

EE CS16A DIS 7A

email: moseswan@berkeley.edu
OH: W 10AM-12PM PST (HWP)

Learning Objectives

- ① Current Divider Derivation + DEMO
- ② More NVA practice
- ③ If time: talk a bit about resistivity + sensing applications

Today's music

① Yoko Kanno - Bunkasai
Medley

② Julian Lage - Nocturne

Suggest @ bit.ly/16ajutebox
(playlist link on form)

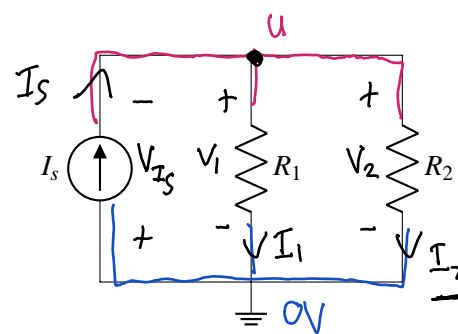
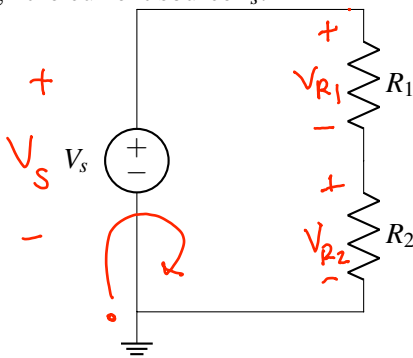
EECS 16A Designing Information Devices and Systems I Discussion 7A
 Fall 2020

1. Current Divider

So far we've shown that for the circuit below to the left, the voltage across the resistor R_2 is expressed

$$V_{R_2} = \left(\frac{R_2}{R_1 + R_2} \right) V_S.$$

Let us derive a similar formula for the right circuit below, relating the current I_{R_2} through R_2 to the current through the current source I_S .



- NVA
- ① Write KCL
 - ② Write I-V relationships
 - ③ Substitute

$$\begin{aligned} -V_S + V_{R_1} + V_{R_2} &= 0 \\ V_{R_1} &= V_S - V_{R_2} \\ &= V_S - \frac{R_2}{R_1 + R_2} V_S \\ &= \frac{R_1 + R_2}{R_1 + R_2} V_S - \frac{R_2}{R_1 + R_2} V_S \\ &= \frac{R_1}{R_1 + R_2} V_S \end{aligned}$$

NVA

- ① $I_S = I_1 + I_2$ (at u node)
 $I_1 + I_2 = I_S$ (at GN node)
- ② $I_1 = \frac{V_1}{R_1} = \frac{u - 0}{R_1}$
 $I_2 = \frac{V_2}{R_2} = \frac{u - 0}{R_2}$
- ③ $I_S = \frac{u}{R_1} + \frac{u}{R_2}$
 $u = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} I_S$

Q: If we use different labelings what changes?
 A: element current/element voltage might change sign

Q: $R_1 \gg R_2$ $I_2 = ?$
 A: $I_2 \approx \frac{R_1}{R_1} I_S \approx I_S$

$$\begin{aligned} \rightarrow I_2 &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} I_S \frac{1}{R_2} \\ &= \frac{R_1}{R_1} \frac{1}{\frac{R_2}{R_1} + 1} I_S = \boxed{\frac{R_1}{R_2 + R_1} I_S} \end{aligned}$$

Q: $R_2 \gg R_1$ $I_2 = ?$
 A: $I_2 \approx \text{small} \cdot I_S \approx 0$

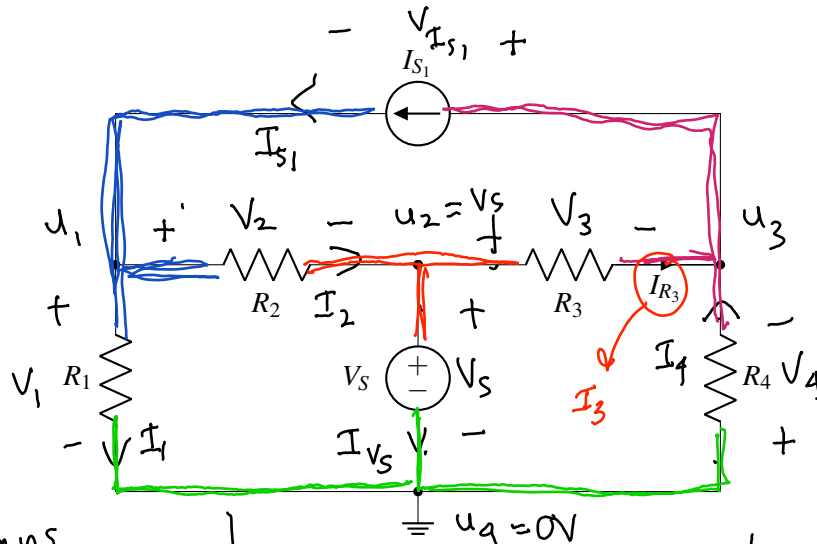
$$I_1 = \frac{R_2}{R_1 + R_2} I_S$$

Unless told otherwise
 • Assume node voltages are unknowns
 • Assume source values (V_s, I_s) and component values (R 's) are knowns

2. Circuit Analysis

Provided the circuit below...

- (a) ...use nodal analysis to solve for all node voltages.
- (b) ...find the current I_{R_3} flowing through resistor R_3 .



$$V_s = u_2 - u_4 = u_2$$

$$u_2 = V_s$$

$$u_4 = 0V$$

$$V = IR$$

$$I = \frac{V}{R}$$

KCL eqns

(a) u_1

$$I_{s1} = I_1 + I_2$$

(b) u_3

$$I_3 + I_4 = I_{s1}$$

I-V relationship (ohm's law)

$$I_1 = \frac{V_1}{R_1} = \frac{u_1 - 0V}{R_1}$$

$$I_2 = \frac{V_2}{R_2} = \frac{u_1 - V_s}{R_2}$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_s - u_3}{R_3}$$

$$I_4 = \frac{V_4}{R_4} = \frac{0V - u_3}{R_4}$$

Substitute into KCL

$$\star I_{s1} = \frac{u_1}{R_1} + \frac{u_1 - V_s}{R_2}$$

$$\square I_{s1} = \frac{V_s - u_3}{R_3} - \frac{u_3}{R_4}$$

Solve for u_3

$$u_1 = \frac{R_1}{R_1 + R_2} V_s + \frac{R_1 R_2}{R_1 + R_2} I_{s1}$$

$$u_2 = \frac{R_4}{R_3 + R_4} V_s - \frac{R_3 R_4}{R_3 + R_4} I_{s1}$$

Algebra for solving for u_1

$$\star I_{s1} = \frac{u_1}{R_1} + \frac{u_1 - V_s}{R_2}$$

$$I_{s1} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) u_1 - \frac{V_s}{R_2}$$

$$I_{s1} + \frac{V_s}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) u_1$$

$$u_1 = \frac{R_1 R_2}{R_1 R_2 \left(\frac{1}{R_1} + \frac{1}{R_2}\right)} I_{s1} + \frac{R_1}{R_1 + R_2} \frac{1}{R_2} V_s$$

Algebra for u_2

$$\square I_{s1} = \frac{V_s - u_3}{R_3} - \frac{u_3}{R_4}$$

$$I_{s1} = \frac{V_s}{R_3} - \left(\frac{1}{R_3} + \frac{1}{R_4}\right) u_3$$

$$I_{s1} - \frac{V_s}{R_3} = -\left(\frac{1}{R_3} + \frac{1}{R_4}\right) u_3$$

$$u_3 = \frac{R_4}{R_3 + R_4} V_s - \frac{R_3 R_4}{R_3 + R_4} I_{s1}$$

$$u_1 = \frac{R_1 R_2}{R_2 + R_1} I_{s1} + \frac{R_1}{R_1 + R_2} V_s$$

(b) Solve for $I_3 = \frac{V_s - u_3}{R_3} = ?$ (substitute)

3. Practice Problem: Bio-Molecule Detector

One application for electronics that has gained a lot of attention over the past several years is in so-called “bio-molecule” detection. The idea is to build a system that detects the presence of specific molecules and/or cells (e.g. specific viruses, proteins, etc.) in a biological sample; if this detection can be performed automatically and using relatively low-cost components, it can have a dramatic impact on a number of areas such as medical diagnosis, drug development, DNA sequencing, etc.

In this problem, we’ll look at how some of the techniques we learned about in the touchscreen module can be applied to realize a hypothetical bio-molecule detector. (Real bio-molecule detection systems involve quite a bit more complexity than what we’ll include here, but in many designs the same basic principles apply.)

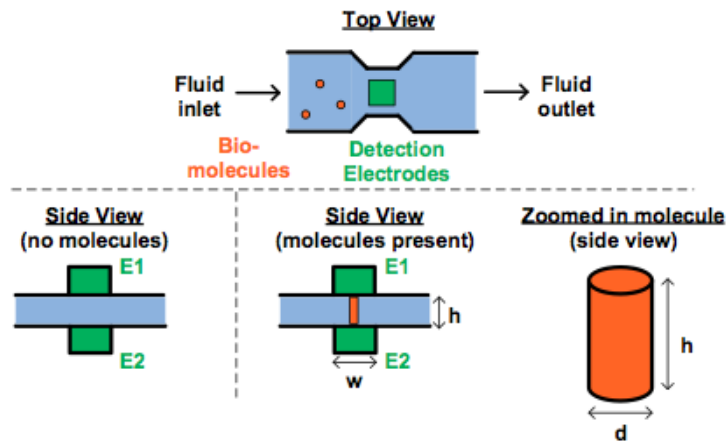
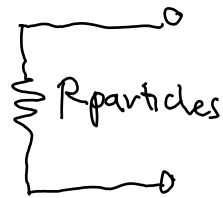
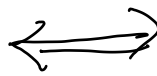
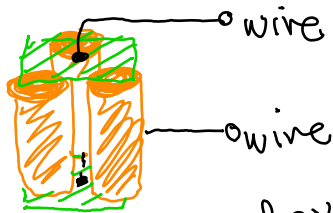
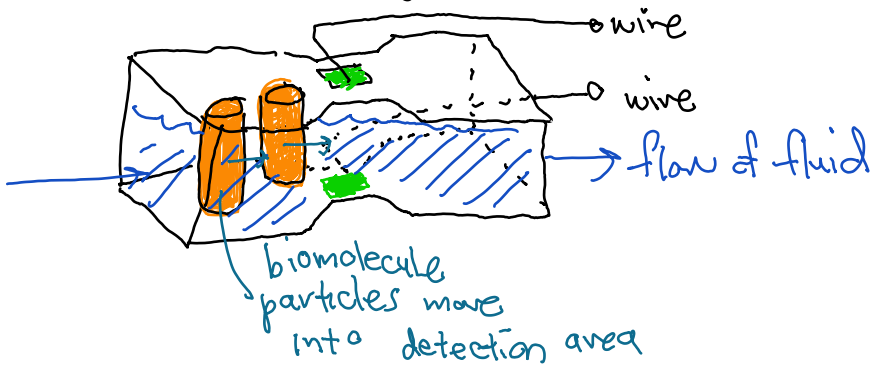


Figure 1: Bio-molecule detector.

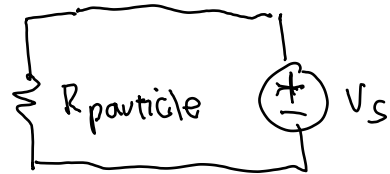
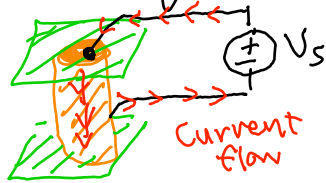
As shown in Figure 1, the detector works by flowing a liquid that may or may not contain the biomolecules through a region in the device that has electrodes on the top and bottom of the liquid channel. The electrodes (E1/E2 in Figure 1) are chemically “functionalized” (using e.g. some appropriately designed antibodies), so that if the specific bio-molecule of interest is present in the fluid sample, one or more of the molecules will get physically trapped between the two electrodes (bottom right of Figure 1). After all of the fluid has been cleared out of the device (i.e., so that if there are no bio-molecules present, there is only air in between the two electrodes E1/E2), we can then figure out whether or not one or more bio-molecules were trapped by measuring the resistance between the two electrodes.

- Let’s first assume that we want to detect the presence of a bio-molecule by measuring resistance. If no bio-molecule is present, what should be the resistance between E1/E2? As shown in Figure 1, if each bio-molecule is a cylinder with diameter $d = 10\text{nm}$, height $h = 100\text{nm}$, and has a resistivity $\rho = 100\mu\Omega\text{m}$, what would be the resistance between E1 and E2 if only a single bio-molecule has been trapped? Note that you can assume that the trapped molecule is exactly vertically oriented when it is trapped – i.e., the top and bottom faces of the molecule are both aligned with surfaces of the electrodes.
- Using the same numbers for d , h , and ρ as part (a), as a function of the number of trapped bio-molecules $N_{\text{molecules}}$, what is the resistance between E1 and E2? (Note that you can assume that $N_{\text{molecules}}$ is small enough that all of the molecules fit within the electrode area and that all of the molecules are still trapped in an exactly vertical orientation.)
- Given your answers to parts (a) and (b), design a circuit that will output a voltage greater than 2.5V if more than 5 molecules are trapped.

Emphasis: translating physical model to circuit



After fluid cleared out, model particles as a single resistor
 If we apply a voltage source, V_s , to a single particle



Q: What is $R_{particle}$? A: Use $R = \rho \frac{L}{A}$!

Q: If many particles, what is $R_{particles}$?

A: Use insight - all same shape so same length + resistivity
 variability is total area!



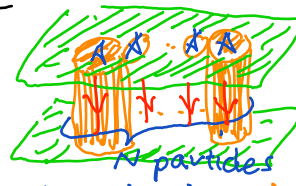
$A_{total} = A$

$R = \rho \frac{L}{A}$



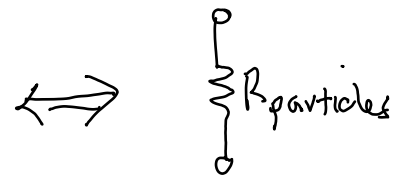
$A_{total} = 2A$

$R = \rho \frac{L}{2A}$



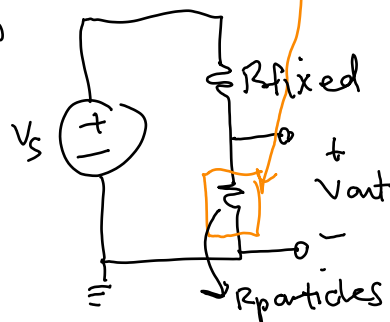
$A_{total} = NA$

$R = \rho \frac{L}{NA}$



If N increases, resistance drops!
 $\uparrow N \downarrow R_{particles}$

Q: How to detect? A: Suggestion Voltage Divider



$V_{out} = \frac{R_{particles}}{R_{fixed} + R_{particles}} V_s$

$\uparrow N \downarrow V_{out}$

$\downarrow N \uparrow V_{out}$
 (Roughly!)