EECS 16A Designing Information Devices and Systems I
Fall 2020 Homework 5

This homework is due Friday, October 2, 2020, at 23:59.

Self-grades are due Tuesday, October 6, 2020, at 23:59.

Solutions for this homework will be released on Wednesday, Sept 30, to give you time to study for the midterm.

Submission Format

Your homework submission should consist of **one** file.

- `hw5.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned) as well as your IPython notebook saved as a PDF.

  If you do not attach a PDF "printout" of your IPython notebook, you will not receive credit for problems that involve coding. Make sure that your results and your plots are visible. Assign the IPython printout to the correct problem(s) on Gradescope.

- **We strongly recommended that you submit your self-grades PRIOR to taking Midterm 1 on October 5, 2020, since looking at the solutions earlier will help you to study for the midterm.**

Submit the file to the appropriate assignment on Gradescope.

1. Reading Assignment

For this homework, please read Note 8 through 9. These notes will give you an overview of matrix subspaces and eigenvalues/eigenvectors. You are always welcome and encouraged to read beyond this as well. Write a paragraph about how this relates to what you have learned before and what is new for you.

2. Introduction to Eigenvalues and Eigenvectors

(Contributors: Adhyyan Narang, Ava Tan, Avi Pandey, Christos Adamopoulos, Gireeja Ranade, Michael Kellman, Panos Zarkos, Titan Yuan, Urmita Sikder, Wahid Rahman)

Learning Goal: *Practice calculating eigenvalues and eigenvectors. The importance of eigenvalues and eigenvectors will become clear in the following problems.*

For each of the following matrices, find their eigenvalues and the corresponding eigenvectors. For simple matrices, you may do this by inspection if you prefer.

(a) $\mathbf{A} = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 22 & 6 \\ 6 & 13 \end{bmatrix}$

(c) $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

(d) Let $\mathbf{A} \in \mathbb{R}^{n \times n}$ be a general square matrix. Show that the set of eigenvectors corresponding to a particular eigenvalue of $\mathbf{A}$ is a subspace of $\mathbb{R}^n$. In other words, show that

$$\{\vec{x} \in \mathbb{R}^n : \mathbf{A}\vec{x} = \lambda\vec{x}, \lambda \in \mathbb{R}\}$$

is a subspace. You have to show that all three properties of a subspace (as mentioned in Note 8) hold.

3. Noisy Images

(Contributors: Ava Tan, Avi Pandey, Gireeja Ranade, Grace Kuo, Matthew McPhail, Michael Kellman, Panos Zarkos, Richard Liou, Titan Yuan, Urmita Sikder, Wahid Rahman)

Learning Goal: *The imaging lab uses the eigenvalues of the masking matrix to understand which masks are better than others for image reconstruction in the presence of additive noise. This problem explores the underlying mathematics.*

In lab, we used a single pixel camera to capture many measurements of an image $\vec{i}$. A single scalar measurement s_i is captured using a mask $\vec{h}_i$ such that $s_i = \vec{h}_i^T \vec{i}$. Many measurements can be expressed as a matrix-vector multiplication of the masks with the image, where the masks lie along the rows of the matrix.

$$\begin{bmatrix} s_1 \\ \vdots \\ s_N \end{bmatrix} = \begin{bmatrix} \vec{h}_1^T \\ \vdots \\ \vec{h}_N^T \end{bmatrix} \vec{i} \tag{1}$$

$$\vec{s} = \mathbf{H}\vec{i} \tag{2}$$

In the real world, noise, $\vec{w}$, creeps into our measurements, so instead we have,

$$\vec{s} = \mathbf{H}\vec{i} + \vec{w}. \tag{3}$$

(a) Express $\vec{i}$ in terms of $\mathbf{H}$ (or its inverse), $\vec{s}$, and $\vec{w}$. Assume $\mathbf{H}$ is invertible. (*Hint:* Think about what you did in the imaging lab.)

(b) It turns out that the eigenvalues of $\mathbf{H}$ and $\mathbf{H}^{-1}$ impact how well we can reconstruct the image from the measurements $\vec{s}$. We will see this in subsequent parts of the problem. First, let us compute the eigenvalues of $\mathbf{H}^{-1}$. The eigenvalues of $\mathbf{H}^{-1}$ are actually related to the eigenvalues of $\mathbf{H}$! Show that if λ is an eigenvalue of a matrix $\mathbf{H}$, then $\frac{1}{\lambda}$ is an eigenvalue of the matrix $\mathbf{H}^{-1}$.
Hint: Start with an eigenvalue λ and one corresponding eigenvector $\vec{v}$, such that they satisfy $\mathbf{H}\vec{v} = \lambda\vec{v}$.

(c) We are going to try different $\mathbf{H}$ matrices in this problem and compare how they deal with noise. Run all of the cells in the attached IPython notebook. Observe the **plots and the printed results.** Which matrix performs best in reconstructing the original image and why? What do you observe regarding the eigenvalues of matrices $\mathbf{H_1}, \mathbf{H_2}$ and $\mathbf{H_3}$? What special matrix is $\mathbf{H}_1$? (*Notice that each plot in the iPython notebook returns the result of trying to image a noisy image as well as the minimum absolute value of the eigenvalue of each matrix.*) Comment on the effect of small eigenvalues on the noise in the image.

(d) Now, because there is noise in our measurements, there will be noise in our recovered image. However, the noise is scaled. From the results of part (a), you know that: $\vec{i} = \mathbf{H}^{-1}\vec{s} - \mathbf{H}^{-1}\vec{w}$, so the impact of the noise on the image $\vec{i}$ is given by $\mathbf{H}^{-1}\vec{w}$.
Let us call this quantity $\hat{\vec{w}}$, often called "w-hat".

$$\hat{\vec{w}} = \mathbf{H}^{-1}\vec{w} \tag{4}$$

To analyze how this transformation alters $\vec{w}$, we reprsent $\vec{w}$ as a linear combination of the eigenvectors of $\mathbf{H}^{-1}$,

$$\vec{w} = \alpha_1 \vec{b}_1 + \ldots + \alpha_N \vec{b}_N, \tag{5}$$

where, $\vec{b}_i$ is $\mathbf{H}^{-1}$'s eigenvector corresponding to eigenvalue $\frac{1}{\lambda_i}$.

Show that we can express the noise in the recovered image as the following linear combination of the vectors$\vec{b}_i$:

$$\hat{\vec{w}} = \mathbf{H}^{-1}\vec{w} = \alpha_1 \frac{1}{\lambda_1}\vec{b}_1 + \ldots + \alpha_N \frac{1}{\lambda_N}\vec{b}_N. \tag{6}$$

Now, if λ_i is very large, will the coefficient of $\vec{b}_i$ be large or small in $\hat{\vec{w}}$? If we want $\hat{\vec{w}}$ to be as small as possible, do we prefer large λ_i's or small λ_i's

4. The Dynamics of Romeo and Juliet's Love Affair

(Contributors: Ava Tan, Avi Pandey, Karina Chang, Vijay Govindarajan, Gireeja Ranade)

Learning Goal: *Eigenvalues and eigenvectors of state transition matrices tend to reveal useful information about the dynamical systems they model. This problem serves as an example of extracting useful information through analysis of the eigenvalues of the state transition matrix of a dynamical system.*

In this problem, we will study a discrete-time model of the dynamics of Romeo and Juliet's love affair—adapted from Steven H. Strogatz's original paper, *Love Affairs and Differential Equations*, Mathematics Magazine, 61(1), p.35, 1988, which describes a continuous-time model.

Let $R[n]$ denote Romeo's feelings about Juliet on day n, and let $J[n]$ denote Juliet's feelings about Romeo on day n, where $R[n]$ and $J[n]$ are **scalars**. The **sign** of $R[n]$ (or $J[n]$) indicates like or dislike. For example, if $R[n] > 0$, it means Romeo likes Juliet. On the other hand, $R[n] < 0$ indicates that Romeo dislikes Juliet. $R[n] = 0$ indicates that Romeo has a neutral stance towards Juliet.

The **magnitude** (i.e. absolute value) of $R[n]$ (or $J[n]$) represents the intensity of that feeling. For example, a larger magnitude of $R[n]$ means that Romeo has a stronger emotion towards Juliet (strong love if $R[n] > 0$ or strong hatred if $R[n] < 0$). Similar interpretations hold for $J[n]$.

We model the dynamics of Romeo and Juliet's relationship using the following linear system:

$$R[n+1] = aR[n] + bJ[n], \quad n = 0, 1, 2, \ldots$$

and

$$J[n+1] = cR[n] + dJ[n], \quad n = 0, 1, 2, \ldots,$$

which we can rewrite as

$$\vec{s}[n+1] = \mathbf{A}\vec{s}[n],$$

where $\vec{s}[n] = \begin{bmatrix} R[n] \\ J[n] \end{bmatrix}$ denotes the state vector and $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ denotes the state transition matrix for our dynamic system model.

The selection of the parameters a, b, c, d results in different dynamic scenarios. The fate of Romeo and Juliet's relationship depends on these model parameters (i.e. a, b, c, d) in the state transition matrix and the initial state ($\vec{s}[0]$). In this problem, we'll explore some of these possibilities.

(a) Consider the case where $a + b = c + d$ in the state-transition matrix

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Show that

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

is an eigenvector of $\mathbf{A}$, and determine its corresponding eigenvalue λ_1.
Show that

$$\vec{v}_2 = \begin{bmatrix} b \\ -c \end{bmatrix}$$

is an eigenvector of $\mathbf{A}$, and determine its corresponding eigenvalue λ_2.
Now, express the first and second eigenvalues and their eigenspaces in terms of the parameters $a, b, c,$ and d.
Hint: *Consider $\mathbf{A}\vec{v}_1$. Is it equal to a scalar multiple of $\vec{v}_1$? Repeat a similar process for $\vec{v}_2$.*

For parts (b) - (e), consider the following state-transition matrix:

$$\mathbf{A} = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

(b) Determine the eigenvalues and corresponding eigenvectors (i.e. $\lambda_1, \vec{v}_1$ and $\lambda_2, \vec{v}_2$) for this system. Note that this matrix is a special case of the matrix explored in part (a), so you can use results from that part to help you.

(c) Determine all of the non-zero *steady states* of the system. That is, find all possible state vectors $\vec{s}_*$ such that if Romeo and Juliet start at, or enter, any of those state vectors, their states will stay in place forever: $\{\vec{s}_* \mid \mathbf{A}\vec{s}_* = \vec{s}_*\}$.

(d) Suppose Romeo and Juliet start from an initial state $\vec{s}[0] \in \operatorname{span}\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$. What happens to their relationship over time? Specifically, what is $\vec{s}[n]$ as $n \to \infty$?

(e) Suppose the initial state is $\vec{s}[0] = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$. What happens to their relationship over time? Specifically, what is $\vec{s}[n]$ as $n \to \infty$?
Hint: *Can you use what you learned about the eigenvectors of $\mathbf{A}$ (in parts c and d) to help you solve this problem? You can represent the starting state as a linear combination of eigenvectors $\vec{v}_1$ and $\vec{v}_2$.*

Now suppose we have the following state-transition matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Use this state-transition matrix for parts (f) - (h).

(f) Determine the eigenvalues and corresponding eigenvectors (i.e. $\lambda_1, \vec{v}_1$ and $\lambda_2, \vec{v}_2$) for this system. Note that this matrix is **a special case** of the matrix explored in part (a), so you can use results from that part to help you.

(g) Suppose Romeo and Juliet start from an initial state $\vec{s}[0] \in \text{span}\left\{\begin{bmatrix}1\\-1\end{bmatrix}\right\}$. What happens to their relationship over time? Specifically, what is $\vec{s}[n]$ as $n \to \infty$?

(h) Now suppose that Romeo and Juliet start from an initial state $\vec{s}[0] \in \text{span}\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}$. What happens to their relationship over time? Specifically, what is $\vec{s}[n]$ as $n \to \infty$?

Finally, we consider the case where we have the following state-transition matrix:

$$\mathbf{A} = \begin{bmatrix}1 & -2\\-2 & 1\end{bmatrix}$$

Use this state-transition matrix for parts (i) - (k).

(i) Determine the eigenvalues and corresponding eigenvectors (i.e. $\lambda_1, \vec{v}_1$ and $\lambda_2, \vec{v}_2$) for this system. Note that this matrix is **a special case** of the matrix explored in part (a), so you can use results from that part to help you.

(j) Suppose Romeo and Juliet start from an initial state $\vec{s}[0] = \begin{bmatrix}R[0]\\J[0]\end{bmatrix}$, where $\vec{s}[0] \in \text{span}\left\{\begin{bmatrix}1\\-1\end{bmatrix}\right\} = \text{span}\left\{\begin{bmatrix}-1\\1\end{bmatrix}\right\}$. What happens to their relationship over time if $R[0] > 0$ and $J[0] < 0$? What about if $R[0] < 0$ and $J[0] > 0$? Specifically, what is $\vec{s}[n]$ as $n \to \infty$?

(k) Now suppose that Romeo and Juliet start from an initial state $\vec{s}[0] \in \text{span}\left\{\begin{bmatrix}1\\1\end{bmatrix}\right\}$. What happens to their relationship over time? Specifically, what is $\vec{s}[n]$ as $n \to \infty$?

5. Traffic Flows

(Contributors: Adhyyan Narang, Ava Tan, Avi Pandey, Gireeja Ranade, Grace Kuo, Laura Brink, Panos Zarkos, Raghav Anand, Richard Liou, Titan Yuan, Vijay Govindarajan)

Learning Objective: *The learning objective of this problem is to see how the concept of nullspaces can be applied to flow problems.*

Your goal is to measure the flow rates of vehicles along roads in a town. It is prohibitively (too) expensive to place a traffic sensor along every road. You realize, however, that the number of cars flowing into an intersection must equal the number of cars flowing out. You can use this "flow conservation" to determine the traffic along all roads in a network by measuring the flow along only some roads. In this problem, we will explore this concept.

(a) Let's begin with a network with three intersections, A, B and C. Define the flow t_1 as the rate of cars (cars/hour) on the road between B and A, flow t_2 as the rate on the road between C and B, and flow t_3 as the rate on the road between C and A.

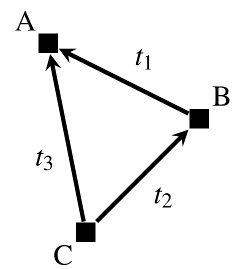


Figure 1: A simple road network.

(Note: The directions of the arrows in the figure are the way that we define positive flow by convention. For example, if there were 100 cars per hour traveling from A to C, then $t_3 = -100$. The flows now are not fractions of water in reservoirs as in the pumps setting, but numbers of cars.)

We assume the "flow conservation" constraints: the net number of cars per hour flowing into each intersection is zero. For example at intersection B, we have the constraint $t_2 - t_1 = 0$. The full set of constraints (one per intersection) is:

$$\begin{cases} t_1 + t_3 = 0 \\ t_2 - t_1 = 0 \\ -t_3 - t_2 = 0 \end{cases}$$

As mentioned earlier, we can place sensors on a road to measure the flow through it, but we have a limited budget, and we would like to determine all of the flows with the smallest possible number of sensors.

Suppose for the network above we have one sensor reading, $t_1 = 10$. Can we figure out the flows along the other roads? (That is, the values of t_2 and t_3). If we can, find the values of t_2 and t_3.

(b) Now suppose we have a larger network, as shown in Figure 2.

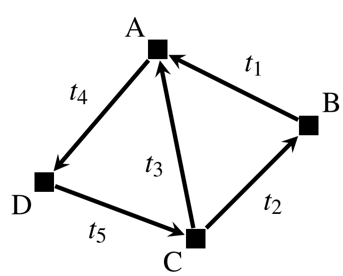


Figure 2: A larger road network.

We would again like to determine the traffic flows on all roads, using measurements from some sensors. A Berkeley student claims that we need two sensors placed on the roads CA (measuring t_3) and DC (measuring t_5). A Stanford student claims that we need two sensors placed on the roads CB (measuring t_2) and BA (measuring t_1). Write out the system of linear equations that represents this flow graph. Is it possible to determine all traffic flows, $\left[t_1, t_2, t_3, t_4, t_5\right]^T$, with the Berkeley student's suggestion? How about the Stanford student's suggestion? *Hint: This can be solved just writing out the relevant equations and reasoning about them.*

(c) We would like a more general way of determining the possible traffic flows in a network. Suppose we write the traffic flow on all roads as a vector $\vec{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix}$. As a first step, let us try to write all the flow conservation constraints (one per intersection) i.e. the system of equations from part (b) as a matrix equation.

Construct a 4×5 matrix $\mathbf{B}$ such that the equation $\mathbf{B}\vec{t} = \vec{0}$:

$$\begin{bmatrix} & & & \\ & & \mathbf{B} & \\ & & & \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

represents the flow conservation constraints for the network in Figure 2.

Hint: You can construct ***B*** *using only* $0, 1,$ *and* -1 *entries. Each row represents the inflow/outflow of an intersection. This matrix is called the* ***incidence matrix****.*

(d) Again, suppose we write the traffic flow on all roads as a vector $\vec{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix}$. Then, determine the subspace of all valid traffic flows for the network of Figure 2. Notice that the set of all vectors $\vec{t}$ that satisfy $\mathbf{B}\vec{t} = \vec{0}$ is exactly the null space of the matrix $\mathbf{B}$. That is, we can find all valid traffic flows by computing the null space of $\mathbf{B}$. What is the dimension of the nullspace?

(e) Notice that we can represent the Berkeley student's measurement as $\mathbf{M}_B\vec{t}$, where:

$$\mathbf{M}_B\vec{t} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vec{t} = \begin{bmatrix} t_3 \\ t_5 \end{bmatrix}$$

Write a matrix $\mathbf{M}_S$ that can be used to represent the Stanford student's measurement.

(f) Now let us analyze more general road networks. Say there is a road network graph G, with incidence matrix $\mathbf{B}_G$. If $\mathbf{B}_G$ has a k-dimensional null space, does this mean measuring the flows along ***any*** k **roads** is always sufficient to recover all of the true flows? In other words, is there ever a possibility of being unable to recover the true flows depending on which k roads you choose?

If you think measuring the flows along any k roads will always work, then prove it showing various possible scenarios. Otherwise give an example showing a scenario where it does not work (such an example is called a counter example).

Hint: Consider the Stanford student's measurement from part (b).

(g) **[Challenge, Optional]** Assume that $\vec{u}$ and $\vec{t}$ are distinct valid flows, that is $\mathbf{B}_G\vec{u} = \mathbf{B}_G\vec{t} = \vec{0}$. Can you recover all of the network's true flows if $(\vec{u} - \vec{t})$ belongs to the nullspace of $\mathbf{M}_S$?

Clarification: A "valid" flow is one that is possible without violating the constraints on the nodes (so flow in must equal to flow out). There may be many valid flows, but only one "true" flow. The "true flow" is one of many the valid flows, which represents the actual number of cars/ hour on each road.

(h) **[Challenge, Optional]** If the incidence matrix $\mathbf{B}_G$ has a k-dimensional null space, does this mean we can **always pick a set of** k **roads** such that measuring the flows along these roads is sufficient to recover the exact flows? If this is true, explain how you would pick these k roads to guarantee that you could recover the missing information. Otherwise, give a counterexample.

6. Subspaces, Bases and Dimension

(Contributors: Moses Won, Urmita Sikder)

For each of the sets $\mathbb{U}$ (which are subsets of $\mathbb{R}^3$) defined below, state whether $\mathbb{U}$ is a subspace of $\mathbb{R}^3$ or not. If $\mathbb{U}$ is a subspace, find a basis for it and state the dimension. You have to show that all three properties of a subspace (as mentioned in Note 8) hold.

(a) $\mathbb{U} = \left\{ \begin{bmatrix} 2(x+y) \\ x \\ y \end{bmatrix} \mid x, y \in \mathbb{R} \right\}$

(b) **(PRACTICE/OPTIONAL)** $\mathbb{U} = \left\{ \begin{bmatrix} x \\ y \\ z+1 \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$

(c) $\mathbb{U} = \left\{ \begin{bmatrix} x \\ y \\ x+1 \end{bmatrix} \mid x, y \in \mathbb{R} \right\}$

(d) **(PRACTICE, OPTIONAL)** $\mathbb{U} = \left\{ \begin{bmatrix} x \\ y \\ x+y^2 \end{bmatrix} \mid x, y \in \mathbb{R} \right\}$

7. Page Rank [OPTIONAL/PRACTICE]

(Contributors: Ava Tan, Avi Pandey, Edward Doyle, Gireeja Ranade, Lydia Lee, Michael Kellman, Panos Zarkos, Raghav Anand, Titan Yuan)

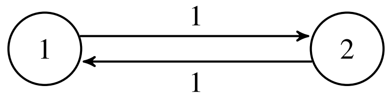
Learning Goal: *This problem highlights the use of transition matrices in modeling dynamical linear systems. Predictions about the steady state of a system can be made using the eigenvalues and eigenvectors of this matrix.*

In homework and discussion, we have discussed the behavior of water flowing in reservoirs and the people flowing in social networks. We now consider the setting of web traffic where the dynamical system can be described with a directed graph, also known as state transition diagram.

As we have seen in lecture and discussion the “transition matrix”, $\mathbf{T}$, can be constructed using the state transition diagram, as follows: entries t_{ji}, represent the *proportion* of the people who are at website i that click the link for website j.

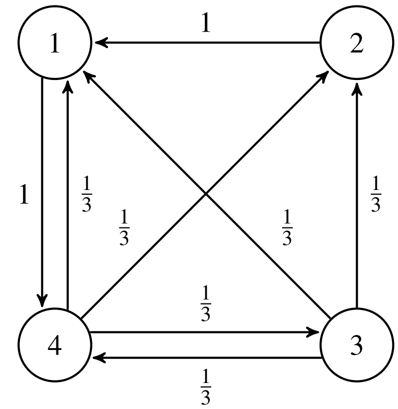
The **steady-state frequency** (i.e. fraction of visitors in steady-state) for a graph of websites is related to the eigenspace associated with eigenvalue 1 for the “transition matrix” of the graph. Once computed, an eigenvector with eigenvalue 1 will have values which correspond to the steady-state frequency for the fraction of people for each webpage. When the elements of this eigenvector are made to **sum to one** (to conserve population), the i^{th} element of the eigenvector will correspond to the fraction of people on the i^{th} website.

(a) For graph A shown below, what are the steady-state frequencies i.e. fraction of visitors in steady-state for the two webpages? Graph A has weights in place to help you construct the transition matrix. Remember to ensure that your steady state-frequencies sum to 1 to maintain conservation.



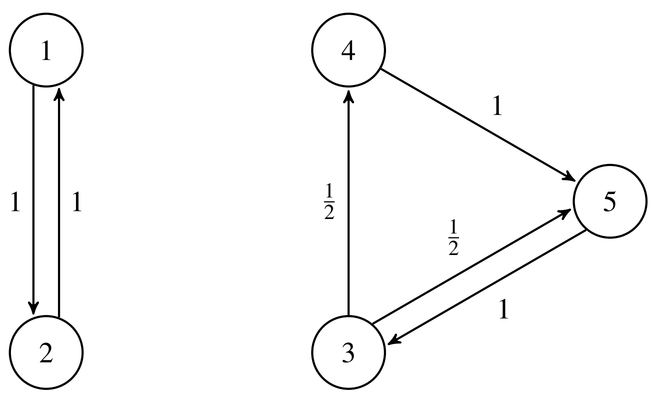
Graph A - weighted

(b) For graph B, what are the steady-state frequencies for the webpages? You may use IPython and the Numpy command `numpy.linalg.eig` for this. It may be helpful to consult the Python documentation for `numpy.linalg.eig` to understand what this function does and what it returns. Graph B is shown below, with weights in place to help you construct the transition matrix.



Graph B - weighted

(c) Find the eigenspace that corresponds to the steady-state for graph C. How many independent systems (disjoint sets of webpages) are there in graph C versus in graph B? What is the dimension of the eigenspace corresponding to the steady-state for graph C? Again, graph C with weights in place is shown below. You may use IPython to compute the eigenvalues and eigenvectors again.



Graph C - weighted

8. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

9. Preparing for Midterm 1. (No submission required.)

(a) Please be sure to again review and be familiar with the entirety of the EECS16A exam proctoring policy (click here) before the first midterm on **October 5, 2020**. We recommend doing this well in advance of the midterm. No submission is required for this question.

(b) Following the instructions outlined in Question 8 of Homework 3, please practice the Zoom proctoring procedures once more so that you are comfortable with setting up and positioning your recording device, recording yourself, saving and uploading your proctoring video, and submitting your exam answers in a timely fashion. You are not required to practice proctoring yourself again, but we recommend it to avoid any technical difficulties the day of the exam. No submission is required for this question.

(c) Prior to **October 5, 2020**, when Midterm 1 will be administered, we will be releasing instructions to prepare your answer sheet template in advance for the exam, so you will know how much paper you will need for the exam. Please be on the lookout for course announcements regarding the preparation of the exam answer sheet template in the coming week. No submission is required for this question.

Link for policy:
`https://docs.google.com/document/d/1EVb4Ca6FWSAykExY7X5ynFW4KdmHd0BI6KZ0ktM8ows/edit?usp=sharing`

