EECS 16A Designing Information Devices and Systems I
Fall 2020 Homework 6

This homework is due Friday, October 9, 2020, at 23:59.
Self-grades are due Monday, October 12, 2020, at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw6.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned)

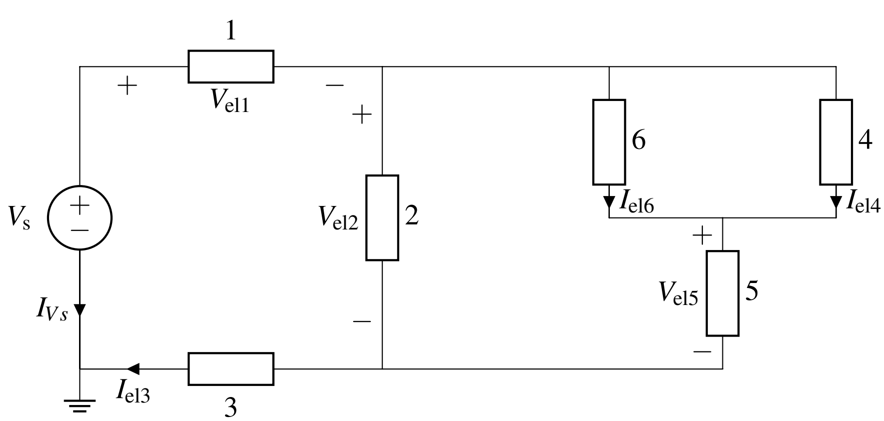
Submit the file to the appropriate assignment on Gradescope.

1. **Reading Assignment**

For this homework, please read Note 11, which introduces the basics of circuit analysis. Also, please review the circuit analysis procedure at the following link: `https://eecs16a.org/lecture/mod2/nva.html`. You are always welcome and encouraged to read beyond this as well. **Question to answer: What is the value of having a systematic procedure for solving circuits?**

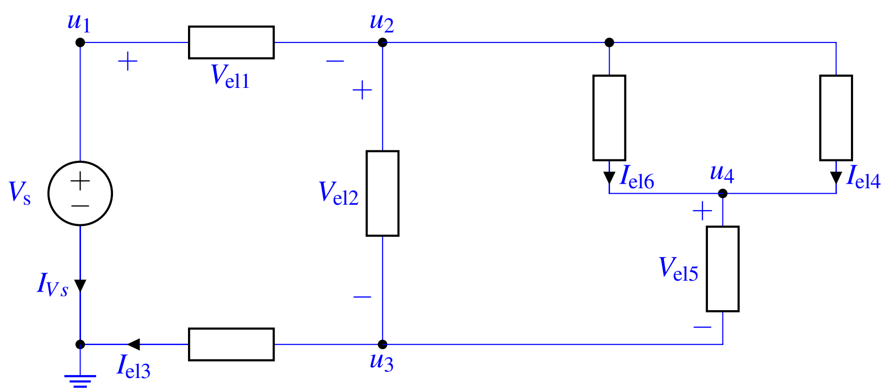
2. **Intro to Circuits**

(Contributors: Panagiotis Zarkos)



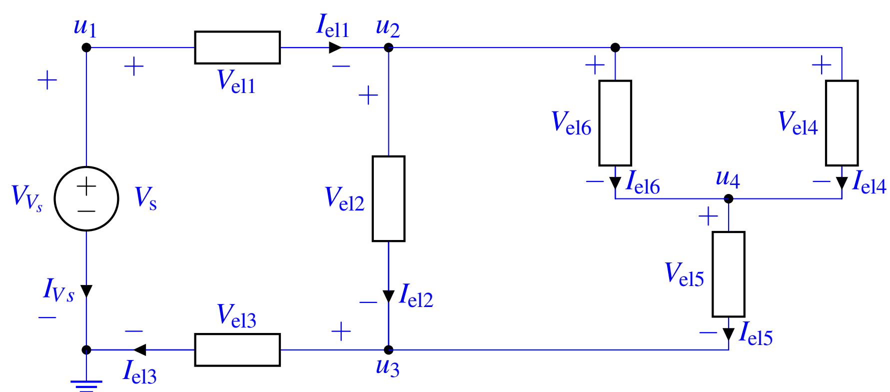
(a) How many nodes does the above circuit have? Label them.
Note: The ground node has been selected for you, so you don't need to label that, but you need to include it in your node count.

Solution: There is a total of 5 nodes in the circuit, including the ground node. They are labeled u_1 - u_4 below:



(b) Notice that elements 1 - 6 and the voltage source V_s have either the *voltage across* or the *current through* them not labeled. Label the missing *voltages across* or *currents through* for elements 1 - 6, and the voltage source V_s, so that they all follow **passive sign convention**.

Solution: The passive sign convention dictates that the current flows from the positive to the negative terminal of the element (or equivalently exiting the negative terminal / entering the positive terminal if you prefer):



(c) Express all element voltages (including the element voltage across the source, V_s) as a function of node voltages. This will depend on the node labeling you chose in part (a).

Solution: For our specific node labeling we can write:

$$V_{V_s} = u_1 - 0 = u_1 (= V_s)$$

$$V_{\text{el1}} = u_1 - u_2$$

$$V_{\text{el2}} = u_2 - u_3$$

$$V_{\text{el3}} = u_3 - 0 = u_3$$

$$V_{\text{el4}} = u_2 - u_4$$

$$V_{\text{el5}} = u_4 - u_3$$

$$V_{\text{el6}} = u_2 - u_4$$

Notice that the element voltage is always of the form: $V_{\text{el}} = u_+ - u_-$.

(d) Write one KCL equation that involves the currents through elements 1 and 2.

Solution: The only node for which we can write a KCL involving elements 1 and 2 is node u_2, since they only intersect on that node:

$$I_{el1} = I_{el2} + I_{el6} + I_{el4}$$

(e) Write a KVL equation for all the loops that contain the voltage source V_s. These equations should be a function of element voltages and the voltage source V_s.

Solution: Notice that there are in fact 3 loops that contain the voltage source V_s, for which we can write the following equations, starting each time from the ground node and ending at the ground node:

$$V_s - V_{\text{el1}} - V_{\text{el2}} - V_{\text{el3}} = 0$$
$$V_s - V_{\text{el1}} - V_{\text{el6}} - V_{\text{el5}} - V_{\text{el3}} = 0$$
$$V_s - V_{\text{el1}} - V_{\text{el4}} - V_{\text{el5}} - V_{\text{el3}} = 0$$

The reason this is not specific to our labeling is that the polarity of all elements is either given or set through the passive sign convention.

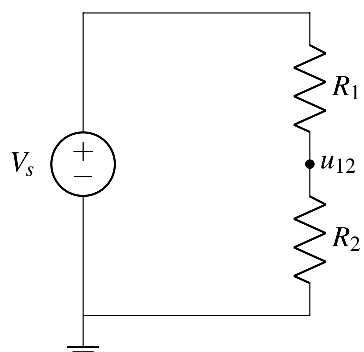
3. Voltage divider

(Contributors: Adhyyan Narang, Panagiotis Zarkos, Sashank Krishnamurthy, Urmita Sikder)

In the following parts, $V_s = 12\,\text{V}$. Choose resistance values such that the current through each element is ≤ 0.8A.

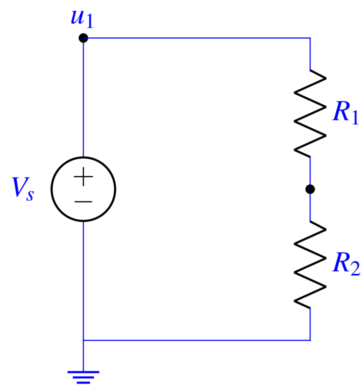
Hint: You can follow the process outlined in `https://eecs16a.org/lecture/mod2/nva.html` *to solve these parts. Other valid circuit solving methods are also acceptable.*

(a) Select values for R_1 and R_2 in the circuit below such that $u_{12} = 6\,\text{V}$. This is a **design problem**, so there can be more than one set of correct answers to this problem.

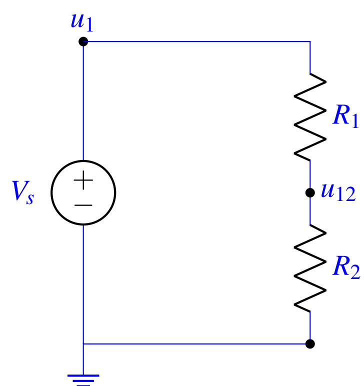


Solution: Step 1: Reference Node
We notice that the ground node has already been selected for us in the question.

Step 2: Label nodes with voltage set by sources.



Step 3: Label other nodes.



Step 4: Label element voltages and currents.
Let V_{R1}, V_{R2} be the voltage drop across R_1 and R_2 respectively. Let I_1 be the current between the voltage source and R_1. Let I_2 be the current from R_2 to the voltage source.
Step 5: KCL Equations
From KCL, at u_{12},

$$I_1 = I_2.$$

Step 6: Find element currents
Using Ohm's law, we have that

$$I_1 = \frac{V_{R1}}{R_1}$$
$$I_2 = \frac{V_{R2}}{R_2}.$$

Writing the element voltages in terms of the node voltages, we have that $V_{R1} = V_s - u_{12}$ and $V_{R2} = u_{12}$.

Step 7: Substitute element currents into KCL Equations.
Substituting back, we obtain $\frac{V_s - u_{12}}{R_1} = \frac{u_{12}}{R_2}$. Solving, we find that $u_{12} = \frac{R_2}{R_1+R_2} V_s$. Plugging in $u_{12} = 6V$ and $V_s = 12V$, we see that $R_1 = R_2$ must be true.

To choose R_1 and R_2 such that the current through each element is ≤ 0.8A, use KVL to write an expression for I_1, I_2 as a function of R_1, R_2:

$V_s - I_1 R_1 - I_2 R_2 = 0$, with $I_1 = I_2 = I_s$
$V_s = I_s(R_1 + R_2)$
$I_s = \frac{V_s}{(R_1+R_2)}$
We need,
$I_s \leq 0.8\text{A}$
Therefore,
$\frac{12\text{V}}{(R_1+R_2)} \leq 0.8\text{A}$
$R_1 + R_2 \geq \frac{12\text{V}}{0.8\text{A}}$
$R_1 + R_2 \geq 15\,\Omega$
As $R_1 + R_2$ must be at least $15\,\Omega$, and $R_1 = R_2$, we choose $R_1 = R_2 = 7.5\,\Omega$. Any other solution with $R_1 = R_2 = R \geq 7.5\Omega$ is also a valid solution.

4. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

Solution:

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group.

XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.

Then I went to homework party for a few hours, where I finished the homework.