EECS16A
Acoustic Positioning System 2

Last Lab! :)

Insert your names here

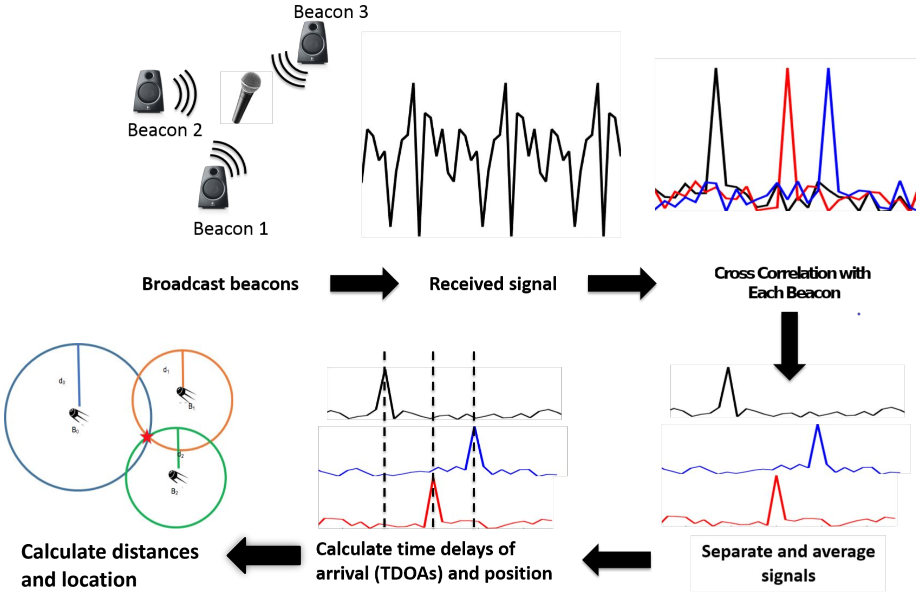
Announcements!

- This is the **last lab!**
- Do APS 1 first if you haven't yet (APS 2 can then be done during buffer)
- Course evaluations: [link]
- APS buffer labs 5/3-5/7 (RRR week)
  - Sign up here: tiny.cc/aps-buffer-sp21
  - Encouraged to attend a Mon-Wed section
- Good luck on the final!

when you finally finish the lab and this shows up

Last lab: APS 1

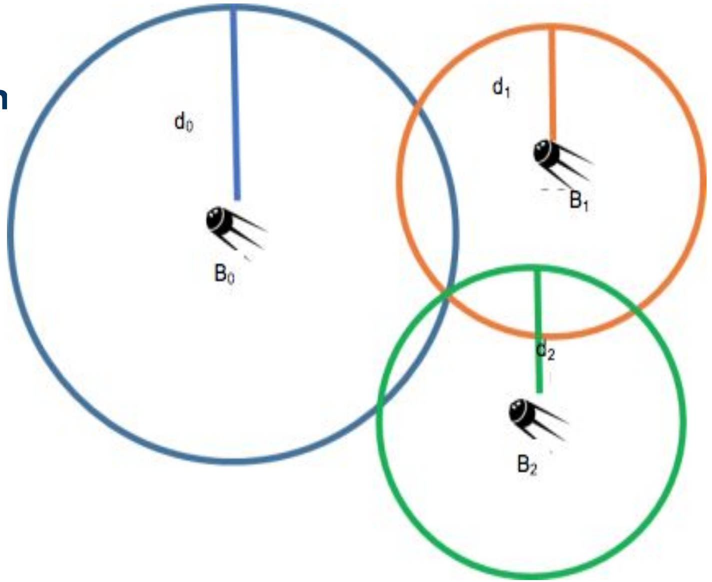
- Cross correlated beacon signals with received signal
- Found the offsets (in samples) between peaks, converted to TDOAs, and calculated distances from each beacon
- **What was the missing piece that we needed to calculate distance?**
  - Hint: we don’t have absolute times of arrival for all the beacons, only relative offsets.



3 Beacon Example

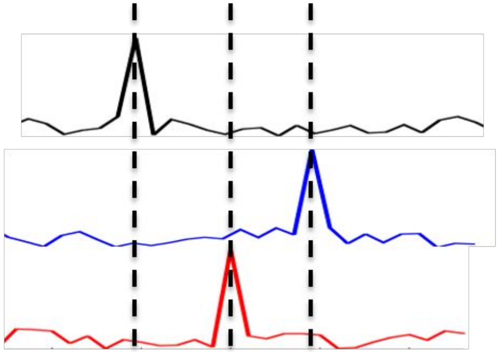
- Let beacon centers be: (x_0, y_0), (x_1, y_1) and (x_2, y_2)
- Time of arrivals: t_0, t_1, t_2
- Distance of beacon m $(m = 0, 1, 2)$ is $d_m = vt_m = R_m$ (circle radii)

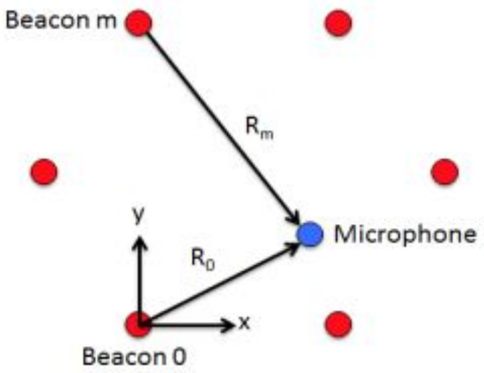
Circle equations: $(x - x_m)^2 + (y - y_m)^2 = d^2{}_m$



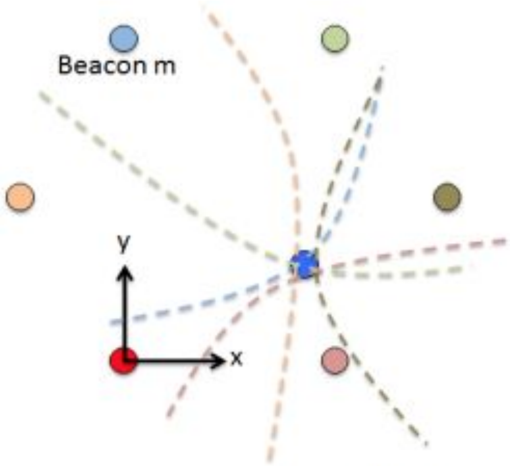
Problem: We don’t know t_0

- Only know time offsets: $\tau_m = t_m - t_0$
- $R_m = \sqrt{(x - x_m)^2 + (y - y_m)^2} = v_s t_m$
- $R_0 = \sqrt{(x)^2 + (y)^2} = v_s t_0$ (Beacon 0 is at origin)
- $R_m - R_0 = v_s (t_m - t_0) = v_s \tau_m$





Setting Up n-1 Hyperbolic Equations



Beacon 0 is not used for locationing since it acts as the reference signal.

$$R_m - R_0 = v_s \tau_m$$

Simplify!

$$v_s \tau_m = \frac{-2x_m x + {x_m}^2 - 2y_m y + {y_m}^2}{v_s \tau_m} - 2\sqrt{x^2 + y^2}$$

- $m \neq 0$ (as $\tau_0 = 0$)
- This is the equation for a hyperbola
- This is hard to solve

Making it Linear

- Same trick: subtract first equation from others

Not linear in x, y :(

$$v_s\tau_m = \frac{-2x_m x + x_m{}^2 - 2y_m y + y_m{}^2}{v_s\tau_m} - 2\sqrt{x^2 + y^2}$$

$$\downarrow$$

$$v_s\tau_m - v_s\tau_1 = [\frac{-2x_m x + x_m{}^2 - 2y_m y + y_m{}^2}{v_s\tau_m} - 2\sqrt{x^2 + y^2}] - [\frac{-2x_1 x + x_1{}^2 - 2y_1 y + y_1{}^2}{v_s\tau_1} - 2\sqrt{x^2 + y^2}]$$

Linear! $\downarrow$ Simplify!

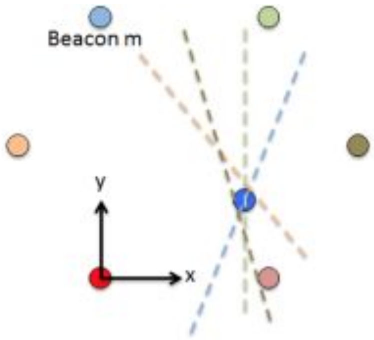
$$(\frac{2x_m}{v_s\tau_m} - \frac{2x_1}{v_s\tau_1})x + (\frac{2y_m}{v_s\tau_m} - \frac{2y_1}{v_s\tau_1})y = (\frac{x_m{}^2 + y_m{}^2}{v_s\tau_m} - \frac{x_1{}^2 + y_1{}^2}{v_s\tau_1}) - (v_s\tau_m - v_s\tau_1) \quad \mathbf{m \neq 0, m \neq 1}$$

Making It Linear

$$m \neq 0,\ m \neq 1$$

$$\left(\frac{2x_m}{v_s\tau_m} - \frac{2x_1}{v_s\tau_1}\right)x + \left(\frac{2y_m}{v_s\tau_m} - \frac{2y_1}{v_s\tau_1}\right)y = \left(\frac{x_m^{\ 2} + y_m^{\ 2}}{v_s\tau_m} - \frac{x_1^{\ 2} + y_1^{\ 2}}{v_s\tau_1}\right) - (v_s\tau_m - v_s\tau_1)$$

- After simplifying, we have n-2 linear equations and 2 unknowns (x,y)
- Can do least-squares regardless of number of beacons



Beacon 1 was sacrificed to make the system of equations linear.

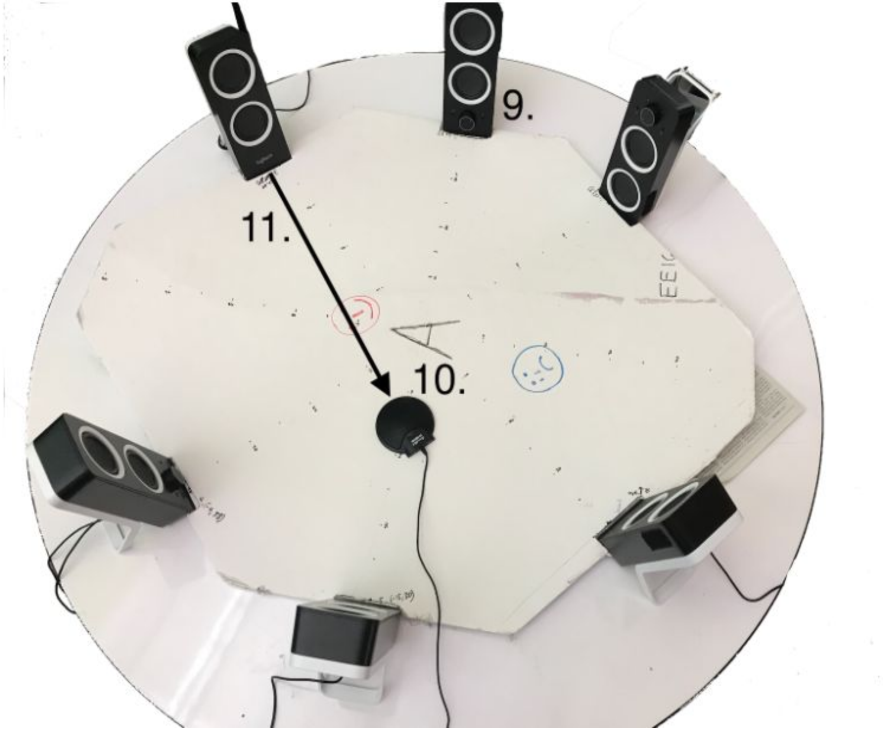
Beacon 0 is not used for locationing since it acts as the reference signal

  - Best estimate of location if measurements are inconsistent
  - If there is no exact point of intersection because of error or noise

$$Ax = b$$

$$A^T A x = A^T b$$

Setup Looks Like:



Important Notes

- Read over the math carefully, We'll be asking you about it!
- Stay safe and good luck with the rest of the semester! *virtual hand wave*
  - Thank you for being part of this remote offering!