EECS 16A Designing Information Devices and Systems I
Fall 2022 Discussion 8B

1. Superposition

For the following circuits, use the superposition theorem to solve for the voltages across the resistor(s).

Solution/Answer:

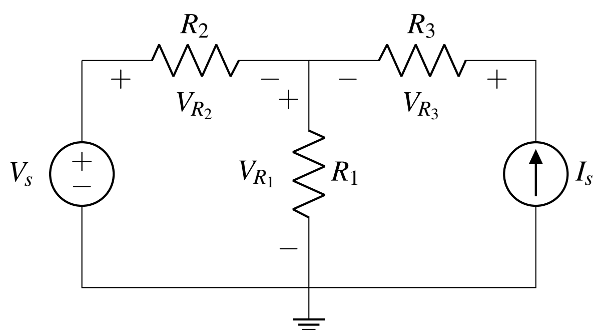
For each circuit;

i. Redraw the circuits with just one source enabled (while disabling/zero-ing all other sources) and solve for circuit voltages and currents.

ii. Repeat for every independent source.

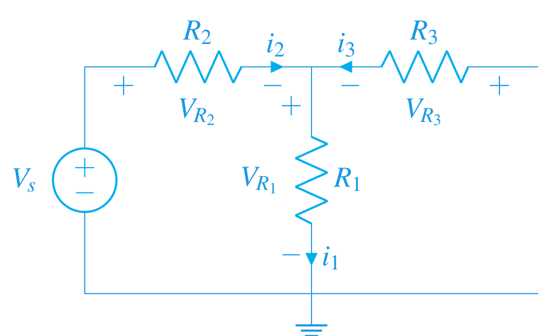
iii. Finally, linearly sum the circuit voltages and currents.

(a)



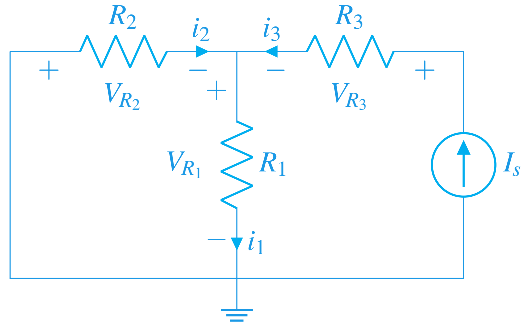
Answer:

Enable/turn-on only the voltage source, V_s, and disable/zero the current source ($I_s = 0$ A so is equivalent to an open circuit). Because of the equivalent open circuit, no current flows through resistor R_3 and it can be neglected. What is left is a classic resistive voltage divider.



$$V_{R_1} = \frac{R_1}{R_1+R_2}V_s$$
$$V_{R_2} = \frac{R_2}{R_1+R_2}V_s$$
$$V_{R_3} = 0$$

Next, enable only the current source, I_s, and disable/zero the voltage source ($V_s = 0\,\mathrm{V}$ so is equivalent to a short circuit).



Intuitively, the current through R_3 is I_s and then splits between R_1 and R_2.
From element I-V characteristics (e.g., Ohm's Law), we know that $V_{R_1} = i_1R_1$, $V_{R_2} = i_2R_2$, and $I_s = i_3$.
We also know from KCL that $i_2 + i_3 = i_1$, and from KVL that $V_{R_1} = -V_{R_2}$.
If we solve this system, for V_{R_1} we find

$$\begin{aligned}
i_3 &= i_1 - i_2\\
I_s &= \frac{V_{R_1}}{R_1} - \frac{V_{R_2}}{R_2}\\
I_s &= \frac{V_{R_1}}{R_1} + \frac{V_{R_1}}{R_2}\\
I_s &= V_{R_1}\cdot\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\\
\implies V_{R_1} &= \frac{R_1R_2}{R_1+R_2}I_s
\end{aligned}$$

for V_{R_2} we find

$$V_{R_2} = -V_{R_1} = -\frac{R_1R_2}{R_1+R_2}I_s$$

and for V_{R_3}

$$V_{R_3} = i_3R_3 = I_sR_3$$

The unknown branch currents i_1 and i_2 can also be quickly found. They could also have been derived from recognizing the circuit as a current divider between R_1 and R_2.

$$i_1 = \frac{R_2}{R_1+R_2}I_s$$
$$i_2 = -\frac{R_1}{R_1+R_2}I_s$$

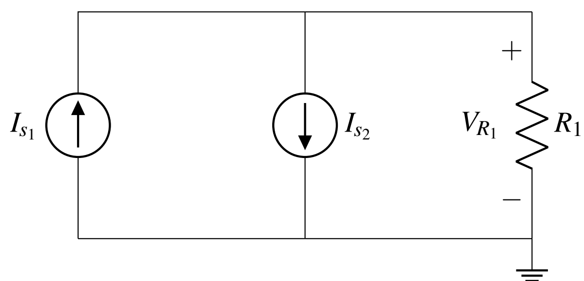
Finally, using superposition we can sum up the contributions from both V_s and I_s to find the resistor voltages of the complete circuit

$$V_{R_1} = V_{R_1}\Big|_{I_s=0} + V_{R_1}\Big|_{V_s=0} = \frac{R_1}{R_1+R_2}V_s + \frac{R_1R_2}{R_1+R_2}I_s$$

$$V_{R_2} = V_{R_2}\Big|_{I_s=0} + V_{R_2}\Big|_{V_s=0} = \frac{R_2}{R_1+R_2}V_s - \frac{R_1R_2}{R_1+R_2}I_s$$

$$V_{R_3} = V_{R_3}\Big|_{I_s=0} + V_{R_3}\Big|_{V_s=0} = I_sR_3$$

(b)



Answer:

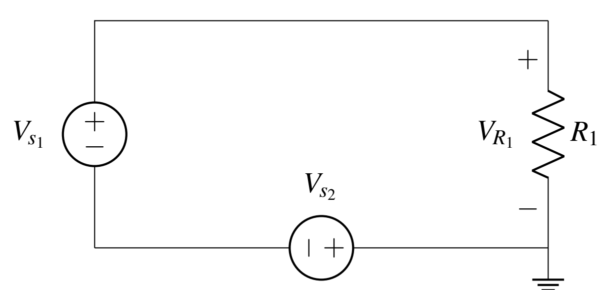
i. *Using superposition:*
Enabling I_{s_1} (and disabling I_{s_2}) gives $V_{R_1} = I_{s_1}R_1$. Enabling I_{s_2} (and disabling I_{s_1}) gives $V_{R_1} = -I_{s_2}R_1$. Finally, the total V_{R_1} is the sum of the individual V_{R_1}'s or

$$V_{R_1} = (I_{s_1} - I_{s_2})R_1$$

ii. *Without superposition:*
Let's approach this holistically. We're looking for the voltage across the resistor, which could be found quickly using Ohm's law if we knew the current. If we were to apply KCL at the node at the top of the circuit, one source is coming in, the other source is leaving, and the current through the resistor is leaving. From KCL, we then know $i_{R_1} = I_{s_1} - I_{s_2}$. Applying Ohm's Law we find:

$$V_{R_1} = (I_{s_1} - I_{s_2})R_1$$

(c) **(PRACTICE)**



Answer:

i. *Using superposition:*
Enabling V_{s_1} (and disabling V_{s_2}) gives $V_{R_1} = V_{s_1}$. Enabling V_{s_2} (and disabling V_{s_1}) gives $V_{R_1} = -V_{s_2}$. Finally, the total V_{R_1} is the sum of the individual V_{R_1}'s or

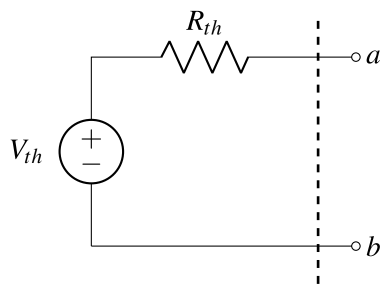
$$V_{R_1} = V_{s_1} - V_{s_2}$$

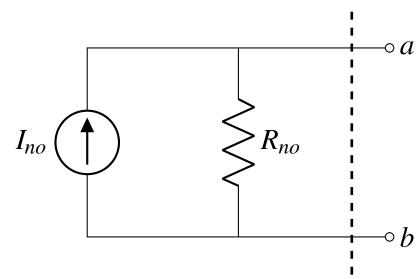
ii. *Without superposition:*
Notice the circuit only has one loop, so use KVL to find the voltage across the resistor.

$$V_{R_1} = V_{s_1} - V_{s_2}$$

2. Thévenin and Norton Equivalence

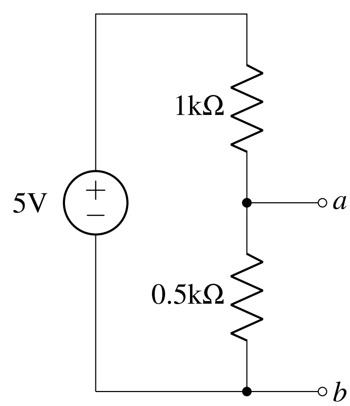
The general Thévenin and Norton equivalent circuits are shown below:





Find the Thévenin and Norton equivalents across terminals a and b for the circuits given below.

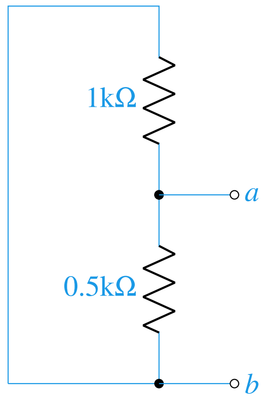
(a)



Answer:
To find V_{th}, simply find the open circuit voltage across terminals ab. Here, that voltage is the voltage across the $0.5\,\text{k}\Omega$ resistor given by the voltage divider equation.

$$V_{th} = 5\text{V}\frac{0.5\,\text{k}\Omega}{1\,\text{k}\Omega + 0.5\,\text{k}\Omega} = 1.67\text{V}$$

To find R_{th}, disable/zero out any independent sources (5V voltage source becomes a short circuit).



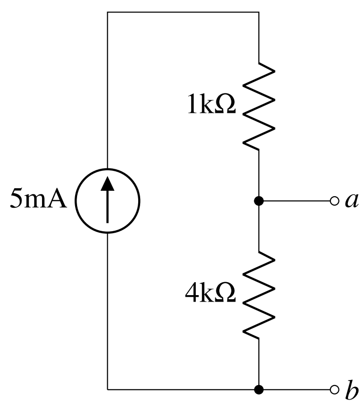
Then, from the open circuit terminals ab, find the equivalent resistance of the rest of the circuit. This is just two resistors in parallel. Elements or groups of elements are in parallel if their terminals share the same two nodes (i.e., have same voltage across them).

$$R_{th} = 1\,\text{k}\Omega \parallel 0.5\,\text{k}\Omega = \frac{1\,\text{k}\Omega \cdot 0.5\,\text{k}\Omega}{1\,\text{k}\Omega + 0.5\,\text{k}\Omega} = 333\,\Omega$$

Note that $R_{th} = R_{no}$ always. Now to find I_{no},

$$I_{no} = \frac{V_{th}}{R_{th}} = \frac{1.67\text{V}}{333\,\Omega} = 5\,\text{mA}$$

(b)

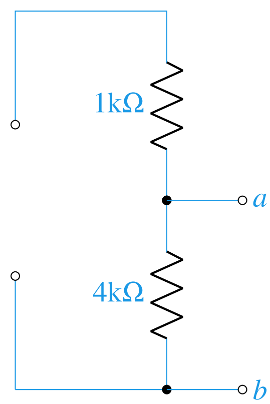


Answer:

The open circuit voltage across terminals ab is given by Ohm's law:

$$V_{th} = 5\,\text{mA} \cdot 4\,\text{k}\Omega = 20\,\text{V}$$

To find R_{th}, disable/zero out any independent sources (5A current source becomes an open circuit).



Then, from the open circuit terminals ab, find the equivalent resistance of the rest of the circuit. This is just a single resistor.

$$R_{th} = 4\,\mathrm{k}\Omega = R_{no}$$

Now to find I_{no},

$$I_{no} = \frac{V_{th}}{R_{th}} = \frac{20\,\mathrm{V}}{4\,\mathrm{k}\Omega} = 5\,\mathrm{mA}$$