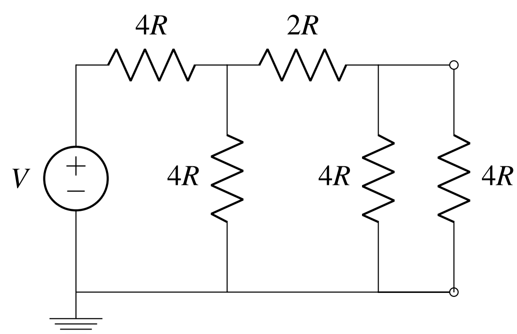
EECS 16A Designing Information Devices and Systems I
Spring 2021 Discussion 11A

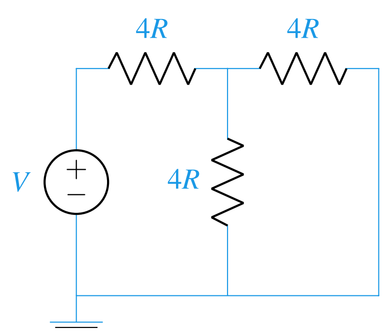
:: Pre-Midterm Discussion ::
Choose which of the following problems you would like to review.
GOOD LUCK!!

1. OPTIONAL: Power to Resist (from Spring 2018 midterm 2)

Find the power dissipated by the voltage source in the circuit below. Be sure to use passive sign convention.



Answer: This circuit can be reduced using techniques similar to those used to analyze the R-2R ladder from homework. We want to find the equivalent resistance across the voltage source in Figure 6.2. Start by reducing the two resistors on the right to $4R \parallel 4R = 2R$. Then combine the other $2R$ resistor with this to get a new resistor of value $4R$ as in the circuit below.



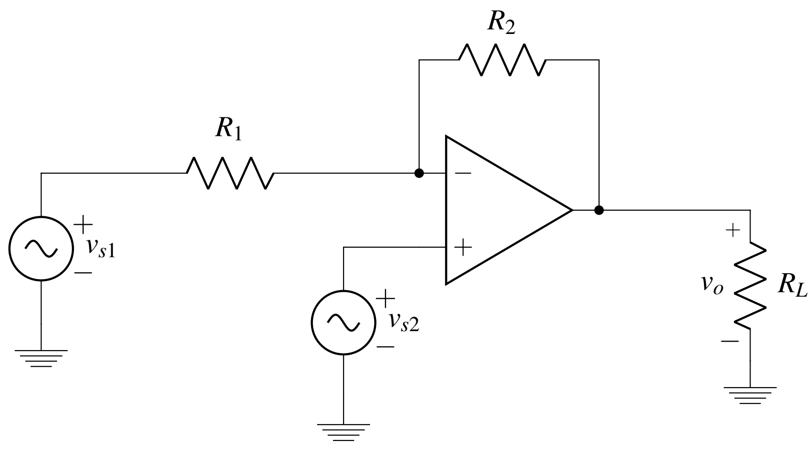
Once again we have $4R \parallel 4R = 2R$. This is finally in series with $4R$ giving us a total resistance of $4R + 2R = 6R$

$$P = VI = V\frac{-V}{6R} = -\frac{V^2}{6R}$$

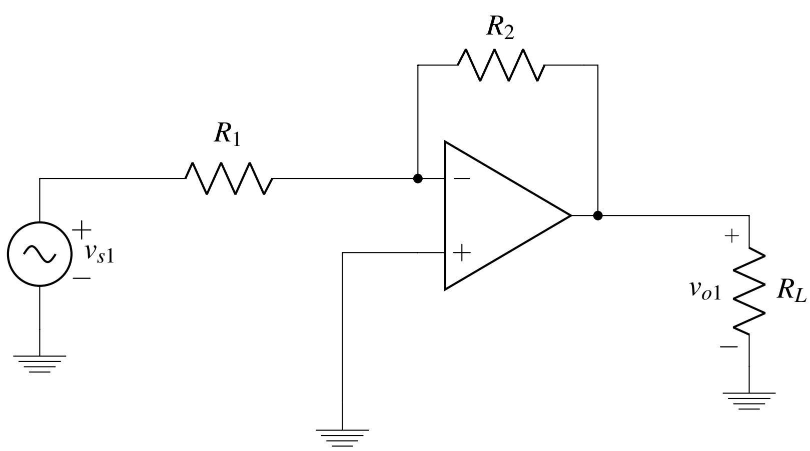
The negative sign is present because the voltage source actually provides power, which can also be seen by using passive sign convention.

2. OPTIONAL: Amplifier with Multiple Inputs

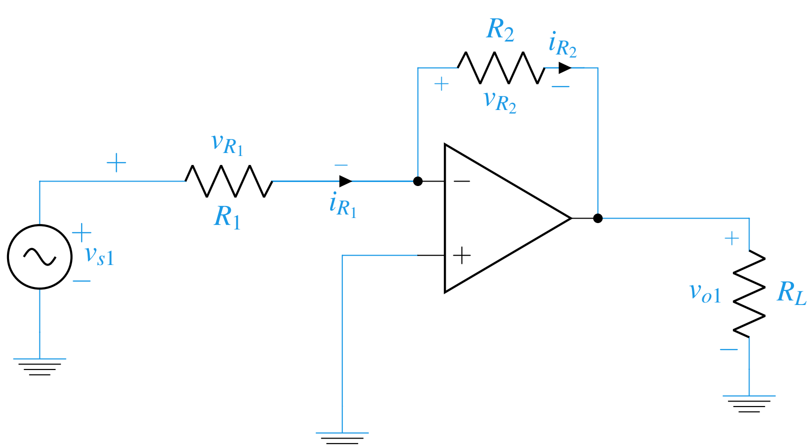
In this problem we will use superposition and the Golden Rules to find the output of the following op amp circuit with multiple inputs:



(a) First, let's turn off v_{s2}. Use the **Golden Rules** to find v_{o1} for the circuit below.



Answer:



Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is 0. In addition, no current flows into the op-amp from the negative terminal due to its infinite input resistance (the negative terminal is connected to an "open" circuit).

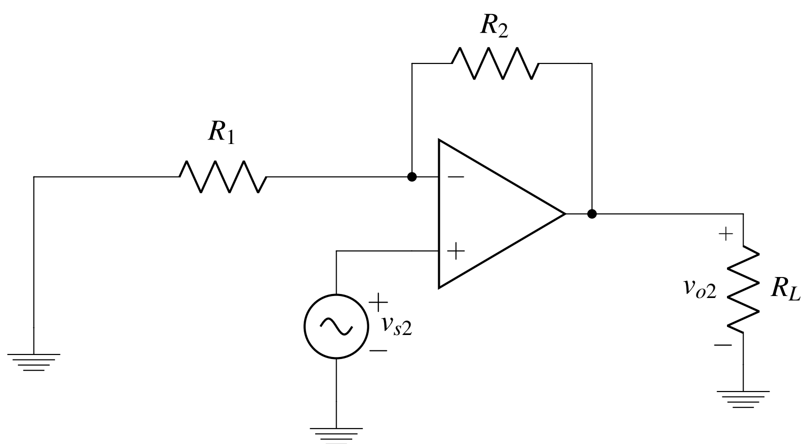
By KCL at the negative terminal of the op-amp, this means that the current going through R_1 and R_2 is $\frac{v_{s1}}{R_1}$. Taking the positive terminal of R_2 to be on the left (following PSC), the voltage drop across R_2 is $-v_{o1}$. By Ohm's law, we conclude:

$$\frac{-}{v_{o1}}R_2 = i_{R_2} = i_{R_1} = \frac{v_{s1}}{R_1}$$

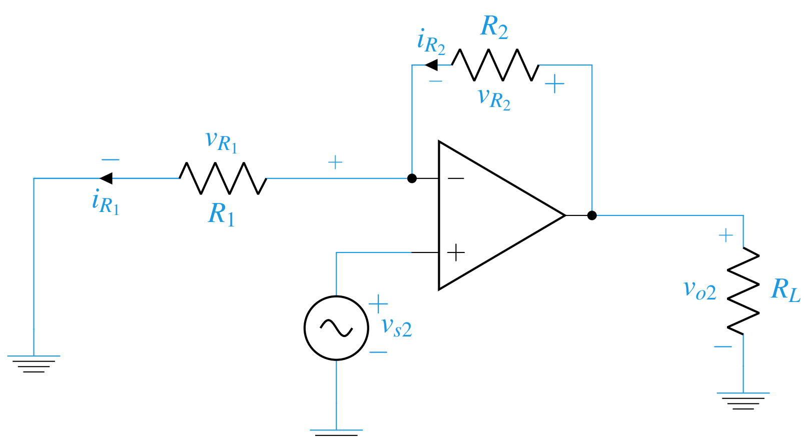
Rearranging we get:

$$v_{o1} = -v_{s1} \cdot \frac{R_2}{R_1}$$

(b) Now let's turn off v_{o1}. Use the **Golden Rules** to find v_{o2} for the circuit below.



Answer:



Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is $V^- = v_{s2}$. In addition, since no current can enter into the negative terminal of the op-amp, R_1 and R_2 are in series. This means that the voltage at the negative terminal of the op-amp can be expressed in terms of v_{o2} using the voltage divider formula:

$$v^- = v_{o2}\left(\frac{R_1}{R_1 + R_2}\right)$$

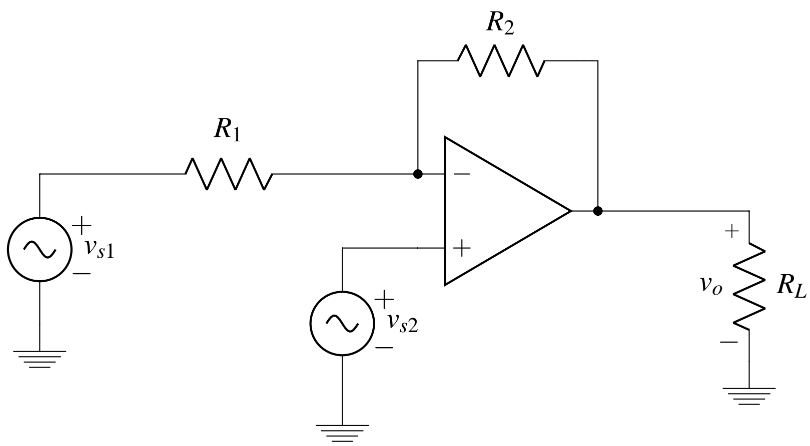
We also know that $v^- = v_{s2}$ and conclude:

$$v_{s2} = v_{o2}\left(\frac{R_1}{R_1 + R_2}\right)$$

After rearranging, we have:

$$v_{o2} = v_{s2}\left(\frac{R_2}{R_1} + 1\right)$$

(c) Use **superposition** to find the output voltage v_o for the circuit shown below.

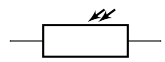


Answer:
Using superposition we can now simply add the results from the two previous part to get the final answer:

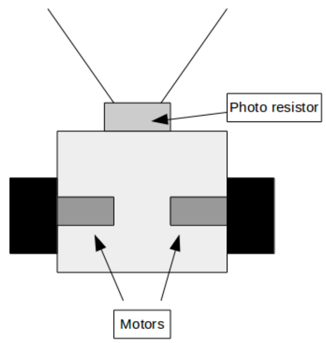
$$v_o = v_{o1} + v_{o2} = -v_{s1} \cdot \frac{R_2}{R_1} + v_{s2}\left(\frac{R_2}{R_1} + 1\right)$$

3. OPTIONAL: PetBot Design (from Fall 2016 Final exam)

In this problem you will design circuits to control PetBot, a simple robot designed to follow light. PetBot measures light using a photoresistor, which is a light-sensitive resistor. *As it is exposed to more light, its resistance decreases.* The diagram below shows the circuit symbol for a photoresistor.



The basic layout of PetBot can be seen below. It is driven by one motor that will be modeled as a resistor. PetBot drives forward (towards the said light source) when a positive voltage is applied across the motor, and conversely a negative applied voltage drives PetBot backward (away from the light source). In this system the light sensor is mounted to the front of the robot, and the speed of PetBot is proportional to the applied voltage to the motor.



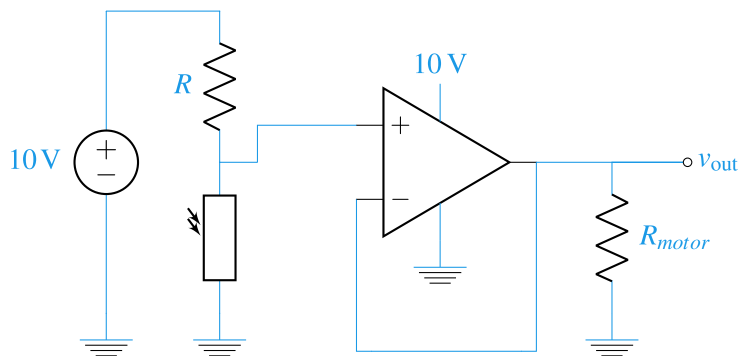
(a) **Speed control**

In our first circuit design, we will begin by making PetBot decrease speed as it drives towards light. **Design a motor-driving circuit that outputs a decreasing positive motor voltage as PetBot drives toward the light source.** The motor voltage should be at least $5\,\text{V}$ when far away from the light. At this far away from the light source, the photoresistor value will be $10\,\text{k}\Omega$, and then drop towards $100\,\Omega$ as it approaches the light.

In your design, you may use any number of resistors and op-amps. You also have access to voltage sources of $10\,\text{V}$ and $-10\,\text{V}$. **Based on your circuit, derive an expression for the motor voltage as a function of the circuit components that you used.**

NOTE! Since the motor is a resistor, the circuit design MUST have a buffer so that the applied voltage to the motor does not depend on its resistance.

Answer: A natural choice for our design is to combine a voltage divider with a buffer circuit, as depicted below.



The output of the above circuit is:

$$v_{out} = \left(\frac{R_p}{R_p + R}\right) 10\,\text{V}$$

R_p represents the photoresistor, and $R \leq 10\,\text{k}\Omega$.

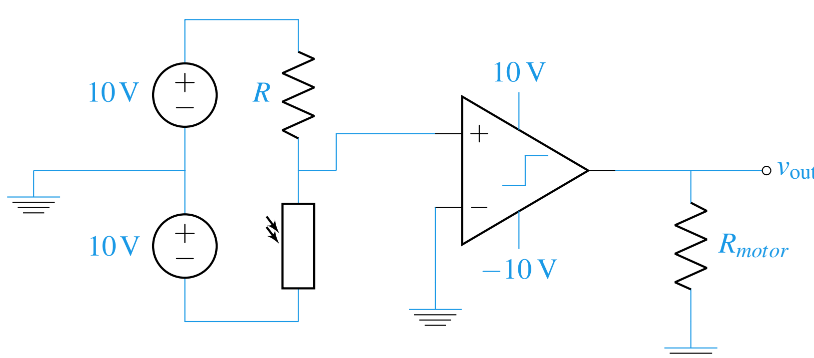
Since we require $v_{out} \geq 5\text{V}$ when far from light ($R_p = 10\,\text{k}\Omega$), the maximum allow value for R is $R = R_p$ as stated.

(b) **Distance control**
When the PetBot stops at a distance of $1\,\text{m}$ away from the light, the photo-resistor has a value $1\,\text{k}\Omega$. We would like to have the PetBot drive away when closer than 1m from the light (so for lower R_p), and drive towards the light when exceeding $1\,\text{m}$ (so for greater R_p).

Design a comparator circuit that outputs a positive motor voltage when the PetBot exceeds $1\,\text{m}$ in distance from the flashlight (making the PetBot move toward it), and a negative voltage when PetBot is within $1\,\text{m}$ of flashlight (making the PetBot back away from the flashlight.

In your design, you may use any number of resistors along with the comparator. You also have access to voltage sources of $10\,\text{V}$ and $-10\,\text{V}$.

Answer: One possible comparator circuit design is shown below:



Since we have grounded the negative input terminal (and there is no feedback in the circuit) we realize the comparator rails high (positive $v_{out} = +10V$ when the input to the positive op-amp terminal has positive voltage.

Using our voltage-divider knowledge (and recognizing this divider is analogous to a simple divider with $V_s = 20V$ and then shifted by 10V) we find the positive node voltage:

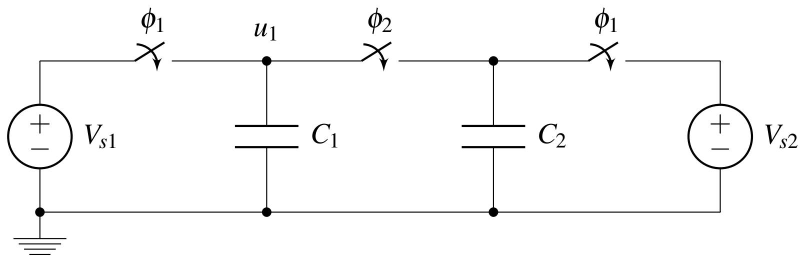
$$u_+ = 20\left(\frac{R_p}{R_p + R}\right) - 10 \;=\; 10\left(\frac{R_p - R}{R_p + R}\right).$$

Notice for $R_p > R$, the node u_+ has positive voltage and so $u_+ > u_-$, thus $v_{out} > 0$. Conversely when $R_p < R$, the u_+ node goes below zero and renders a negative v_{out} on the motor.

This means our circuit successfully applies positive motor voltage (drives PetBot forward) when far away from the light source with large R_p, and applies negative voltage when too close to the light. Our exact design criteria are met when $R = R_p = 1\,\text{k}\Omega$.

4. OPTIONAL: Capacitive Charge Sharing (from Spring 2020 Midterm 2)

Consider the circuit below with $C_1 = C_2 = 1\,\mu\text{F}$ and two switches ϕ_1, ϕ_2. Suppose that initially the switch ϕ_1 is closed and ϕ_2 is open such that C_1 and C_2 are charged through the corresponding voltage sources $V_{s1} = 1\,\text{V}$ and $V_{s2} = 2\,\text{V}$.



(a) How much charge is on C_1 and C_2? How much energy is stored in each of the capacitors? What is the total stored energy?

Answer:

$$q_1 = C_1 V_1 = 1\,\mu\text{C}$$
$$q_2 = C_2 V_2 = 2\,\mu\text{C}$$

Energy:

$$E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

Therefore, $E_1 = \frac{1}{2}C_1V_1^2 = 0.5\,\mu\text{J}$, $E_2 = \frac{1}{2}C_2V_2^2 = 2\,\mu\text{J}$, and the total energy is $2.5\,\mu\text{J}$.

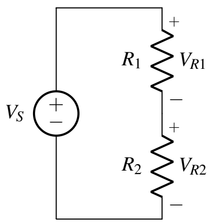
(b) Now suppose that some time later, switch ϕ_1 opens and switch ϕ_2 closes. What is the value of voltage u_1 at steady state?

Answer: The total charge on capacitors C_1 and C_2 will be conserved after switch S_1 is opened. That charge is $Q_tot = q_1 + q_2$. Also note that during phase 2 the capacitors are connected in parallel so they will both have $V_{C1} = V_{C2} = u_1$.

$$Q_{tot} = C_1 u_1 + C_2 u_1$$
$$u_1 = \frac{q_1 + q_2}{C_1 + C_2} = 1.5\,\text{V}$$

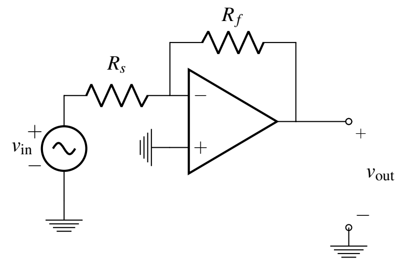
Reference Circuits

Voltage Divider



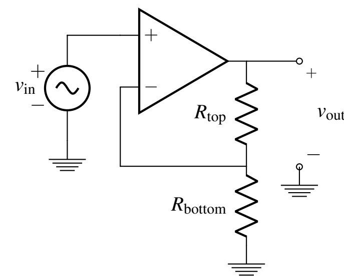
$$V_{R_2} = V_S\left(\frac{R_2}{R_1+R_2}\right)$$

Inverting Amplifier



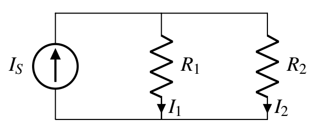
$$v_{out} = v_{in}\left(-\frac{R_f}{R_s}\right)$$

Noninverting Amplifier



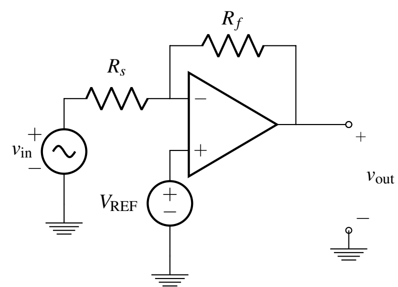
$$v_{out} = v_{in}\left(1+\frac{R_{top}}{R_{bottom}}\right)$$

Current Divider



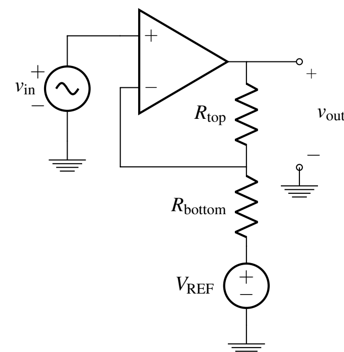
$$I_1 = I_S\left(\frac{R_2}{R_1+R_2}\right)$$

Inverting Amplifier with Reference



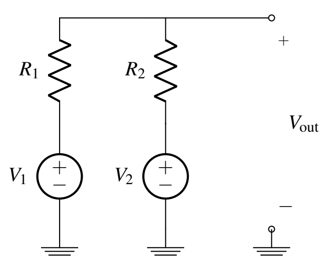
$$v_{out} = v_{in}\left(-\frac{R_f}{R_s}\right)+V_{REF}\left(\frac{R_f}{R_s}+1\right)$$

Noninverting Amplifier with Reference



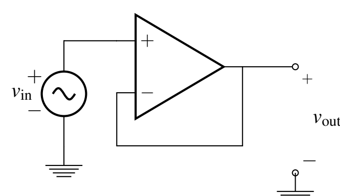
$$v_{out} = v_{in}\left(1+\frac{R_{top}}{R_{bottom}}\right)-V_{REF}\left(\frac{R_{top}}{R_{bottom}}\right)$$

Voltage Summer



$$V_{out} = V_1\left(\frac{R_2}{R_1+R_2}\right)+V_2\left(\frac{R_1}{R_1+R_2}\right)$$

Unity Gain Buffer



$$v_{out} = v_{in}$$