EECS 16A Designing Information Devices and Systems I
Fall 2022 Homework 11

This homework is due November 11, 2021, at 23:59.
Self-grades are due November 14, 2021, at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw11.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned)

Submit each file to its respective assignment on Gradescope.

1. Reading Assignment

For this homework, please read Notes 18 and 19. They will provide an overview on operational amplifiers (op-amps), negative feedback, the "golden rules" of op-amps, and various op-amp configurations (non-inverting, inverting, buffers, etc). You are always encouraged to read beyond this as well.

(a) What are the two "golden rules" of ideal op-amps? When do these rules hold true?

Solution: The golden rules are the following:

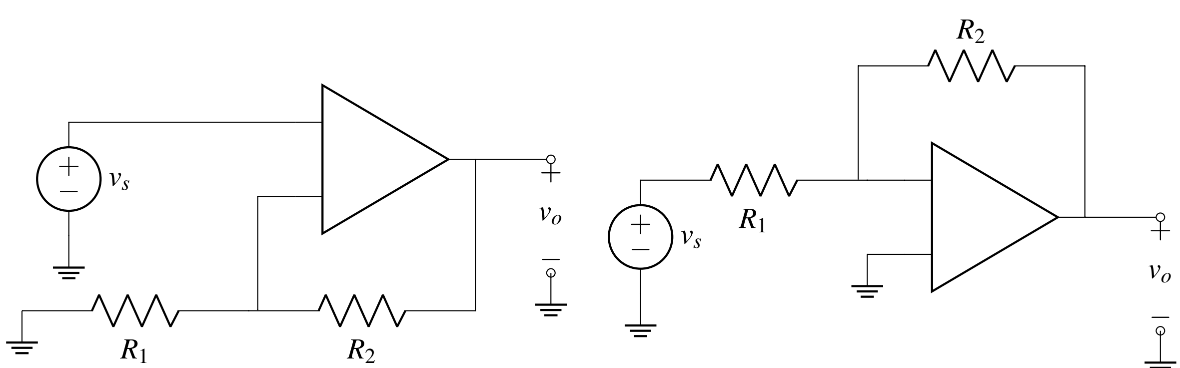
- The error signal going into the op-amp must be zero, i.e. $u_+ = u_-$. This rule only holds when there is negative feedback.
- The currents into the input terminals of the op-amp are zero, i.e. $I_+ = I_- = 0$. This rule holds regardless of whether there is negative feedback or not.

(b) What does the internal gain of an op-amp, A, mean? What is its value for an ideal op-amp? What about for a non-ideal one?

Solution: The internal gain of an op-amp, A is the ratio of the output voltage to the error voltage, i.e. A is given by $\frac{v_{out}}{u_+ - u_-}$. For ideal op-amps, $A \to \infty$. For non-ideal op-amps, A is finite.

2. Basic Amplifier Building Blocks

The following amplifier stages are used often in many circuits and are well known as (a) the non-inverting amplifier and (b) the inverting amplifier.

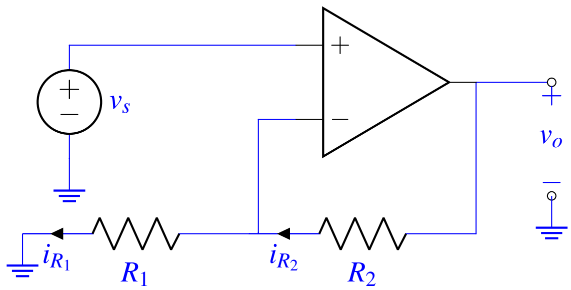


(a) (b)

(a) Label the input terminals of the op-amp with $(+)$ and $(-)$ signs in Figure (a), so that it is in negative feedback. Then derive the voltage gain ($G = \frac{v_o}{v_s}$) of the non-inverting amplifier in Figure (a) using the Golden Rules. Why do you think this circuit is called a non-inverting amplifier?

Solution:

For labeling negative feedback, we must analyze how $v_o = A(u_+ - u_-)$ behaves when we wiggle it by increasing it. If u_- increases in direct proportion to the wiggled output, then we can assume our circuit is in negative feedback and begin applying both golden rules for our circuit analysis.



The $+$, $-$ should be labeled on the top and bottom of the op-amp, respectively, in order to put the circuit in negative feedback. By inspection, we see that by doing this, u_- is in some way connected to the output of the op-amp and we can resume our circuit analysis. To show this more rigourously, if we move the negative input of the op-amp u_- upward, $v_o = Av_{\text{error}} = A(u_+ - u_-)$ moves downward and as a result $u_- = \frac{R_1}{R_1+R_2}v_o$ moves downward. So the result of the initial stimulus goes in the opposite direction of the initial stimulus, which satisfies negative feedback.

By the Golden Rules, the voltage at the positive input terminal v_s must also set the voltage at the negative input terminal to be v_s. Our Golden Rules also tell us that no current can flow into the input terminals of the op-amp. Therefore, we can write a single KCL equation at the input node of the negative terminal as follows:

$$\begin{aligned} u_- &= u_+ = v_s \\ i_{R_1} &= i_{R_2} \\ \implies \frac{v_s}{R_1} &= \frac{v_o - v_s}{R_2} \end{aligned}$$

Rearranging and solving for v_o, we therefore obtain:

$$\begin{aligned} R_2 v_s &= R_1 v_o - R_1 v_s \\ \implies v_o &= \left(\frac{R_1 + R_2}{R_1}\right) v_s \end{aligned}$$

Note also that you may be familiar already with a faster way of solving this problem! Because no current flows into the negative input terminal of the op-amp, you may recognize R_1 and R_2 as simply forming a *voltage divider* over v_o. Therefore, the potential at the negative terminal is:

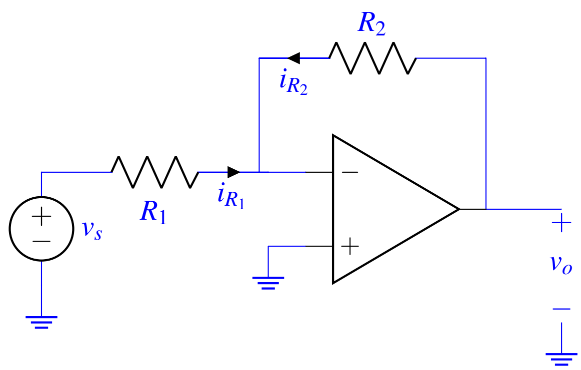
$$\begin{aligned} u_- = v_s &= v_o \left(\frac{R_1}{R_1 + R_2}\right) \\ \implies v_o &= \left(\frac{R_1 + R_2}{R_1}\right) v_s \\ \implies G = \frac{v_o}{v_s} &= \left(\frac{R_1 + R_2}{R_1}\right) \end{aligned}$$

This is called an *non-inverting amplifier* because the gain G is positive – it does not invert the input signal (in contrast to the amplifier in the next part of this problem).

(b) Label the input terminals of the op-amp with $(+)$ and $(-)$ signs in Figure (b), so that it is in negative feedback. Then derive the voltage gain ($G = \frac{v_o}{v_s}$) of the inverting amplifier using the Golden Rules. Can you explain why this circuit is called an inverting amplifier?

Solution:

Just as in part (a), for labeling negative feedback, we must analyze how $v_o = A(u_+ - u_-)$ behaves when we wiggle it by increasing it. If u_- increases in direct proportion to the wiggled output, then we can assume our circuit is in negative feedback and begin applying both golden rules for our circuit analysis.



The $+$, $-$ should be labeled on the bottom and top of the op-amp, respectively. Now if we move the negative input of the op-amp u_- upward, $v_o = Av_{\text{error}} = A(u_+ - u_-)$ moves downward and as a result u_- moves downward because of the following relationship:

$$\frac{v_o - u_-}{R_2} = \frac{u_- - v_s}{R_1}$$
$$\implies u_- = \frac{R_1}{R_1 + R_2} v_o + \frac{R_2}{R_1 + R_2} v_s$$

So the result of the initial stimulus goes in the opposite direction of the initial stimulus, which satisfies negative feedback.

Since the potential at the positive input terminal is $u_+ = 0$, the op-amp will act such that the potential at the negative input terminal is $u_- = 0$ as well (by the Golden Rules). Now, by KCL at the node with potential u_-:

$$i_{R_1} = i_{R_2}$$
$$\frac{v_s - 0}{R_1} + \frac{v_o - 0}{R_2} = 0$$

Solving this yields:

$$v_o = -\left(\frac{R_2}{R_1}\right) v_s$$

Thus, the voltage gain of this amplifier circuit is:

$$G = \frac{v_o}{v_s} = -\frac{R_2}{R_1}$$

This is called an *inverting amplifier* because the voltage gain G is *negative*, meaning it "inverts" its input signal.

(c) Using your toolkit of circuit topologies, design blocks that implement the following equations. Feel free to reference Discussion 11B for circuit topologies:

i. $v_o = 2v_s$

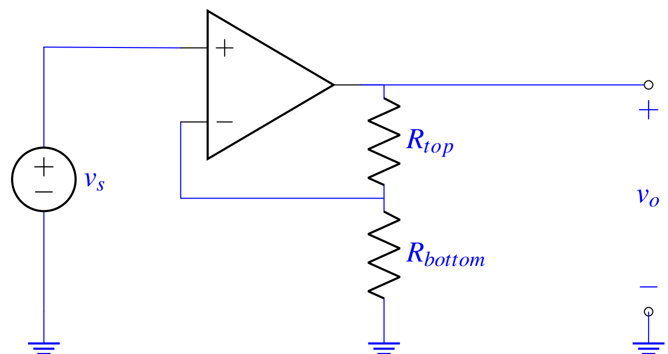
ii. $v_o = -3v_s + 8$

Solution:

i. We can use a non-inverting amplifier with gain 2.

$$\frac{v_o}{v_s} = \left(1 + \frac{R_{top}}{R_{bottom}}\right) = 2$$
$$\frac{R_{top}}{R_{bottom}} = 1$$

Any value for R_{top} and R_{bottom} is correct as long as they have the same resistance.



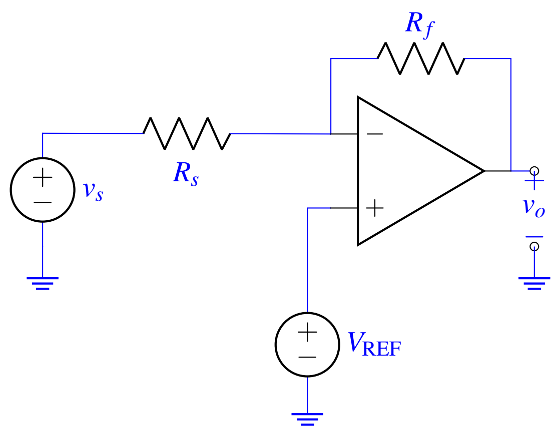
ii. We can use an inverting amplifier with reference voltage source. We need to determine values for R_f, R_s and V_{REF}.

$$v_o = v_s\left(-\frac{R_f}{R_s}\right) + V_{REF}\left(\frac{R_f}{R_s} + 1\right) = -3v_s + 8$$

Matching the coefficients in this equation:

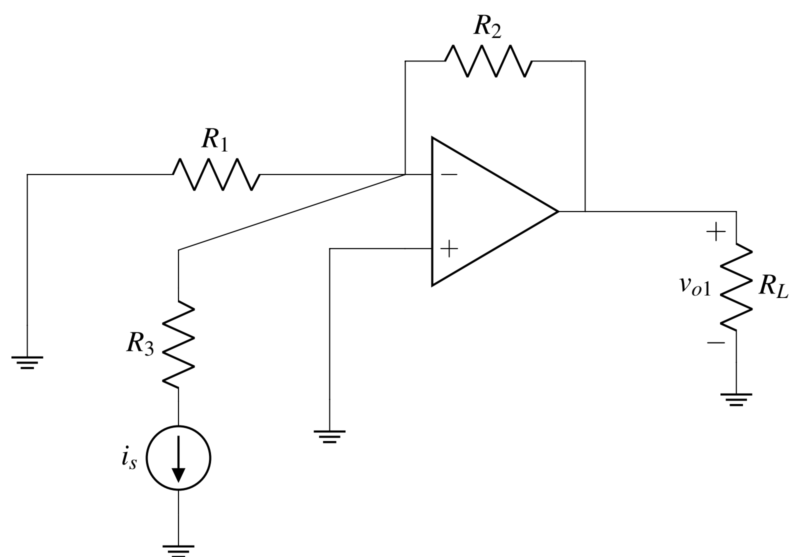
$$\frac{R_f}{R_s} = 3$$
$$V_{REF}\left(\frac{R_f}{R_s} + 1\right) = 8$$
$$V_{REF}(3+1) = 8$$
$$V_{REF} = 2V$$

Any values for R_f and R_s are correct as long as $\frac{R_f}{R_s} = 3$.

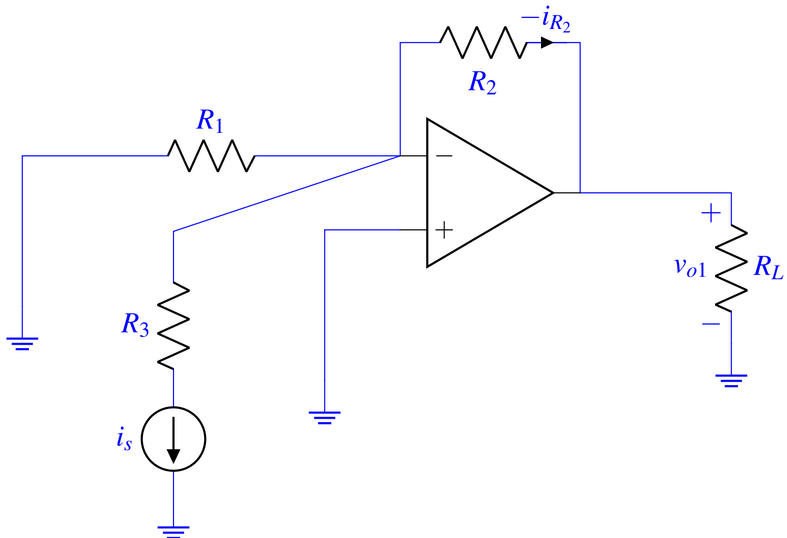


3. Amplifier with Multiple Inputs

(a) Use the Golden Rules to find v_{o1} for the circuit below.



Solution:



Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is 0. The voltage drop across R_1 is 0 and no current flows through it. In addition, no current flows into the op-amp from the negative terminal due to its infinite input resistance (the negative terminal is connected to an "open" circuit).

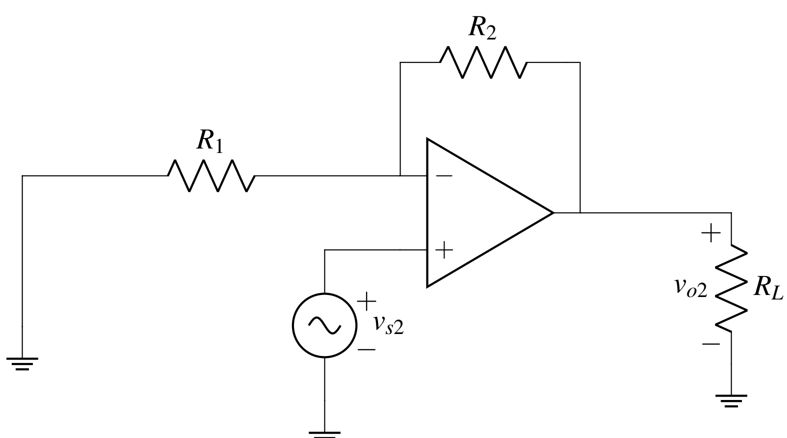
By KCL at the negative terminal of the op-amp, this means that the current going through R_3 and R_2 is i_s. Taking the positive terminal of R_2 to be on the right, the voltage drop across R_2 is v_{o1}. By Ohm's law, we conclude:

$$\frac{v_{o1}}{R_2} = i_s$$

Rearranging we get:

$$v_{o1} = i_s \cdot R_2$$

(b) Use the Golden Rules to find v_{o2} for the circuit below.



Solution:

Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is $u^- = v_{s2}$. In addition, since no current can enter into the negative terminal of the op-amp, R_1 and R_2 are in series. This means that the voltage at the negative terminal of the op-amp can be expressed in terms of v_{o2} using the voltage divider formula:

$$u^- = v_{o2}\left(\frac{R_1}{R_1+R_2}\right)$$

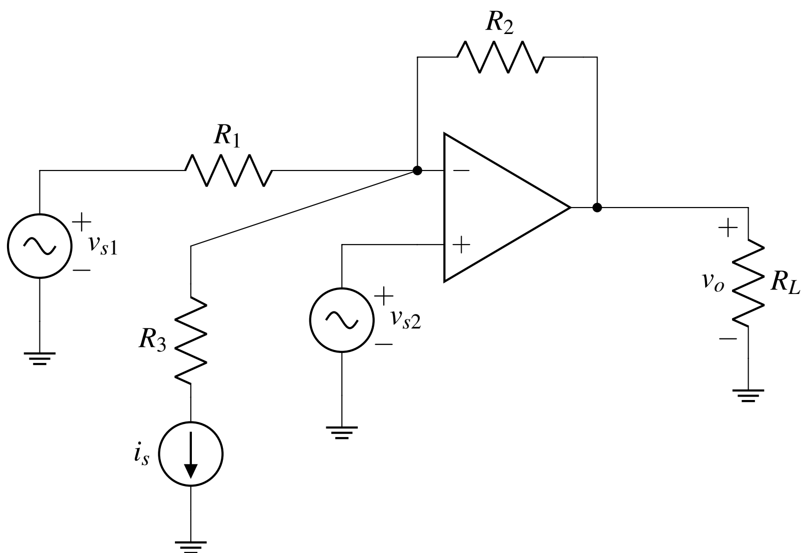
We also know that $u^- = v_{s2}$ and conclude:

$$v_{s2} = v_{o2}\left(\frac{R_1}{R_1+R_2}\right)$$

After rearranging, we have:

$$v_{o2} = v_{s2}\left(\frac{R_2}{R_1}+1\right)$$

(c) Use the Golden Rules to find the output voltage v_o for the circuit shown below.



Solution:

Applying the Golden Rules we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is $u^- = v_{s2}$. Then we write a KCL equation at the node connected to the minus terminal of the op-amp (recalling that no current flows into or out of the op-amp's terminals). All currents are defined as flowing out of the node:

$$i_{R_1} + i_{R_2} + i_{R_3} = 0$$

Because of the independent current source, we know:

$$i_{R_3} = i_s$$

By Ohm's law, we know:

$$i_{R_1} = \frac{u^- - v_{s1}}{R_1}$$

and

$$i_{R_2} = \frac{u^- - v_o}{R_2}$$

Then, substituting back into the original KCL equation, we have:

$$\frac{u^- - v_{s1}}{R_1} + \frac{u^- - v_o}{R_2} + i_s = 0$$

and substituting $u^- = v_{s2}$, we have:

$$\frac{v_{s2} - v_{s1}}{R_1} + \frac{v_{s2} - v_o}{R_2} + i_s = 0$$

which we rearrange to find v_o, giving:

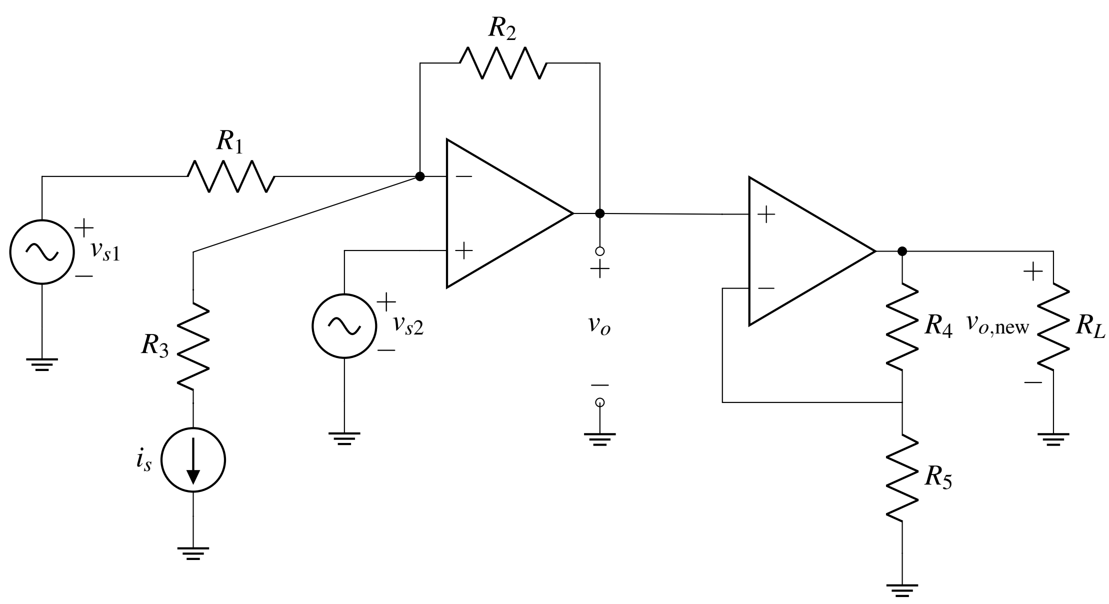
$$v_o = v_{s2}\left(1 + \frac{R_2}{R_1}\right) + i_s \cdot R_2 - \left(\frac{R_2}{R_1}\right) v_{s1}$$

(d) Use superposition and the answers to the first few parts of this problem to verify your answer to part c. *Hint: See if you can generate some combination of the circuits in a & b that is equivalent to the one in c.*

Solution: Using superposition we can analyze the circuit leaving only one source on at a time. If we leave on v_{s1} and turn off v_{s2} and i_s, then we have an inverting amplifier. If we leave on i_s and turn off v_{s1} and v_{s2}, then we have the circuit in (a). If we leave on v_{s2} and turn off v_{s1} and i_s, then we have the circuit in (b). From this we can see that v_o is the sum from the solutions.

$$v_o = -\frac{R_2}{R_1} v_{s1} + i_s R_2 + v_{s2} \frac{R_2 + R_1}{R_1}$$

(e) Now add a second stage as shown below. What is $v_{o,\text{new}}$? Does v_o change between part (c) and this part? Does the voltage $v_{o,\text{new}}$ depend on R_L?



Solution:

Adding the second stage does not change the voltages in the first stage. This is because the circuit connected to the positive and negative terminals of the first stage op-amp "sees" an open circuit/infinite input resistance in the op-amp.

Hence v_o remains unchanged from part (c).

$$v_o = -\left(\frac{R_2}{R_1}\right) v_{s1} + i_s \cdot R_2 + v_{s2}\left(\frac{R_2+R_1}{R_1}\right)$$

By the Golden Rules, the negative terminal of the second op-amp must have the same voltage as the plus terminal, which is v_o. No current can flow into the negative terminal, so R_4 and R_5 are in series and have the same current, so we know:

$$\frac{v_o}{R_5} = \frac{v_{o,new} - v_o}{R_4}$$

Therefore:

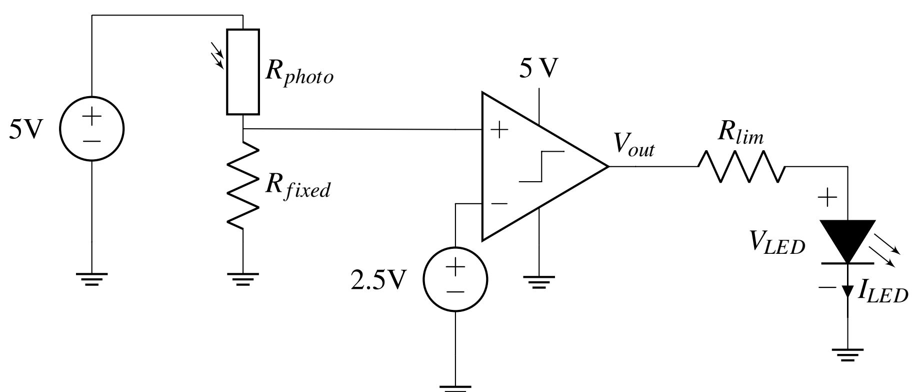
$$v_{o,new} = \left(\frac{R_4+R_5}{R_5}\right) v_o = \frac{R_4+R_5}{R_5}\left(-\frac{R_2}{R_1} \cdot v_{s1} + i_s \cdot R_2 + v_{s2} \cdot \frac{R_2+R_1}{R_1}\right)$$

Note that you could have directly used the non-inverting amplifier gain formula $(1+\frac{R_4}{R_5})$for this extra stage.

The output voltage **does not** depend on the load resistance R_L, since it is set by the dependent voltage source inside the op-amp. Remember that a voltage source will provide any amount of current necessary while maintaining its voltage constant. **That is the beauty of op-amps:** they provide isolation between stages because of the open circuit at the input and they get rid of the loading effect, since they can maintain the output voltage constant regardless of the load value.

4. LED Alarm Circuit

One day, you come back to your dorm to find that your favorite candy has been stolen. Determined to catch the perpetrator red-handed, you decide to put the candy inside a kitchen drawer. Using the following circuit design, you would like to turn on a light-emitting diode (LED) "alarm" if the kitchen drawer is opened.



Note R_{photo} is a photoresistor, which acts like a typical resistor but changes resistance based on the amount of light it is exposed to. This photoresistor is located inside the kitchen drawer, so we can tell when the drawer is opened or closed.

V_{LED} indicates the voltage across the LED; we will guide you through the IV behavior of this element later in the problem. The LED is located in your room (and connected to a long wire going to the kitchen), so that you can remotely tell when the kitchen drawer has been opened.

(a) What is V_+, the voltage at the positive voltage input of the comparator? Your answer should be written in terms of R_{photo} and R_{fixed}.

Solution: V_+ is the output of a voltage divider:

$$V_+ = \frac{R_{fixed}}{R_{fixed} + R_{photo}} \cdot 5\,\text{V}$$

(b) We now want to choose a value for R_{fixed}. From the photoresistor's datasheet, we see the resistance in "light" conditions (i.e. drawer open) is $1\,\text{k}\Omega$. In "dark" conditions (i.e. drawer closed), the resistance is $10\,\text{k}\Omega$.

To ensure the comparator detects the light condition with more tolerance, we decide to design R_{fixed} so that V_+ is $3\,\text{V}$ under the "light" condition. Solve for the value of R_{fixed} to meet this specification.

Solution: We start from the voltage divider equation we derived in the previous part:

$$V_+ = \frac{R_{fixed}}{R_{fixed} + R_{photo}} \cdot 5\,\text{V}$$

Now we plug in the known values, $V_+ = 3\,\text{V}$ and $R_{photo} = 1\,\text{k}\Omega$.

$$3\,\text{V} = \frac{R_{fixed}}{R_{fixed} + 1000\Omega} \cdot 5\,\text{V}$$

Solving this equation, we get $R_{fixed} = 1.5\,\text{k}\Omega$.

(c) Write down V_{out} with any conditions in terms of V_+. For simplicity, consider the case when $V_+ \neq V_-$ and assume the comparator is ideal.

Solution:

Since the comparator is ideal, we know that V_{out} will be the voltage at either the positive rail ($5\,\text{V}$) or at the negative rail ($0\,\text{V}$) when $V_+ \neq V_-$. Which voltage depends on if V_+ is greater than V_- or not. Since V_- is $2.5\,\text{V}$, we get the following piecewise equation for V_{out}:

$$V_{out} = \begin{cases} 5\,\text{V}, & V_+ > 2.5\,\text{V} \\ 0\,\text{V}, & V_+ < 2.5\,\text{V} \end{cases}$$

(d) Using your answers to the previous parts, write down V_{out} with the conditions on its output **in terms of** R_{photo}. You can substitute the value of R_{fixed} you found in part (b). As before, you can assume that $V_+ \neq V_-$ and the comparator is ideal.

Solution:

We substitute the equations for V_+ into the equation for V_{out}:

$$V_{out} = \begin{cases} 5\,\text{V}, & \frac{R_{fixed}}{R_{fixed}+R_{photo}} \cdot 5\,\text{V} > 2.5\,\text{V} \\ 0\,\text{V}, & \frac{R_{fixed}}{R_{fixed}+R_{photo}} \cdot 5\,\text{V} < 2.5\,\text{V} \end{cases}$$

Plugging in $R_{fixed} = 1.5\,\text{k}\Omega$ from part (b), we can get the following in terms of R_{photo}:

$$V_{out} = \begin{cases} 5\,\text{V}, & R_{photo} < 1.5\,\text{k}\Omega \\ 0\,\text{V}, & R_{photo} > 1.5\,\text{k}\Omega \end{cases}$$

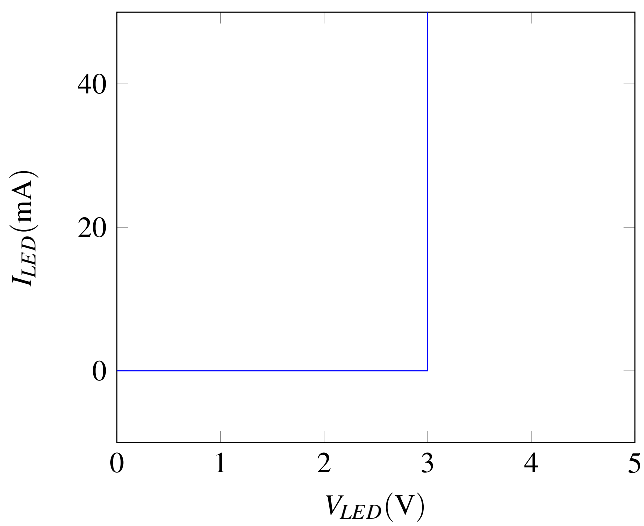
(e) From the design steps in the previous parts, we have designed a circuit that outputs non-zero voltage when the photoresistor is exposed to light (i.e. kitchen drawer open). We now want to design the LED portion of the circuit, so we get a visual alarm when the drawer is open.

From the LED's datasheet, the forward voltage, V_F is $3\,\text{V}$. Essentially, if V_{LED} is less than this voltage, the LED won't light up and I_{LED} will be $0\,\text{A}$.

Here is an idealized IV curve of this LED. The LED behaves in one of the following two modes:

i. If the voltage across the LED is less than $V_F = 3\,\text{V}$ or if $I_{LED} < 0\,\text{A}$, then the LED acts like an open circuit.

ii. If the voltage across the LED is $V_F = 3\,\text{V}$, then the LED acts like a voltage source, except that it only allows positive current flow (i.e. only in the direction of current marked on the circuit diagram).



To avoid exceeding the power rating of the LED (and having it burn out), the recommended value for I_{LED} is $20\,\text{mA}$.

Find the value of the current-limiting resistor, R_{lim}, such that when the photoresistor is in the "light" condition, $I_{LED} = 20\,\text{mA}$.

Solution: When the photoresistor is in the "light" condition, $R_{photo} = 1\,\text{k}\Omega$, and based on our analysis in the previous part, $V_{out} = 5\,\text{V}$. This implies that $V_{LED} = V_F$ and the LED acts like a power supply with positive current flow when in the "light" condition.

Using Ohm's Law and noting that the same current passes through R_{lim} and the LED itself,

$$V_{out} - V_F = I_{LED} R_{lim}$$

Rearranging and plugging in values when in the "light" condition:

$$R_{lim} = \frac{V_{out} - V_F}{I_{LED}}$$

$$R_{lim} = \frac{5 - 3\text{ V}}{0.02\text{ A}}$$

$$R_{lim} = 100\,\Omega$$

Note that when $V_{out} < 3\,\text{V}$, the LED will not light up and I_{LED} will be $0\,\text{mA}$. Thus by our design of the voltage divider, we were able to ensure the LED lights up only if the drawer is opened.

5. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

Solution:

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group.

XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.

Then I went to homework party for a few hours, where I finished the homework.