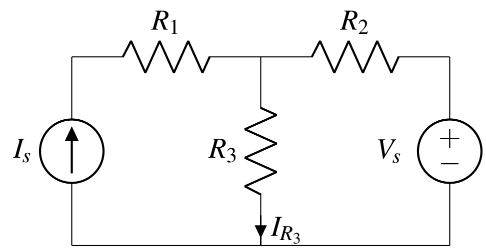
EECS 16A Designing Information Devices and Systems I
Fall 2022 Discussion 12A

1. Superposition

Consider the following circuit:



(a) With the current source turned on and the voltage source turned off, find the current I_{R_3}.

Answer:

We note that I_s is split between R_2 and R_3, and therefore we can use the current divider relationship:

$$I_{R_3}\big|_{V_s=0} = \frac{I_s R_2}{R_2 + R_3}$$

(b) With the voltage source turned on and the current source turned off, find the voltage drop V_{R_3} across R_3. **Answer:**

We note that when the current source is turned off, it becomes an open circuit. Thus, we are left with a voltage divider.

$$V_{R_3}\big|_{I_s=0} = \frac{V_s R_3}{R_2 + R_3}$$

(c) Find the power dissipated by R_3.

Answer:

We first find the missing quantities. The voltage drop across R_3 when only the current source is on and the current through R_3 when only the voltage source is on is given by:

$$V_{R_3}\big|_{V_s=0} = I_{R_3}\big|_{V_s=0} R_3 = \frac{I_s R_2 R_3}{R_2 + R_3}$$

$$I_{R_3}\big|_{I_s=0} = \frac{V_{R_3}\big|_{I_s=0}}{R_3} = \frac{V_s}{R_2 + R_3}$$

Thus, we have:

$$V_{R_3} = V_{R_3}\big|_{I_s=0} + V_{R_3}\big|_{V_s=0} = \frac{V_s R_3 + I_s R_2 R_3}{R_2 + R_3}$$

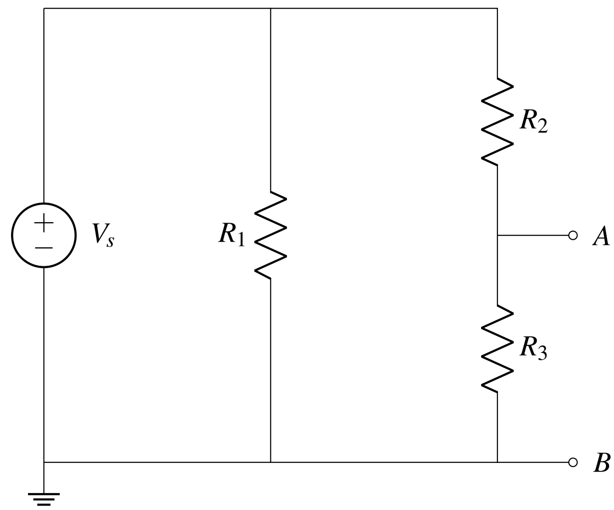
$$I_{R_3} = I_{R_3}\big|_{I_s=0} + I_{R_3}\big|_{V_s=0} = \frac{I_s R_2 + V_s}{R_2 + R_3}$$

Thus, the power dissipated is:

$$P_{R_3} = I_{R_3} V_{R_3} = \frac{R_3 (I_s R_2 + V_s)^2}{(R_2 + R_3)^2}$$

2. Thévenin/Norton Equivalence

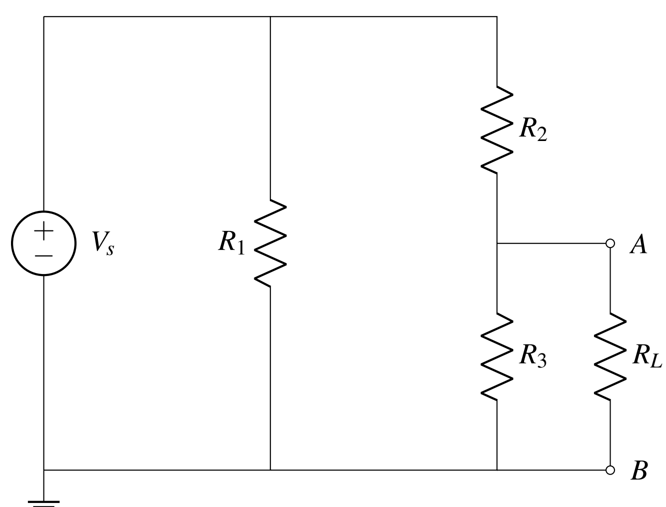
(a) Find the Thévenin resistance R_{th} of the circuit shown below, with respect to its terminals A and B.



Answer: To find the Thévenin resistance, we null out the voltage source (which shorts out R_1) and find the equivalent resistance, which is simply:

$$R_{th} = R_2 \parallel R_3$$

(b) Now, a load resistor, $R_L = R$, is connected across terminals A and B, as shown in the circuit below. Find the power dissipated in the load resistor in terms of the given variables.

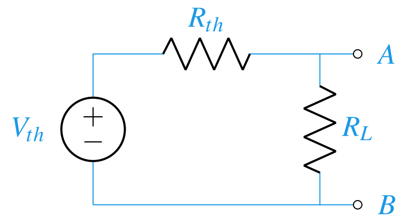


Answer:

To help simplify the analysis, we replace the circuit with its Thévenin equivalent circuit. In order to do so, we first need to find the Thévenin voltage. That is the open circuit voltage, V_{AB}, in the original circuit, which is simply a voltage divider:

$$V_{th} = V_s \frac{R_3}{R_2 + R_3}$$

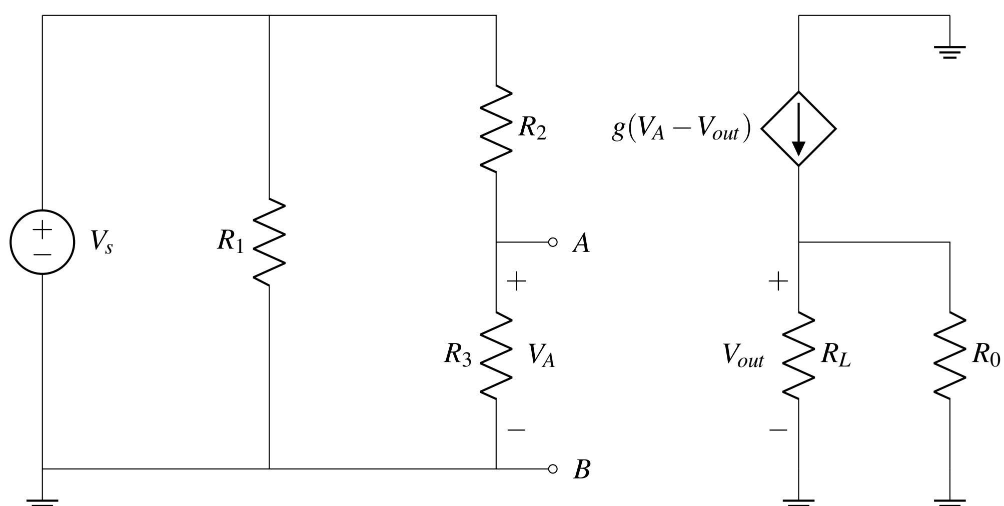
Thus, the circuit can be simplified to:



The power through the load resistor is given by:

$$P_{R_L} = I_{R_L}^2 * R_L = (\frac{V_{th}}{R_L + R_{th}})^2 R_L = (V_s \frac{R_3}{R_2 + R_3} \cdot \frac{1}{R_L + R_{th}})^2 R_L$$

(c) We modify the circuit as shown below, where g is a known constant:



Find a symbolic expression for V_{out} as a function of V_s.
Hint: Redraw the left part of the circuit using its Thévenin equivalent.

Answer:

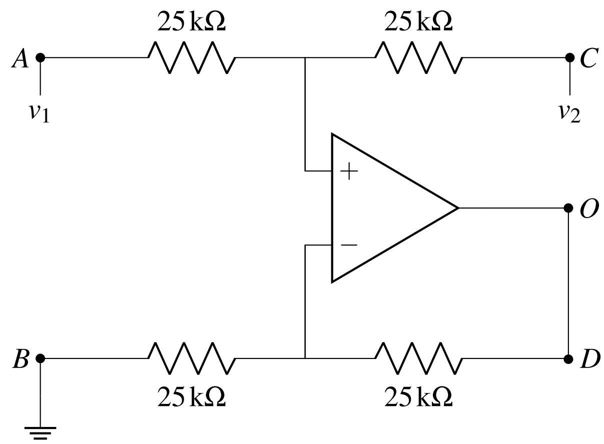
We note that V_{AB} simply equals $V_{th} = \frac{R_3}{R_2+R_3} V_s$. Then, noting that R_0 and R_L are in parallel, we have that $V_{out} = g(V_{th} - V_{out})(R_0 \parallel R_L)$. Solving for V_{out}, we get:

$$V_{out} = \frac{R_2}{R_2 + R_3} V_s \frac{gR_L \parallel R_0}{1 + gR_L \parallel R_0}$$

3. A Versatile Opamp Circuit

For each subpart, determine the voltage at O, given that v_1 and v_2 are voltage sources.

(a) Configuration 1:



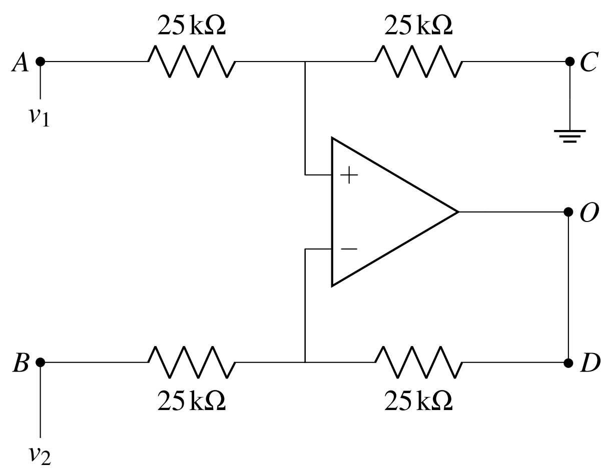
Answer:
By superposition, we note that the voltage at v^+ is given by:

$$v^+ = \frac{v_1 + v_2}{2}$$

The rest of the circuit looks like a non-inverting amplifier with a gain of $1 + \frac{25\,\mathrm{k}\Omega}{25\,\mathrm{k}\Omega} = 2$. Therefore, the output voltage is:

$$v_O = 2v^+ = v_1 + v_2$$

(b) Configuration 2:



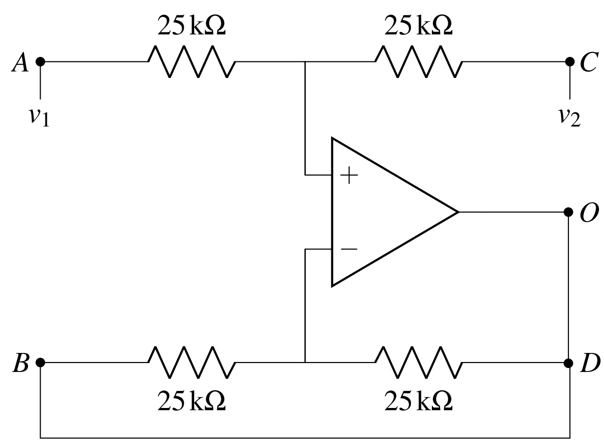
Answer:
Through the voltage divider equation, we note that the voltages at the input terminals of the op-amp

are given by:

$$v^+ = \frac{v_1}{2}$$
$$v^- - v_2 = \frac{25\,\text{k}\Omega}{25\,\text{k}\Omega + 25\,\text{k}\Omega} * v_O$$
$$v^- = \frac{v_2 + v_O}{2}$$

By the Golden Rules, these must be equal to each other since the op-amp is in negative feedback ($v^+ = v^-$). Therefore, $v_O = v_1 - v_2$.

(c) Configuration 3:



Answer:

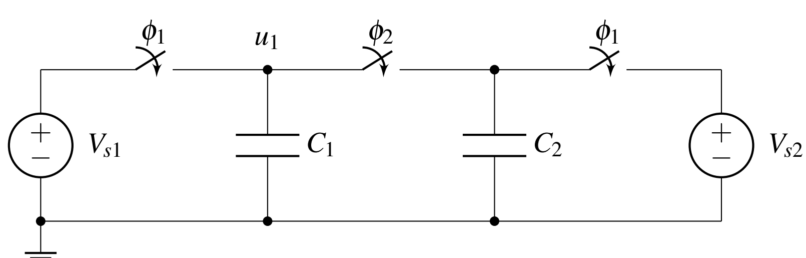
Like in part (i), by superposition, we note that the voltage at v^+ is given by:

$$v^+ = \frac{v_1 + v_2}{2}$$

We note that the resistors connected to B and D do not affect the circuit as no current is flowing through them. Therefore, $v_O = v^- = v^+ = \frac{v_1+v_2}{2}$.

4. Capacitive Charge Sharing (from Spring 2020 Midterm 2)

Consider the circuit below with $C_1 = C_2 = 1\,\mu\text{F}$ and three switches ϕ_1, ϕ_2. Suppose that initially the switches ϕ_1 are closed and ϕ_2 is open, such that C_1 and C_2 are charged through the corresponding voltage sources $V_{s1} = 1\,\text{V}$ and $V_{s2} = 2\,\text{V}$.



(a) How much charge is on C_1 and C_2?

$$q_1 = C_1 V_1 = C_1 V_{s1} = 1\,\mu\text{C}$$
$$q_2 = C_2 V_2 = C_2 V_{s2} = 2\,\mu\text{C}$$

(b) Now suppose that some time later, switch ϕ_1 opens and switch ϕ_2 closes. What is the value of the voltage u_1 at steady state?

Answer: The total charge on node u_1 will be conserved after switch ϕ_1 is opened. That charge is $Q_{tot} = q_1 + q_2$. Also, note that during phase 2, the capacitors are connected in parallel so they will both have $V_{C1} = V_{C2} = u_1$.

$$Q_{tot} = C_1 u_1 + C_2 u_1$$
$$u_1 = \frac{q_1 + q_2}{C_1 + C_2} = 1.5\,\text{V}$$

