EECS16A DIS9B

email: moseswan@berkeley.edu

OH: W 10AM-12PM (HWP)

Logistical details

1. Don't forget MT2 next week
2. Review sessions on Friday, Saturday

Learning Objectives

1. How to identify floating nodes (where does charge conservation apply?)
2. Charge sharing algorithm (finding voltages and charges in a switch-capacitor circuit)

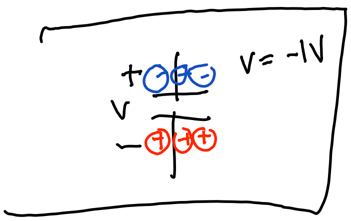
Playlist (suggest @ 16a jukebox)

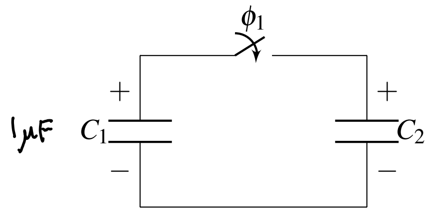
1. Troyboi - ili (suggested by Lucy)
2. H.E.R. - Best Part ft. Daniel Caesar
3. MONSUNE - Outta my mind

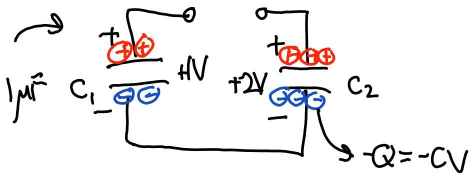
EECS 16A Fall 2020 — Designing Information Devices and Systems I — Discussion 9B

1. Capacitors and Charge Conservation

(a) Consider the circuit below with $C_1 = C_2 = 1\,\mu\text{F}$ and an open switch. Suppose that C_1 is initially charged to $+1\,\text{V}$ and that C_2 is charged to $+2\,\text{V}$. How much charge is on C_1 and C_2?





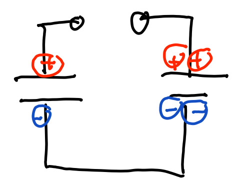


$$\rightarrow Q_1 = \underset{C_1}{(1\mu F)}\,\underset{V_1}{(1V)} = \underline{1\mu C}$$

$$Q_2 = \underset{C_2}{(1\mu F)}(2V) = \underline{2\mu C}$$

$\underline{Q} = CV$

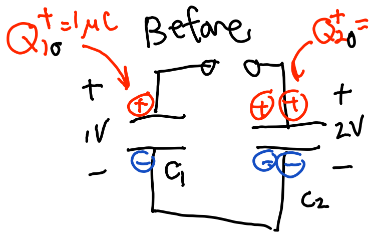
positive

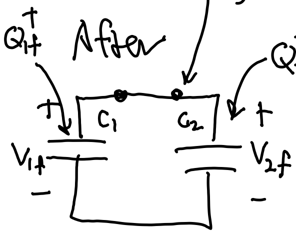


(b) Now the switch is closed (i.e. the capacitors are connected together.) What are the voltages across and the charges on C_1 and C_2?

floating node (charge cannot leave or enter)

V_{1f} ← final





Since charge cannot leave the top node, then the total charge must stay the same

$$Q_{10}^+ + Q_{20}^+ = Q_{1f}^+ + Q_{2f}^+$$

$Q_{10}^+ \neq Q_{1f}^+$ (not always)

$Q_{20}^+ \neq Q_{2f}^+$ (not always)

$Q_{1f}^+ \neq Q_{2f}^+$ (not always)

$V_{1f} = V_{2f}$ (why? parallel)

$$V_{1f} = \frac{Q_{1f}^+}{C_1} \quad (Q = CV)$$

$$V_{2f} = \frac{Q_{2f}^+}{C_2}$$

$$\Rightarrow \frac{Q_{1f}^+}{C_1} = \frac{Q_{2f}^+}{C_2}$$

$C_1 = C_2 = 1\mu F$ simplifies

$$Q_{1f}^+ = Q_{2f}^+$$

Charge conservation

$$Q_{10}^+ + Q_{20}^+ = Q_{1f}^+ + Q_{2f}^+$$

$$1\mu C + 2\mu C = Q_{1f}^+ + Q_{2f}^+$$

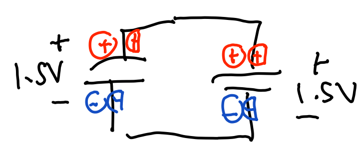
$$3\mu C = Q_{1f}^+ + Q_{2f}^+ = 2Q_{1f}^+ \Rightarrow Q_{1f}^+ = \frac{3\mu C}{2} = \boxed{1.5\mu C}$$

$$\Rightarrow Q_{2f}^+ = \boxed{1.5\mu C}$$

$$V_{1f} = V_{2f} = \frac{1.5\mu C}{1\mu F} = \boxed{1.5V}$$

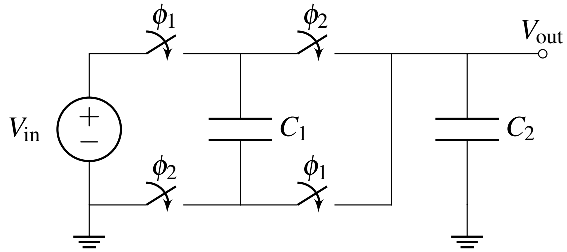
Floating node

- node from which charge cannot escape or enter
- connected to opens or only capacitor plates
- Charge conservation applies

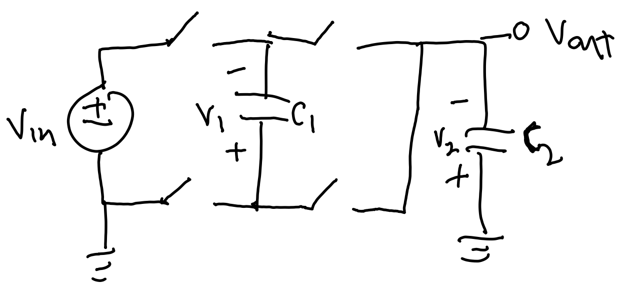


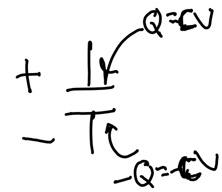
2. Charge Sharing

Consider the circuit shown below. In phase ϕ_1, the switches labeled ϕ_1 are on while the switches labeled ϕ_2 are off. In phase ϕ_2, the switches labeled ϕ_2 are on while the switches labeled ϕ_1 are off.



(a) Draw the polarity of the voltage (using $+$ and $-$ signs) across the two capacitors C_1 and C_2. (It doesn't matter which terminal you label $+$ or $-$; just remember to keep these consistent through phase 1 and 2!)



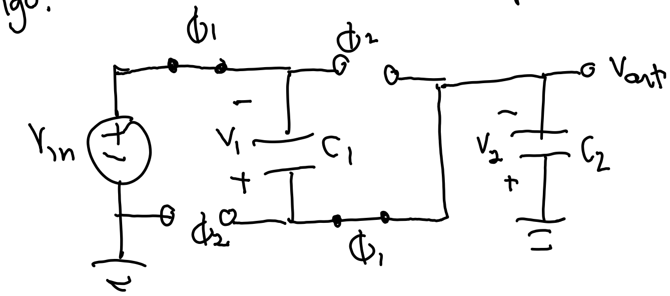


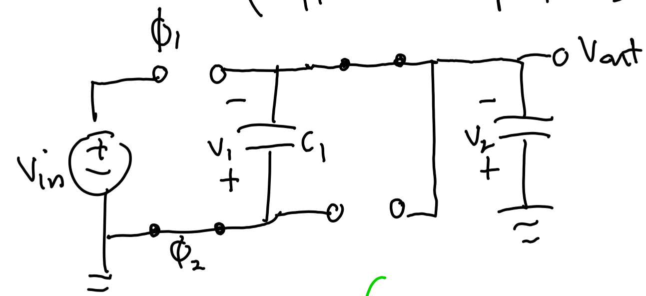
(b) Redraw the circuit in phase ϕ_1 and phase ϕ_2. Keep your polarity from part (a) in mind.

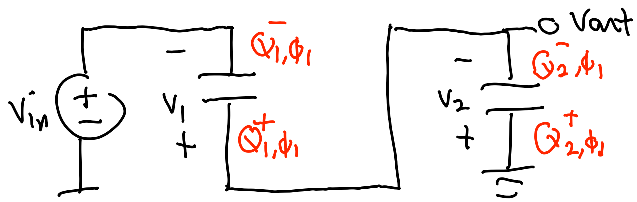
II C_{in} algo.

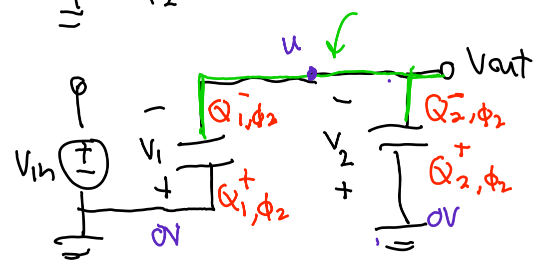
Phase ϕ_1 (ϕ_1 switches closed - shorts/wire) ϕ_2 switches open

Phase ϕ_2 (ϕ_2 switches closed/shorts/wires ϕ_1 switches open)









(c) Find V_{out} as a function of V_{in}, C_1, and C_2 in phase ϕ_2

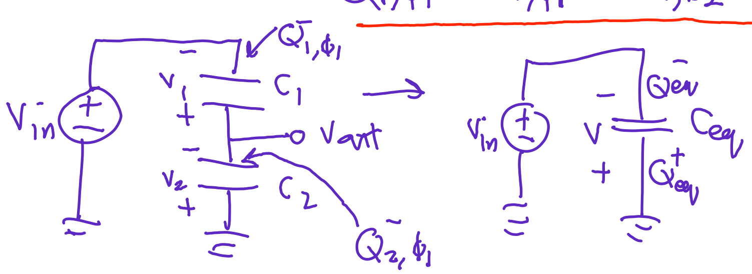
III Identify all floating nodes in phase ϕ_2 $\qquad Q = CV$

IV For each floating node compute charges and write conservation equations

$$Q^-_{1,\phi_1} + Q^-_{2,\phi_1} = Q^-_{1,\phi_2} + Q^-_{2,\phi_2} \quad (1)$$

charge conservation. at floating node with node voltage u.

Series equivalent computation in phase ϕ_1 to compute $Q^-_{1,\phi_1}, Q^-_{2,\phi_1}$



$V = -V_{in}$

$Q^-_{eq} = -C_{eq}V$

$= -C_{eq}(-V_{in})$

$= C_{eq}V_{in}$

$C_{eq} = C_1 || C_2 = \frac{C_1C_2}{C_1+C_2}$

☆ All capacitors are discharged in phase ϕ_1 before V_{in} charges them

$$Q^-_{eq} = Q^-_{1,\phi_1} = Q^-_{2,\phi_1} = \frac{C_1C_2}{C_1+C_2}V_{in} \quad (2)$$

Coming from series equivalence in ϕ_1

$Q^-_{1,\phi_2} = -C_1V_1$

$Q^-_{2,\phi_2} = -C_2V_2$

$$2\frac{C_1C_2}{C_1+C_2}V_{in} = Q^-_{1,\phi_2} + Q^-_{2,\phi_2}$$

(1) has (2) substituted into it

$$= -C_1(0V-u) - C_2(0V-u)$$

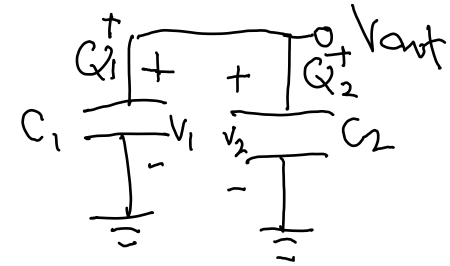
☆ use node voltages $V_1 = V_2 = 0V - u$

$$= C_1u + C_2u$$

$$\frac{2C_1C_2}{C_1+C_2}V_{in} = u(C_1+C_2) \Rightarrow \boxed{u = \frac{2C_1C_2}{(C_1+C_2)^2}V_{in} = V_{out}}$$

(d) How will the charges be distributed in phase ϕ_2 if we assume $C_1 \gg C_2$?

$$V_{out} = \frac{2C_1C_2}{(C_1+C_2)^2} V_{in}$$



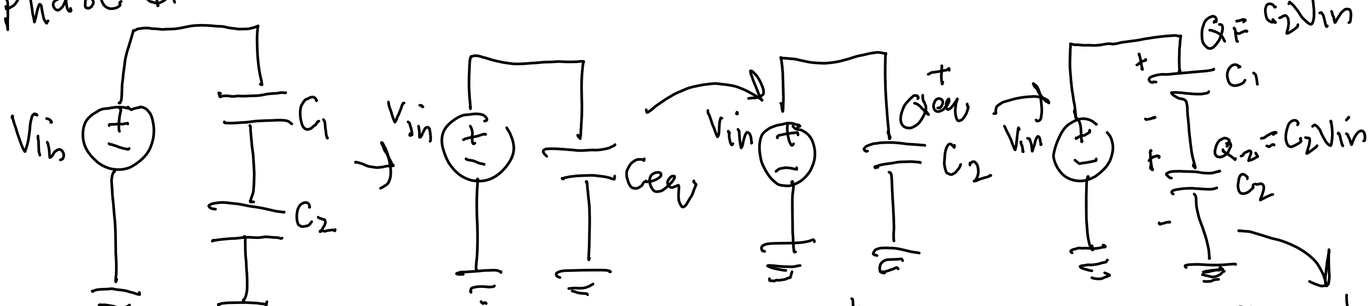
$$V_1 = V_2 = V_{out} \qquad Q_1^+ = C_1V_1 = \frac{2C_1^2C_2}{(C_1+C_2)^2}V_{in} \qquad Q_2^+ = C_2V_2 = \frac{2C_2^2C_1}{(C_1+C_2)^2}V_{in}$$

$$\lim_{C_1\to\infty} Q_1^+ = \lim_{C_1\to\infty} \frac{2C_1^2C_2}{(C_1+C_2)^2}V_{in} = \lim_{C_1\to\infty} \frac{2C_2}{(1+\frac{C_2}{C_1})^2}V_{in} = \frac{2C_2}{(1+0)^2}V_{in} = \boxed{2C_2V_{in}}$$

$$\lim_{C_1\to\infty} Q_2^+ = \lim_{C_1\to\infty} \frac{2C_2^2C_1}{(C_1+C_2)^2}V_{in} = \lim_{C_1\to\infty} \frac{2\frac{C_2^2}{C_1}}{(1+\frac{C_2}{C_1})^2}V_{in} = \frac{2\cdot 0}{(1+0)^2}V_{in} = \boxed{0}$$

Intuition for $C_1 >> C_2 \Rightarrow$ The charge mostly goes on the larger capacitor! (C_1)

Phase ϕ_1

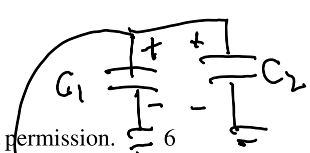


$$C_{eq} = C_1 \| C_2 = \frac{C_1C_2}{C_1+C_2}$$

$$Q_{eq}^+ = C_{eq}V_{in} = C_2V_{in}$$

$$\lim_{C_1\to\infty} C_{eq} = C_2$$

Phase ϕ_2



$Q_{total} = 2C_2V_{in}$

$\left(V_1 = \frac{Q_1}{C_1}\right)$ C_1 looks like a short to C_2.