EECS 16A DIS 9A

email: moseswan@berkeley.edu
OH: W 10AM-12PM PT (HWP)

Logistical notes

1. MT 2 next Monday, logistics post coming out on piazza this week
2. Circuits Review Session 3 moved to Friday 2PM-4PM PT (10/30/20)

Learning objectives

1. Using notion of steady state in looking at charges and voltages in capacitive circuits, using equivalence to solve for charges and voltages.
2. Capacitors discharging/charging by current sources
3. Charge in capacitors, what does $Q = CV$ mean?

Suggest @ bit.ly/16ajukebox

Playlist

1. June Marieezy - Fly (FKJ Remix)
2. Prince - Mini Kid
   Hiatus Kaiyote
3. Tom Misch - It Runs Through Me (feat. De La Soul)

EECS 16A Fall 2020 — Designing Information Devices and Systems I — Discussion 9A

1. Voltages Across Capacitors

For the circuits given below, determine the voltage across each capacitor and calculate the charge and energy stored on each capacitor (assume all capacitors start *uncharged*, and then we've let the system reach steady state). We are also given $C_1 = 1\,\mu\text{F}$, $C_2 = 3\,\mu\text{F}$, and $V_s = 1\,\text{V}$.

Recall charge has units of Coulombs (C), and capacitance is measured in Farads (F) $= \frac{\text{Coulomb}}{\text{Volt}}$.
It may also help to note metric prefix examples: $3\,\mu\text{F} = 3 \times 10^{-6}\text{F}$.

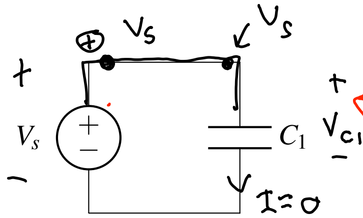
cap voltage

charge → $Q = CV$ ⇐

↓ $\frac{d}{dt}$ current $I = C\frac{dV}{dt}$

energy $E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

(a)

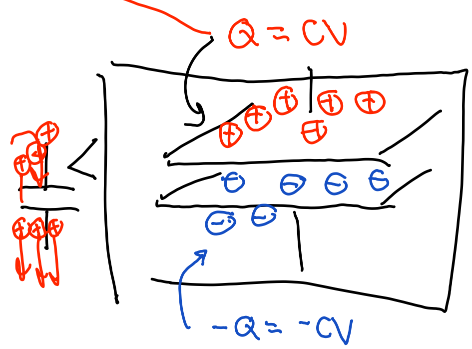


$V_s = 1\text{Volt}$

$\Rightarrow V_s = \boxed{V_{C1} = 1\text{Volt}}$

$I = C\frac{d}{dt}V = C\frac{d}{dt}(1V) = 0A$

(at steady state caps have zero current ⇒ constant voltage ⇒ 0 current)



$\Rightarrow Q = CV = (1\mu F)(1V) = \boxed{1\mu C}$

$\mu = 10^{-6}$

$E = \frac{1}{2}CV^2 = \frac{1}{2}(1\mu F)(1V)^2 = \boxed{\frac{1}{2}\mu J}$ ⇐

Units $[F][V]^2 = \left[\frac{C}{\cancel{V}}\right][V]^{\cancel{2}} = [C][V]$

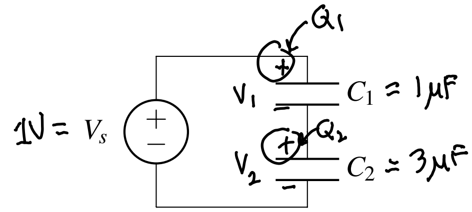
$= [\cancel{C}]\left[\frac{J}{\cancel{C}}\right] = [J]$

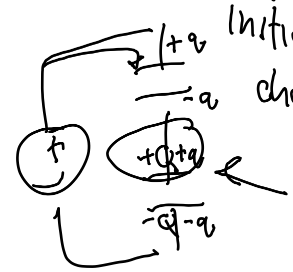
joules (energy)

(b)

$I = C\frac{dV}{dt}$

make NVA hard $\Rightarrow$ $\cancel{I = C\frac{dV}{dt}} = V$

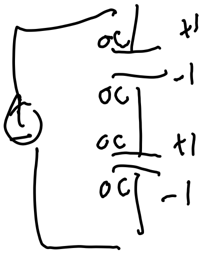


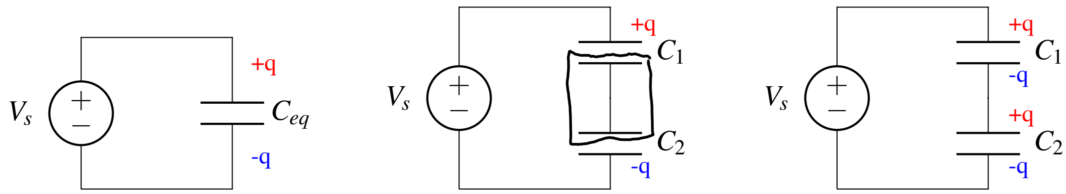


Initially charged

Helpful diagrams for considering the charges capacitors linked in series: (without any initial charges)

when can we simplify in series





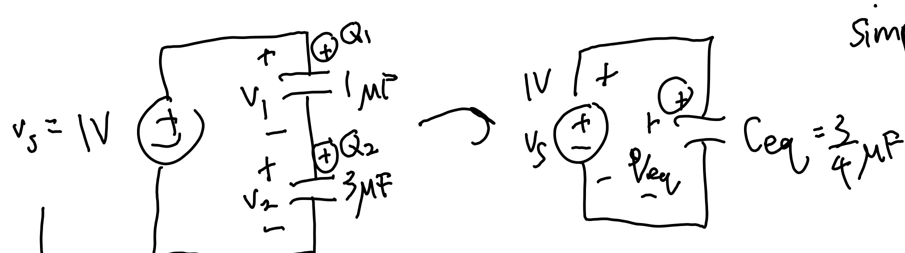
Left: Our series capacitors may be modeled as one equivalent capacitor C_{eq}, which after some time is charged up by V_s to have $+q$ on the top plate and $-q$ on the bottom plate.

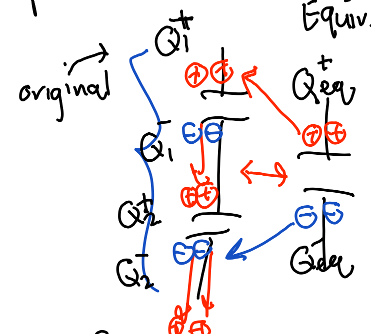
Middle: We return to the 2-capacitor picture, but carry this insight of equivalent charge with us. Now the charge $+q$ is on the top plate *of capacitor* C_1, and $-q$ is on the bottom plate *of capacitor* C_2.

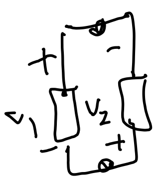
Right: Since capacitor plates have opposite & equal charges, we attain this final right digram.

As another conceptual check, we notice that the node between C_1 and C_2 is isolated from any other connections and should always remain *charge neutral*. From the diagram right we see this is maintained since $(+q)+(-q) = 0$.

Simplifying capacitors in series







$V_1 = -V_2$

$\boxed{Q_1 = \frac{3}{4}\mu C}$

$\boxed{Q_2 = \frac{3}{4}\mu C}$

$C_{eq} = 1\mu F \,||\, 3\mu F$

$= \frac{3(\mu F)^{\cancel{2}}}{4\,\cancel{\mu F}} = \frac{3}{4}\mu F$

$Q_{eq}^+ = C_{eq}V_{eq} = \frac{3}{4}\mu F \cdot 1V$

V_s

$= \frac{3}{4}\mu C$

$\begin{cases} Q_{eq}^+ = Q_1^+ = Q_2^+ \\ Q_{eq}^- = Q_1^- = Q_2^- \end{cases}$

$a \,||\, b < a, b$

$\frac{ab}{a+b} < a, b$

$a\left(\frac{b}{a+b}\right) < a$

$b\left(\frac{a}{a+b}\right) < b$

$C_1V_1 = Q_1$ $\quad$ $C_2V_2 = Q_2$

$V_1 = \frac{Q_1}{C_1}$ $\quad$ $V_2 = \frac{Q_2}{C_2}$

$= \frac{\frac{3}{4}\mu C}{1\mu F}$ $\quad$ $= \frac{\frac{3}{4}\mu C}{3\mu F}$

$= \boxed{\frac{3}{4}V}$ $\quad$ $= \boxed{\frac{1}{4}V}$

$E_1 = \frac{1}{2}C_1V_1^2$

$= \frac{1}{2}(1\mu F)\left(\frac{3}{4}V\right)^2$

$= \boxed{\frac{9}{32}\mu J}$

$E_2 = \frac{1}{2}C_2V_2^2$

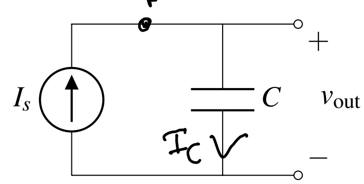
$= \frac{1}{2}(3\mu F)\left(\frac{1}{4}V\right)^2$

$= \boxed{\frac{3}{32}\mu J}$

2. Current Sources And Capacitors

Given the circuit below, find an expression for $v_{out}(t)$ in terms of I_s, C, V_0, and t, where V_0 is the initial voltage across the capacitor at $t = 0$.

KCL: $I_C = I_s$



$I_C = C\frac{dV_{out}}{dt}$

$\frac{I_s}{C} = \frac{dV_{out}}{dt}$

$\int_0^t \frac{I_s}{C}\,dt = \int_0^t \frac{dV_{out}}{dt}\,dt$

$\frac{I_s}{C}t = V_{out}(t) - V_{out}(0)$

Then plot the function $v_{out}(t)$ over time on the graph below for the following conditions detailed below. Use the values $I_s = 1\text{mA}$ and $C = 2\,\mu\text{F}$.

(a) Capacitor is initially uncharged $V_0 = 0$ at $t = 0$.

(b) Capacitor has been charged with $V_0 = +1.5\text{V}$ at $t = 0$.

(c) **Practice:** Swap this capacitor for one with half the capacitance $C = 1\,\mu\text{F}$, which is initially uncharged $V_0 = 0$ at $t = 0$.

$V_{out}(t) = V_{out}(0) + \frac{I_s}{C}t$

HINT: Recall the calculus identity $\int_a^b f'(x)dx = f(b) - f(a)$, where $f'(x) = \frac{df}{dx}$.

$V_{out}(t) = \frac{1V}{1ms}t + 0V$

$V_{out}(t) = \frac{1V}{2ms}t + 0V$

$V_{out}(t) = \frac{1V}{2ms}t + 1.5V$

(a) $\frac{I_s}{C} = \frac{1mA}{2\mu F}$

(b) $= \frac{1}{2}\frac{V}{ms}$

$A = \frac{C}{s}$

$F = \frac{C}{V}$