EECS 16A Designing Information Devices and Systems I
Fall 2022 Discussion 9B

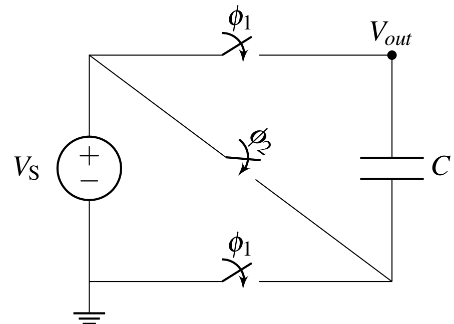
Mid Semester Survey

Please fill out the mid semester survey: https://tinyurl.com/midsemester16a

We highly appreciate your feedback!

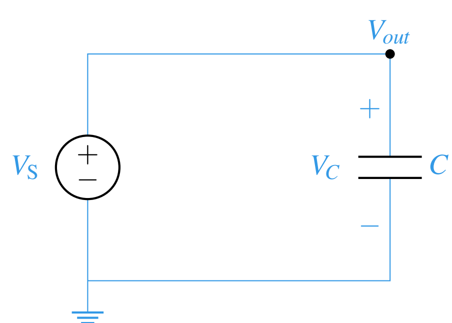
1. Voltage Booster

We have made extensive use of resistive voltage dividers to reduce voltage. What about a circuit that boosts voltage to a value greater than the supply $V_S = 5\,\mathrm{V}$? We can do this with capacitors!



(a) In the circuit above switches ϕ_1 are initially closed and switch ϕ_2 is initially open. Calculate the value of the output voltage, V_{out} with respect to ground, and the amount of charge stored on capacitor, C, at that state (phase 1).

In this setting we have the following equivalent circuit:

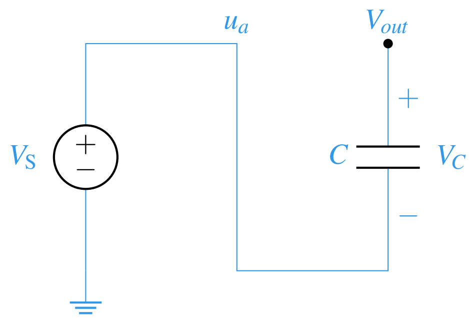


Hence,

$$V_{S=V_C=V_{out}}, \quad Q = CV_C = CV_S.$$

(b) Now, after the capacitors are charged, switches ϕ_1 are opened and switch ϕ_2 is closed. Calculate the new voltage output voltage, V_{out}, at steady state (phase 2).

Phase 2 equivalent ckt:



In phase 2 notice that the voltage source is connected to the *negative* plate of capacitor C, while the positive plate is left floating (since it is open). Hence, charge is going to be conserved on the top plate of C. However, in phase 2:

$$V_S = u_a - 0$$
$$V_C^{\phi_2} = V_{out} - u_a$$

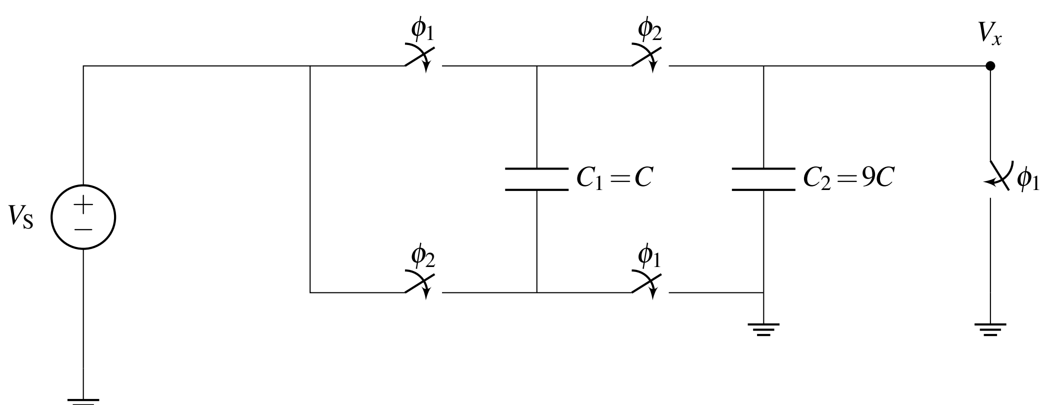
Therefore,

$$V_C^{\phi_2} = V_{out} - V_S$$
$$Q_C^{\phi_1} = Q_C^{\phi_2} \Rightarrow CV_S = C \cdot (V_{out} - V_S) \Rightarrow V_{out} = 2V_S = 10\,\mathrm{V}$$

We have created a voltage doubler!

2. Charge Sharing

Consider the following circuit:



In the first phase, all of the switches labeled ϕ_1 will be closed and all switches labeled ϕ_2 will be open. In the second phase, all switches labeled ϕ_1 are opened and all switches labeled ϕ_2 are closed.

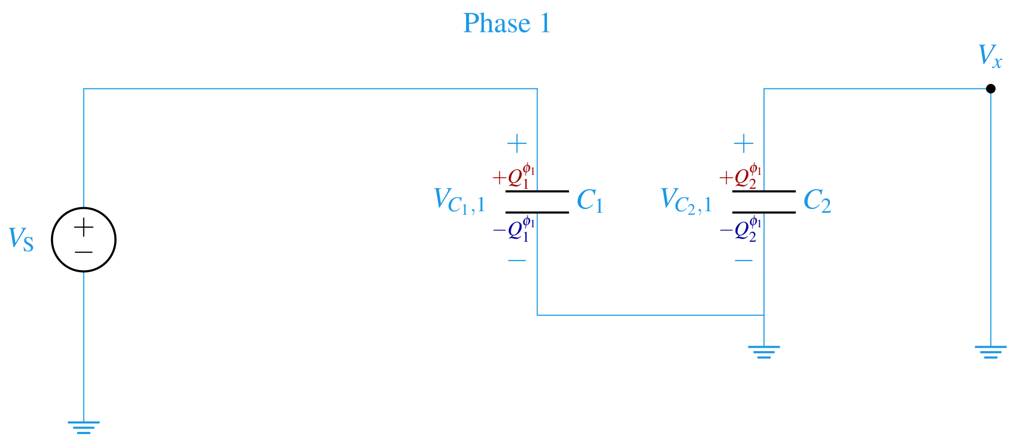
(a) Draw the polarity of the voltage (using + and − signs) across the two capacitors C_1 and C_2. It doesn't matter which terminal you label + or −; just remember to keep these consistent through phase 1 and 2! Also, label the charge on at each plate: $+Q_{C_1}$, $-Q_{C_1}$, $+Q_{C_2}$, and $-Q_{C_2}$.

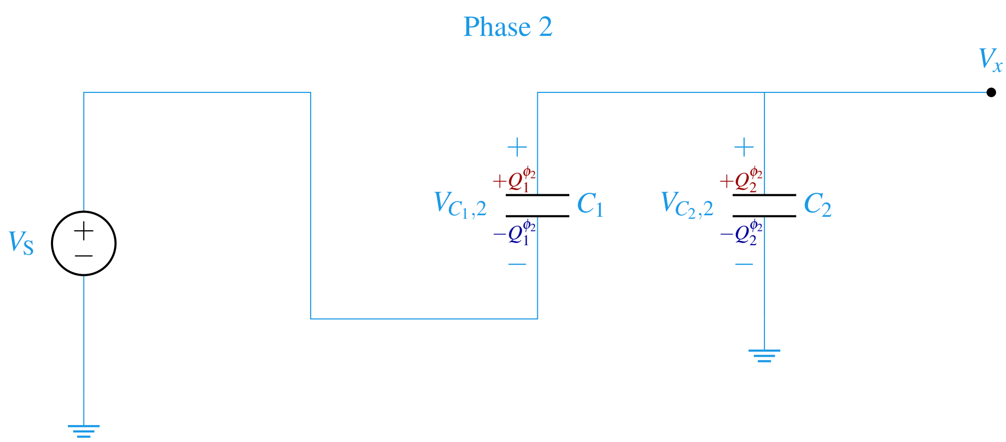
Answer:

One way of marking the polarities is $+$ on the top plate and $-$ on the bottom plate of both C_1 and C_2, and label the charges accordingly ($+Q_{C_1}$ and $-Q_{C_1}$ on the $+$ and $-$ plate of C_1, respectively; and $+Q_{C_2}$ and $-Q_{C_2}$ on the $+$ and $-$ plate of C_2, respectively). Let's call the voltage drop across capacitor C_1, V_{C_1} and across capacitor C_2, V_{C_2}.

(b) Draw the circuit in the first phase and in the second phase. Keep your polarity from part (a) in mind.

Answer:





In phase 1, all the switches marked as ϕ_1 are closed and switches marked as ϕ_2 are open. In phase 2, all the switches marked as ϕ_2 are closed and switches marked as ϕ_1 are open. Draw both the circuits separately, side by side, with the switches in their respective positions.

(c) Find the voltages and charges on C_1 and C_2 in phase 1. Be sure to keep the polarities of the voltages the same!

Answer:

In phase 1,

$$V_{C_1}^{\phi_1} = V_S - 0 = V_S$$

and

$$V_{C_2}^{\phi_1} = 0 - 0 = 0$$

Answer:

Next, we find the charge on each capacitor:

$$Q_{C_1}^{\phi_1} = V_{C_1}^{\phi_1} C_1 = V_S C_1 = V_S C$$

Note that the positive plate has a charge of $+CV_S$, while the negative plate has a charge of $-CV_S$.

$$Q_{C_2}^{\phi_1} = V_{C_2}^{\phi_1} C_2 = 0$$

(d) Now, in the second phase, find the voltage V_x.

Answer:

Where is charge conserved? To answer this, look at the top plates of C_1 and C_2. In phase 2, they are both "floating" because they are not connected to V_S or ground. And in phase 1, they are not connected to each other, but in phase 2, they are connected by the switch. Therefore, in phase 2, the charges on the top plates of C_1 and C_2 will be *shared*, or distributed, because they simply cannot go anywhere else. The total charge will remain the same as in phase 1. Let's find the voltages across C_1 and C_2 in phase 2 (same polarities as in phase 1!):

$$V_{C_1}^{\phi_2} = V_x - V_S$$

and

$$V_{C_2}^{\phi_2} = V_x$$

Now, let's find the charge stored in top plates of C_1 and C_2:

$$Q_{C_1}^{\phi_2} = C \cdot (V_x - V_S)$$

and

$$Q_{C_2}^{\phi_2} = 9CV_x$$

Next, let's write the equation for charge conservation:

$$+Q_{C_1}^{\phi_1} + Q_{C_2}^{\phi_1} = +Q_{C_1}^{\phi_2} + Q_{C_2}^{\phi_2},$$

giving

$$CV_S + 0 = C \cdot (V_x - V_S) + 9CV_x,$$

which results in

$$V_x = \frac{V_S}{5}.$$

(e) **Practice Problem:** If the capacitor C_2 did not exist (i.e. had a capacitance of $0\,\mathrm{F}$), what would the voltage V_x be?

Answer:

We could always go back to the equations above, plug in $C_2 = 0$, and derive $V_x = 2V_S$. It might be worthwhile to go over what this means for the circuit, though. If $C_2 = 0\,\mathrm{F}$, the capacitor is actually an open circuit. (*Why?*) So we can pretend, as the question says, that C_2 does not exist. In phase 1, as before, C_1 has a voltage drop of V_S across it (from top to bottom) and is charged up to CV_S. Now, in phase 2, the top plate of C_1 is left dangling (floating). This means that the charge on the top plate of C_1 is going to be the same just like the charge on the bottom plate. We will therefore get

$$V_x = V_S - (-V_S) = 2V_S$$