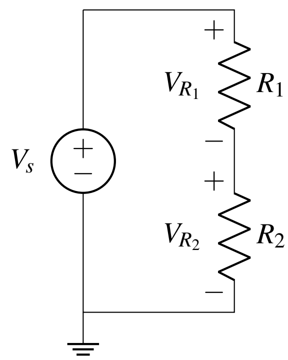
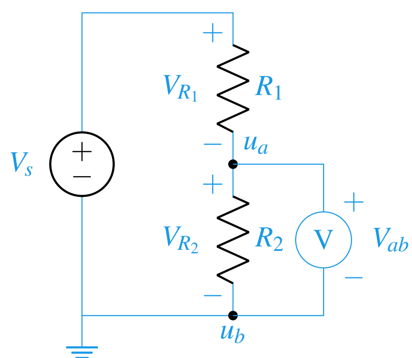
EECS 16A Designing Information Devices and Systems I
Fall 2022 Discussion 7B

1. Volt and Ammeter

(a) For the voltage divider below, how would we connect a voltmeter to the circuit to measure the voltage V_{R_2}?

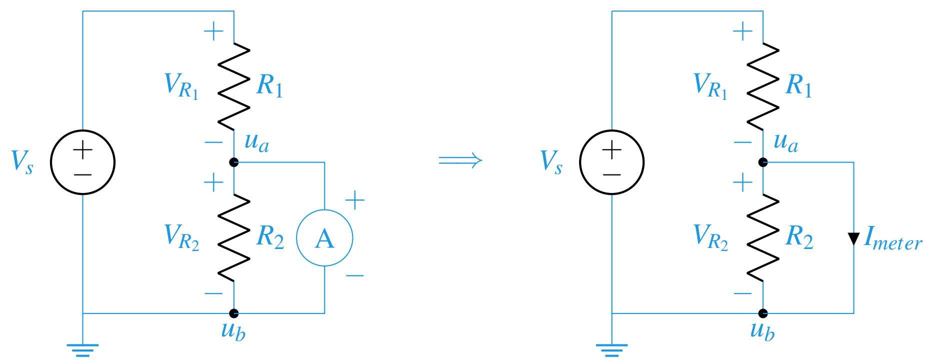


Answer: We connect our voltmeter to the voltage divider such that the voltage across the voltmeter nodes is equal to the voltage we want to measure.
i.e. $V_{ab} = u_a - u_b = V_{R_2}$



(b) What would happen if we accidentally connected an ammeter in the same configuration instead? Assume our ammeter is ideal.

Answer: An ideal ammeter behaves like a wire or a short between two nodes as depicted below:

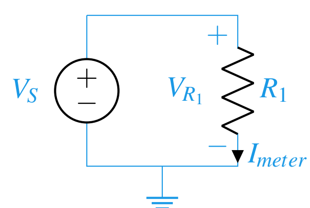


When we have a short, or a new lower resistance path for the current to travel, current will choose to take the path of least resistance. In this case, the ideal wire has no resistance so all of the current leaving R_1 will flow through the wire instead of R_2.

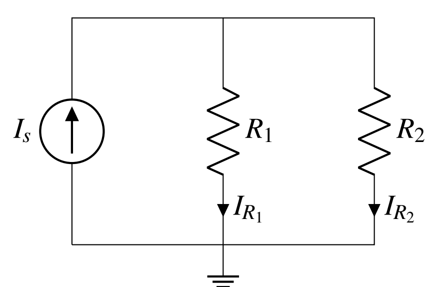
Mathematically, we can see this because this wire has now combined the nodes u_a and u_b into one node. In other words, $u_a = u_b$. We can use this to show that no current is flowing through R_2.

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{u_a - u_b}{R_2} = \frac{0}{R_2} = 0$$

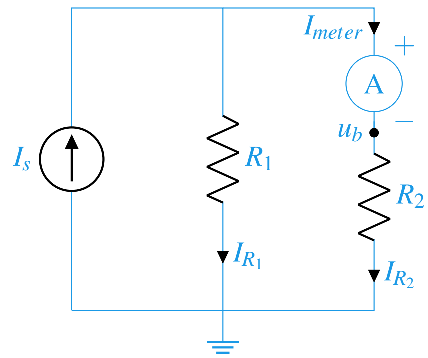
Therefore, an equivalent circuit can be drawn as shown below:



(c) For the current divider below, how would we connect an ammeter to the circuit to measure the current I_{R_2}?

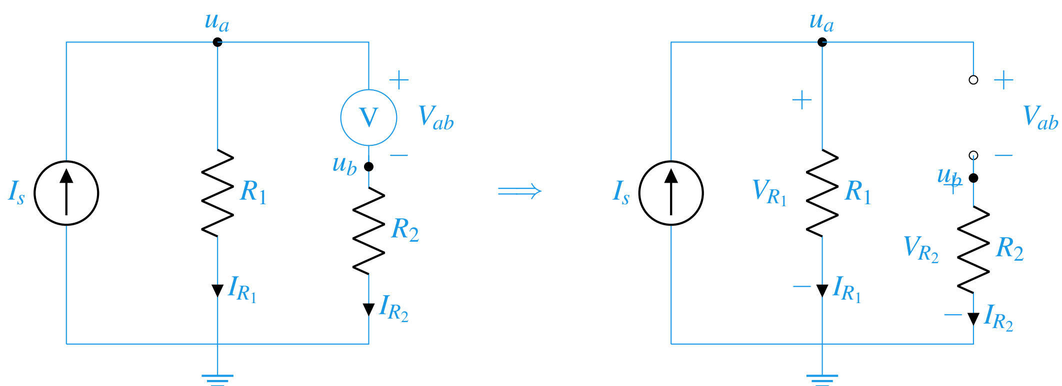


Answer: We connect our ammeter to the current divider such that the current going through the ammeter is equal to the current we want to measure. By doing a KCL at node u_b we can see that $I_{meter} = I_{R_2}$.



(d) What would happen if we accidentally connected a voltmeter in that configuration instead? Assume the voltmeter is ideal.

Answer: An ideal voltmeter behaves like an open circuit as depicted below:



The open circuit creates a dead end and prevents any current from flowing through therefore $I_{R_2} = 0$. As a result, we will notice that the voltage at node u_b is now equal to the reference node voltage.

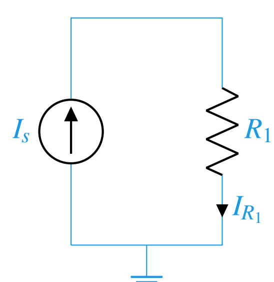
$$V_{R_2} = I_{R_2} R_2 = 0 = u_b - 0$$

$$u_b = 0$$

With this knowledge, we can conclude that the voltmeter will actually read the voltage across the resistor R_1.

$$V_{ab} = u_a - u_b = u_a - 0 = V_{R_1}$$

Since $I_{R_2} = 0$ the resistor R_2 has no effect on our circuit and an equivalent circuit can be drawn below:



2. Cell Phone Battery

As great as smartphones are, one of their drawbacks is that their batteries don't last a long time. For example, a typical smartphone, under average usage conditions (internet, a few cat videos, etc.) uses 0.3W. We will model the battery as an ideal voltage source (which maintains a constant voltage across its terminals regardless of current) except that we assume that the voltage drops abruptly to zero when the battery is discharged (in reality, the voltage drops gradually, but let's keep things simple).

Battery capacity is specified in mAh, which indicates how many mA of current the battery can supply for one hour before it needs to be recharged. Suppose the phone's battery has a capacity of 2770mAh at 3.8V. For example, this battery could provide 1000mA (or 3.8W) for 2.77 hours before the voltage abruptly drops from 3.8V to zero.

(a) How long will the phone's full battery last under average usage conditions?

Answer:

Using out power relation $P = IV$ we see that $300\,\text{mW}$ of power at $3.8\,\text{V}$ is about $79\,\text{mA}$ of current. Our $2770\,\text{mA h}$ battery can supply $79\,\text{mA}$ for $\frac{2770\,\text{mA h}}{79\,\text{mA}} = 35\,\text{h}$, or about a day and a half.

(b) How many coulombs of charge does the battery contain? How many usable electrons worth of charge are contained in the battery when it is fully charged? (An electron has $1.602 \times 10^{-19}\,\text{C}$ of charge.)

Answer: Recall that $1\,\text{C} = 1\,\text{A} \times 1\,\text{s}$, which implies that $1\,\text{mC} = 1\,\text{mAs}$. One hour has 3600 seconds, so the battery's capacity can be written as $2770\,\text{mA h} \times 3600\,\text{s h}^{-1} = 9.972 \times 10^6\,\text{mA s} = 9972\text{As} = 9972\text{C}$.

An electron has a charge of approximately $1.602 \times 10^{-19}\,\text{C}$, so $9972\,\text{C}$ is $\frac{9972\,\text{C}}{1.602 \times 10^{-19}\,\text{C}} \approx 6.225 \times 10^{22}$ electrons. That's a lot!

(c) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? Recall that a J is equivalent to a W s.

Answer:

The battery capacity is $2770\,\text{mA h}$ at $3.8\,\text{V}$. Using $P = IV$ we see that the battery has a total stored energy of $2770\,\text{mA h} \cdot 3.8\,\text{V} = 10.5\,\text{W h} = 10.5\text{Wh} \cdot 3600\text{s} = 37.9\text{kJ}$.

(d) The battery has internal circuitry that prevents it from getting overcharged (and possibly exploding!). We will model the battery and its internal circuitry as a resistor R_{bat}. We now wish to charge the battery by plugging it into a wall plug. The wall plug can be modeled as a $5\,\text{V}$ voltage source and $200\,\text{m}\Omega$ resistor, as pictured in Figure 1. What is the power dissipated across R_{bat} for $R_{\text{bat}} = 1\,\text{m}\Omega$, $1\,\Omega$, and $10\,\text{k}\Omega$? (i.e. how much power is being supplied to the phone battery as it is charging?). How long will the battery take to charge for each of those values of R_{bat}?

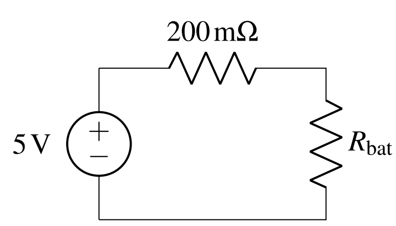


Figure 1: Model of wall plug, wire, and battery.

Answer:

As per the last part, the energy stored in the battery is $2770\,\text{mA h}$ at $3.8\,\text{V}$, which is $2.77\,\text{A h} \cdot 3.8\,\text{V} = 10.5\,\text{W h}$. We can find the time to charge by dividing this energy by power dissipated across R_{bat} (in W) to get time in hours. To find the dissipated power, we first need to find the voltage across and current through R_{bat}. We can recognize this circuit as a voltage divider and so we can find the voltage across R_{bat} using our voltage divider equation:

$$V_{bat} = \frac{R_{bat}}{200\,\text{m}\Omega + R_{bat}} * 5\,\text{V},$$

and the current via Ohm's law. With these we can use $P = IV$ to get the dissipated power across R_{bat}.

For $\underline{R_{\text{bat}} = 1\,\text{m}\Omega}$:
Let's plug in to our voltage divider equation: $V_{bat} = \frac{1\,\text{m}\Omega}{201\,\text{m}\Omega} * 5\,\text{V} = 0.024\,88\,\text{V}$. Then by Ohm's law, $I_{bat} = 0.024\,88\,\text{V}/0.001\,\Omega = 24.88\,\text{A}$. Now we can find the power $P_{bat} = 0.024\,88\,\text{V} \cdot 24.88\,\text{A} = 0.619\,\text{W}$ and finally the time $t = \frac{E}{P_{bat}} = \frac{10.5\,\text{W h}}{0.619\,\text{W}} = 17\,\text{h}$.

For $\underline{R_{\text{bat}} = 1\,\Omega}$:
Let's plug in to our voltage divider equation: $V_{bat} = \frac{1\,\Omega}{1.2\,\Omega} * 5\,\text{V} = 4.167\,\text{V}$. Then by Ohm's law, $I_{bat} = 4.167\,\text{V}/1\,\Omega = 4.167\,\text{A}$. Now we can find the power $P_{bat} = 4.167\,\text{V} \cdot 4.167\,\text{A} = 17.36\,\text{W}$ and finally the time $t = \frac{E}{P_{bat}} = \frac{10.5\,\text{W h}}{17.36\,\text{W}} = 0.6\,\text{h}$ or about $36\,\text{min}$.

For $\underline{R_{\text{bat}} = 10\,\text{k}\Omega}$:
Let's plug in to our voltage divider equation: $V_{bat} = \frac{10\,000\,\Omega}{10000.2\,\Omega} * 5\,\text{V} \approx 5\,\text{V}$. Then by Ohm's law, $I_{bat} = 5\,\text{V}/10\,000\,\Omega \approx 0.5\,\text{mA}$. Now we can find the power $P_{bat} = 5\,\text{V} \cdot 0.5\,\text{A} = 2.5\,\text{mW}$ and finally the time $t = \frac{E}{P_{bat}} = \frac{10.5\,\text{W h}}{0.0025\,\text{W}} = 4210\,\text{h}$.

(e) (Bonus) Suppose you forgot to charge your phone overnight, and you're in a hurry to charge it before you leave home for the day. What should we set R_{bat} to be if we want to charge our battery as quickly as possible? How much current will this draw? How long will it take to charge?
Hint: what choice of R_{bat} maximizes the power dissipated across the resistor?
To minimize the time it takes to charge the battery, we want to minimize the time it takes to accumulate the amount of energy to fill the battery. For constant power (which we have because the circuit is not time varying), $E = PT$, so to minimize time T we should maximize power P.
We know $P = I_R^2 R_{\text{bat}}$, so we want to find R_{bat} that maximizes that expression. Then by combining our voltage divider equation from the previous part with Ohm's law $I_R = \frac{5\text{V}}{200\text{m}\Omega + R_{\text{bat}}}$. We then maximize

$$P = \left(\frac{5}{0.2 + R_{\text{bat}}}\right)^2 \times R_{\text{bat}}$$

where R_{bat} is in Ohms and P is in Watts.
The R_{bat} that maximizes P is $R_{\text{bat}} = 200\text{m}\Omega$. To show this, find the derivative of P with respect to R_{bat} and set it equal to zero.

$$\frac{\mathrm{d}P}{\mathrm{d}R_{\text{bat}}} = 25\left(\frac{1}{(0.2 + R_{\text{bat}})^2} + \frac{-2R_{\text{bat}}}{(0.2 + R_{\text{bat}})^3}\right) = 25\frac{0.2 - R_{\text{bat}}}{(0.2 + R_{\text{bat}})^2}$$

Then $\frac{\mathrm{d}P}{\mathrm{d}R_{\text{bat}}} = 0$ when $R_{\text{bat}} = 0.2\Omega$. From calculus, we know that this could be a maximum- you can check the sign of $\frac{\mathrm{d}^2P}{\mathrm{d}R_{\text{bat}}^2}$ or plot P to convince yourself that it actually is a maximum.
Choosing $R_{\text{bat}}$to be $200\text{m}\Omega$ will cause the resistance of the wire and R_{bat} in series to be $400\text{m}\Omega$, so by Ohm's law the current through the battery will be $5/0.4 = 12.5\text{A}$.
The battery has 8.36Wh, and the power dissipated across the battery is

$$P_{R_{\text{bat}}} = I_{R_{\text{bat}}}^2 \times R_{\text{bat}} = 12.5^2 \times 0.2 = 31.25\text{W}$$

so to get charging time we divide energy by power and get $8.36/31.25 = 0.2675$ hours, or about 16 minutes.

