EECS16A DIS 12A

Music (bit.ly/16ajukebox)

① Maroon 5
Don't wanna know

② G-Dragon - Black

(① suggested by Ichchitaa)

Op amps with superposition

Inner products (how to compute, properties)

Correlation: shifted inner products, how to compute, interpretation

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Terms / Definitions / Notation

① standard inner product / Dot product / Euclidean inner product

$\vec{x}, \vec{y} \in \mathbb{R}^n$ → $\underline{\langle \vec{x}, \vec{y} \rangle} \in \mathbb{R}$ → $\underset{1\times n}{\vec{x}^T}\ \underset{n\times 1}{\vec{y}}$ $\quad \underline{\vec{x} \cdot \vec{y}}$

② signals

$s[n]$

name of signal

time

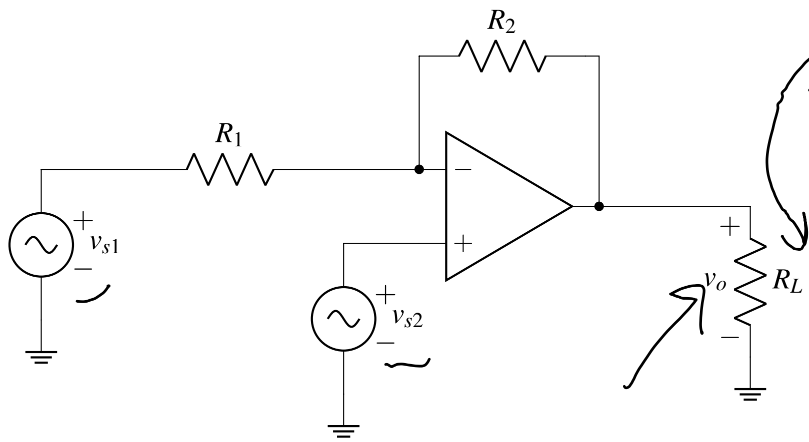
signal: a function

a quantity that varies (with time/position)

EECS 16A Designing Information Devices and Systems I
Fall 2020 Discussion 12A

1. Amplifier with Multiple Inputs

In this problem we will use superposition and the Golden Rules to find the output of the following op amp circuit with multiple inputs:

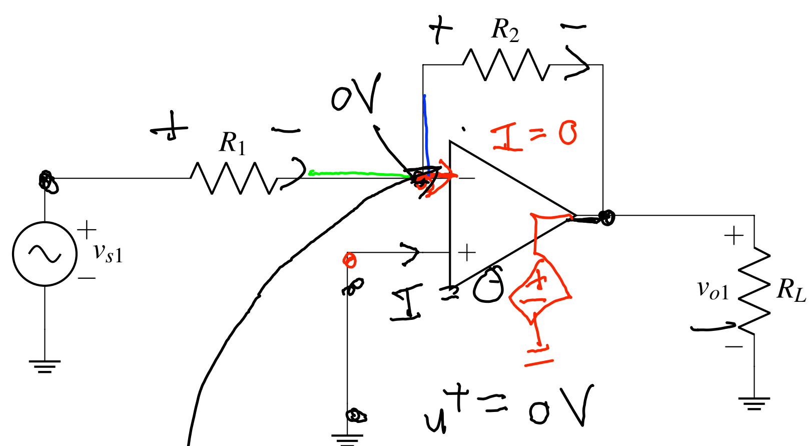


$V_{out} = A(v^+ - v^-)$

Analyzing op amp use GR's

NVA

(a) First, let's turn off v_{s2}. Use the **Golden Rules** to find v_{o1} for the circuit below.

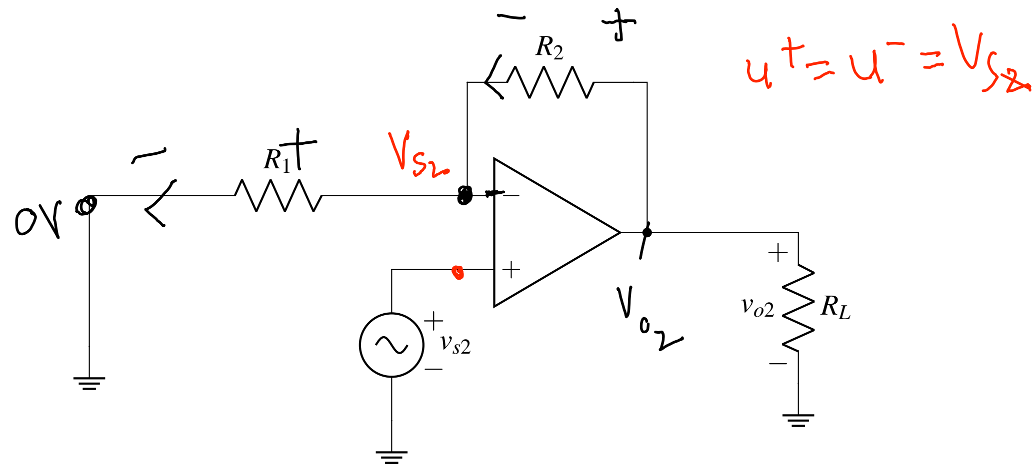


$u^+ = 0V$

$u^- = u^+ = 0V$

$$\frac{V_{s_1} - 0V}{R_1} = \frac{0V - V_{o1}}{R_2} \Rightarrow V_{o1} = -\frac{R_2}{R_1} V_{s_1}$$

(b) Now let's turn off v_{o1}. Use the **Golden Rules** to find v_{o2} for the circuit below.



$$\Rightarrow \frac{V_{o2} - V_{S2}}{R_2} = \frac{V_{S2} - 0V}{R_1}$$

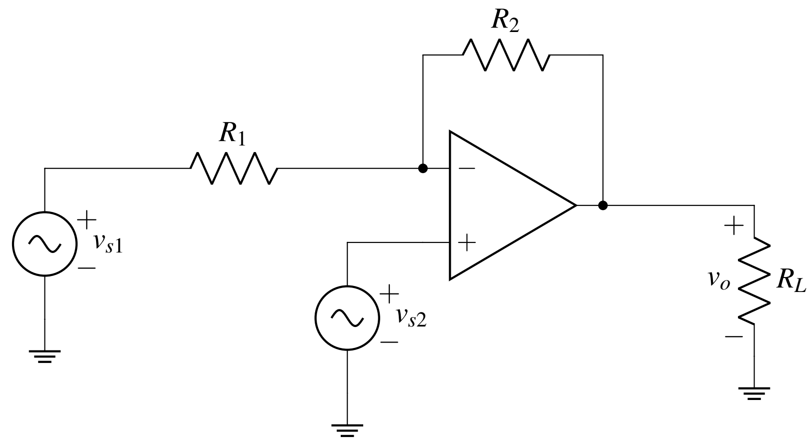
$$-V_{S2} + V_{o2} = \frac{R_2}{R_1} V_{S2}$$

$$V_{o2} = V_{S2} + \frac{R_2}{R_1} V_{S2}$$

$$= \left(1 + \frac{R_2}{R_1}\right) V_{S2}$$

(c) $V_o = V_{o1} + V_{o2} = \left(1 + \frac{R_2}{R_1}\right) V_{S2} - \frac{R_2}{R_1} V_{S1}$

(c) Use **superposition** to find the output voltage v_o for the circuit shown below.



$(AB)^T = B^T A^T$

Reference: Inner products

Let $\vec{x}$, $\vec{y}$, and $\vec{z}$ be vectors in real vector space $\mathbb{V}$. A mapping $\langle \cdot, \cdot \rangle$ is said to be an inner product on $\mathbb{V}$ if it satisfies the following three properties:

(a) Symmetry: $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle$

(b) Linearity: $\langle \vec{x}, \vec{y} + \vec{z} \rangle = \langle \vec{x}, \vec{y} \rangle + \langle \vec{x}, \vec{z} \rangle$ and $\langle c\vec{x}, \vec{y} \rangle = c\langle \vec{x}, \vec{y} \rangle$

(c) Non-negativeness: $\langle \vec{x}, \vec{x} \rangle \geq 0$, with equality if and only if $\vec{x} = \vec{0}$.

$(\vec{x}^T \vec{y})^T$

$\vec{y}^T \vec{x} = \vec{x}^T \vec{y}$

We define the norm of $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ as $\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}$.

$\vec{x}^T(\vec{y} + \vec{z})$

2. Mechanical Inner Products

For the following pairs of vectors, find the Euclidean inner product $\langle \vec{x}, \vec{y} \rangle = \vec{x}^T \vec{y}$.

(a)

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\vec{x}^T \vec{x}$

$\langle \vec{x}, \vec{y} \rangle = \vec{x}^T \vec{y}$

$= x_1^2 + x_2^2 + \cdots$

$\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} =$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$1 \cdot 1 + 0 \cdot 2 + 3 \cdot 1 = 4$

b $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 4$

(b)

$$\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$2\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}2\\4\\6\end{bmatrix}$$

$$\left\langle \begin{bmatrix}2\\4\\6\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix} \right\rangle = \begin{bmatrix}2 & 4 & 6\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} = \underline{8}$$

$$\left\langle 2\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix} \right\rangle = 2\underbrace{\left\langle \begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix} \right\rangle}$$

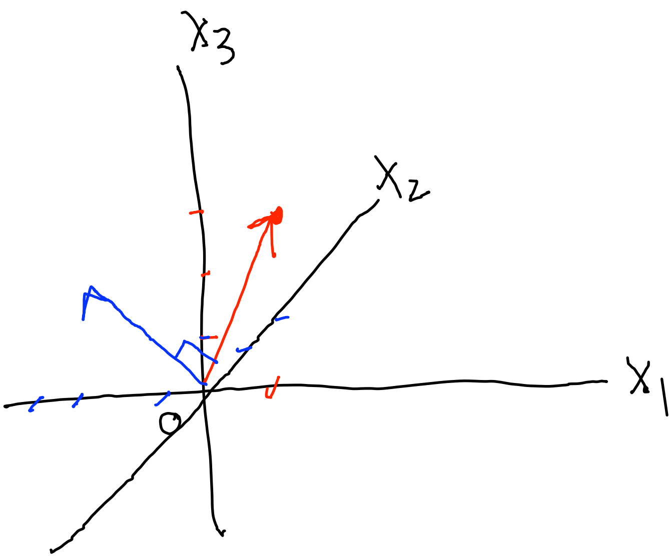
$$= 2 \cdot 4$$

(c)

$$\begin{bmatrix}1\\0\\3\end{bmatrix}, \begin{bmatrix}-3\\2\\1\end{bmatrix}$$

$$\left\langle \begin{bmatrix}1\\0\\3\end{bmatrix}, \begin{bmatrix}-3\\2\\1\end{bmatrix} \right\rangle = (+1)(-3)+(3)(1)$$

$$= 0$$ (inner product is zero so vectors are orthogonal)



Q: Norm vs. Mag.

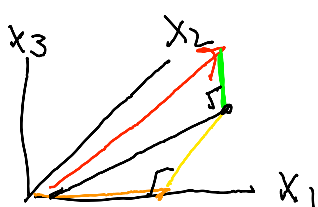
$\|\vec{x}\|$ - length of a vector

↳ norm = magnitude

$\langle \vec{x}, \vec{x} \rangle = \|\vec{x}\|^2$ (norm squared)

magnitude squared

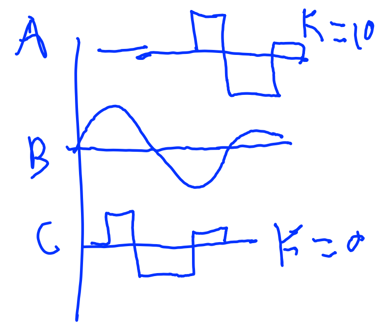
$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$



$$\sqrt{x_1^2+x_2^2+x_3^2}$$

3. Correlation

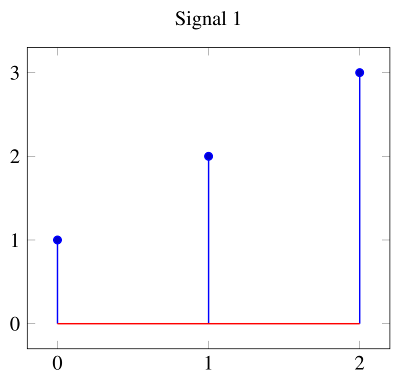
We are given the following two signals, $s_1[n]$ and $s_2[n]$ respectively.

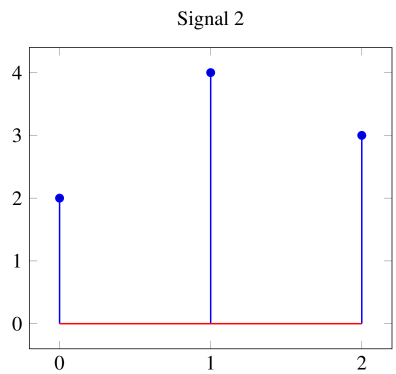


$s_1[0] = 1$

$s_1[-1] = 0$

$s_1[3] = 0$





$\vec{x}, \vec{y}$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

Find the cross correlations, $\text{corr}_{s_1}(s_2)$ and $\text{corr}_{s_2}(s_1)$ for signals $s_[n]$ and $s_2[n]$. Recall

$$\text{corr}_x(y)[k] = \sum_{i=-\infty}^{\infty} x[i]y[i-k].$$

$n-(-2)$

$\text{corr}_{\vec{s}_1}(\vec{s}_2)[k]$

$\text{corr}_{\vec{s}_1}(\vec{s}_2)[-2]$

$\langle \vec{x}, \vec{y} \rangle = \sum_{i=1}^{n} x_i y_i$

n	-2	-1	0	1	2	3	4
$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n+2]$	2	1	3	0	0	0	0
$\langle \vec{s}_1, \vec{s}_2[n+2] \rangle$	0 +	0 +	3 +	0 +	0 +	0 +	0 = 3

$K = -1$

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n+1]$							
$\langle \vec{s}_1, \vec{s}_2[n+1] \rangle$	+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n]$							
$\langle \vec{s}_1, \vec{s}_2[n] \rangle$	+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n-1]$							
$\langle \vec{s}_1, \vec{s}_2[n-1] \rangle$	+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n-2]$							
$\langle \vec{s}_1, \vec{s}_2[n-2] \rangle$	+	+	+	+	+	+	=

$\text{corr}_{\vec{s}_2}(\vec{s}_1)[k]$

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n+2]$														
$\langle\vec{s}_2,\vec{s}_1[n+2]\rangle$		$+$		$+$		$+$		$+$		$+$		$+$		$=$

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n+1]$														
$\langle\vec{s}_2,\vec{s}_1[n+1]\rangle$		$+$		$+$		$+$		$+$		$+$		$+$		$=$

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n]$														
$\langle\vec{s}_2,\vec{s}_1[n]\rangle$		$+$		$+$		$+$		$+$		$+$		$+$		$=$

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n-1]$														
$\langle\vec{s}_2,\vec{s}_1[n-1]\rangle$		$+$		$+$		$+$		$+$		$+$		$+$		$=$

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n-2]$														
$\langle\vec{s}_2,\vec{s}_1[n-2]\rangle$		$+$		$+$		$+$		$+$		$+$		$+$		$=$