec{b}$
↳ This will be our optimal (best guess) solution!

denotes estimate → $\hat{b} = A\vec{x}$ is directly below b
"orthogonal (perpendicular) projection" of $\vec{b}$ onto col(A)

The error is $\vec{e} = \vec{b} - \hat{b}$

Example: 1D Projection

$A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ (2x1) $\quad \hat{x}$ (1x1; 1 unknown, 2 equations) $\quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ (2x1)



col(A) is span($\vec{a}$) line

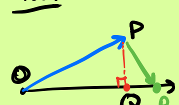
closest point on line to $\vec{b}$ is the dropped perpendicular

Want min $\|\vec{e}\|^2 = \|\vec{b} - \hat{b}\|^2 = \|\vec{b} - A\hat{x}\|^2$ ← estimate of $\vec{x}$

$\hat{b} = \alpha \vec{a}$ ← want to find α
↳ find 'projection' of $\vec{b}$ onto $\vec{a}$

Theorem: shortest dist. b/w a point (vector) and a line is given by the perpendicular

Proof: PQR form a right angle triangle w/ ∟ at Q



$PQ^2 + \underbrace{QR^2}_{\text{always} > 0} = PR^2$ Pythagorus

$\therefore PQ < PR$ always larger! ✓ ☺